SKP Engineering College

Tiruvannamalai – 606611

A Course Material

on

Principles Of Digital Signal Processing

By

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Quality Certificate

This is to Certify that the Electronic Study Material

Subject Code:EC6502

Subject Name:Principles of Digital Signal Processing

Year/Sem:III/V

Being prepared by me and it meets the knowledge requirement of the University curriculum.

Signature of the Author

Name: R.Rajesh

Designation: Assistant Professor

This is to certify that the course material being prepared by Mr.R.Rajesh is of the adequate quality. He has referred more than five books and one among them is from abroad author.

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Seal:

EC6502 PRINCIPLES OF DIGITAL SIGNAL PROCESSING L T P C 3 1 0 4

OBJECTIVES

- To learn discrete Fourier transform and its properties
- To know the characteristics of IIR and FIR filters learn the design of infinite and finite impulse response filters for filtering undesired signals
- To understand Finite word length effects
- To study the concept of Multirate and adaptive filters

UNITI DISCRETE FOURIER TRANSFORM 9

Discrete Signals and Systems- A Review – Introduction to DFT – Properties of DFT – Circular Convolution - Filtering methods based on DFT – FFT Algorithms –Decimation in time Algorithms, Decimation in frequency Algorithms – Use of FFT in Linear Filtering.

UNIT II IIR FILTER DESIGN 9

Structures of IIR – Analog filter design – Discrete time IIR filter from analog filter – IIR filter design by Impulse Invariance, Bilinear transformation, Approximation of derivatives – (LPF, HPF, BPF, BRF) filter design using frequency translation.

UNIT III FIR FILTER DESIGN 9

Structures of FIR – Linear phase FIR filter – Fourier Series - Filter design using windowing techniques (Rectangular Window, Hamming Window, Hanning Window), Frequency sampling techniques – Finite word length effects in digital Filters: Errors, Limit Cycle, Noise Power Spectrum.

UNIT IV FINITE WORDLENGTH EFFECTS 9

Fixed point and floating point number representations – ADC –Quantization- Truncation and Rounding errors - Quantization noise – coefficient quantization error – Product quantization error - Overflow error – Roundoff noise power - limit cycle oscillations due to product round off and overflow errors – Principle of scaling

UNIT V DSPAPPLICATIONS 9

Multirate signal processing: Decimation, Interpolation, Sampling rate conversion by a rational factor – Adaptive Filters: Introduction, Applications of adaptive filtering to equalization.

TOTAL (L:45+T:15): 60 PERIODS

TEXT BOOK:

1. John G. Proakis & Dimitris G.Manolakis, "Digital Signal Processing – Principles, Algorithms & Applications", Fourth Edition, Pearson Education / Prentice Hall, 2007.

CONTENTS

Unit – I

Discrete Fourier Transform

Part – A

1. What is the relation between DTFT and DFT? [CO1-L1]
Let x(n) be a discrete time sequence.
Now DTFT[x(n)] = X(n) or FT[x(n)] = X(n)
& DFT[x(n)] = X(k).

The X(n) is a periodic continuous function of n and X(k) is an N – point periodic sequence.

The N –point sequence x(k) is actually N samples of X(n) which can be obtained by sampling one period of X(n) at N equal intervals.

2..Distinguish between Discrete Time Fourier Transform and Discrete Fourier Transform.

(Or) Distinguish between DFT and DTFT. [CO1-L2-May/June 2010]

DFT	DTFT
1. Obtained by performing sampling operation in both the time and frequency domains.	1. Sampling is performed only in time domain.
2. Used to convert Continuous function of n. to discrete function of n.	2. It is a Continuous function of n.

3. What is the draw back in Fourier Transform and how it is overcome? [CO1-L1]
The drawback in Fourier Transform is that it is a continuous function of n and so it cannot be processed by digital system. This drawback is overcome by using Discrete Fourier transform. The DFT converts the continuous function of n to a discrete function of n.

4. Write two applications of DFT. [CO1-L2]
(a)The DFT is used for **spectral analysis** of signals using a digital computer.
(b)The DFT is used to perform **linear filtering** operations on signals using digital computer.

(c)**Correlation**

5.When an N- point periodic sequence is said to be even or odd sequence.?

[CO1-L1-May/June 2011]

An N – point periodic sequence is called even if it satisfies the condition.

$X(n-N) = x(n)$; for $0\leq n\leq (N-1)$

An N – point periodic sequence is called odd if it satisfies the condition.

$X(n-N) = -x(n)$; for $0\leq n\leq (N-1)$

6.. List any four properties of DFT. [CO1-L1-May/June 2009]

Let DFT{x(n)} = X(k), DFT{$x_1(n)$} = $X_1(k)$ and DFT{$x_2(n)$} = $X_2(k)$

a. **Periodicity:** X(k+N) =X(k); for all k

b**. Linearity:** DFT{$a_1x_1(n)+a_2\ x_2(n)$} =$a_1X_1(k)+a_2X_2(k)$; where a_1 and a_2 are constants.

c. **DFT of time revised sequence**: DFT{x(N-m)} =X(N-k)

d. **Circular Convolution:** DFT {$x_1(n)©x_2(n)$} = $X_1(k)\ X_2(k)$

7. Why linear convolution is important in DSP? [CO1-L1-May/June 2008]

The response or output of LTI discrete time system for any input x(n) is given by linear convolution of the input x(n) and the impulse response h(n) of the system. This means that if the impulse response of a system is known, then the response for any input can be determined by convolution operation.

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8. Write the properties of Linear Convolution. [CO1-L1-May/June 2008]

a. Commutative Property: x(n) * h(n) = h(n) * x(n)

b. Associative Property: [x(n) * $h_1(n)$] * $h_2(n)$ = x(n) * [$h_1(n)$ * $h_2(n)$]

c. Distributive Property: x(n) * [$h_1(n)$ + $h_2(n)$] = [x(n) * $h_1(n)$] + [x(n) * $h_2(n)$]

9.What is Zero Padding? Why it is needed. [CO1-L1]

Appending Zeros to a sequence in order to increase the size or length of the sequence is called Zero Padding.

In circular convolution, when the two input sequences are different size, then they are converted to equal size by zero padding.

10. What is FFT? [CO1-L1-Nov/Dec 2012]

The Fast Fourier Transform is needed to compute DFT with reduced number of calculations. The DFT is spectrum analysis & filtering operations on the signals using digital computers.

11. How many multiplications and additions are required to compute N point DFT using radix-2 FFT? [CO1-L1]

The number of multiplications and additions required to compute N point DFT using radix-2 FFT are $N\log_2N$ and $N/2\log_2N$ respectively,

12. What is DIT radix – 2 FFT? [CO1-L1-May/June 2008]

The Decimation in Time (DIT) radix – 2 FFT is an efficient algorithm for computing DFT.

In DIT radix – 2 FFT, the time domain N – point sequence is decimated into 2 – point sequences. This process is continued until we get N – point DFT.

13. What is phase factor or twiddle factor? CO1-L1]

The complex number $\mathbf{W_N}$ is called phase factor or twiddle factor. $\mathbf{W_N = e^{-j2\pi/N}}$. It also represents an $\mathbf{N^{th}}$ root of unity.

14. What is DIF radix – 2 FFT? [CO1-L1-Nov/Dec 2011]

The Decimation in Frequency (DIF) radix – 2 FFT is an efficient algorithm for computing DFT. In this algorithm the N – point time domain sequence is converted in to two numbers of N/2 point sequences. Here the equations forming N/2 point sequences, N/4 point sequences etc.; are obtained by decimation of frequency domain sequences. This process is continued until we get N – point DFT.

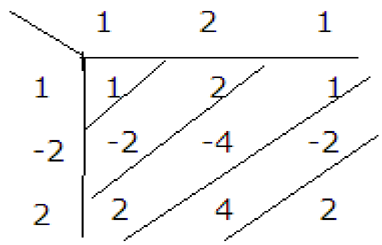
15. Compare the DIT and DIF radix – 2 FFT. [CO1-L2-May/June 2006]

What are the differences & similarities between DIT & DIF?

S. No	**DIT radix – 2 FFT.**	**DIF radix – 2 FFT.**
1.	The time domain sequence is decimated.	The frequency domain sequence is decimated.
2.	When the input is in bit reversed order, the output will be in normal order and vice versa.	When the input is in bit normal order, the output will be in bit reversed order and vice versa.
3.	In each stage of computations, the phase factors are multiplied before add and subtract operations.	In each stage of computations, the phase factors are multiplied after add and subtract operations.
4.	The value of N should be expressed such that N = 2m and this algorithm consists of m stages of computations.	The value of N should be expressed such that N = 2m and this algorithm consists of m stages of computations.
5.	Total number of arithmetic operations is Nlog2N complex additions and (N/2) log2N complex multiplications.	Total number of arithmetic operations is Nlog2N complex additions and (N/2) log2N complex multiplications.

16.Obtain the circular convolution of the following sequence x(n)={1,2,1}; h(n)={1,-2,2}.

[CO1-L1-May/June 2007]



y(n)={1,0,-1,2,2}

17.State the advantages of FFT over DFTs[CO1-L1]

Efficient computation of DFT
Reduced time required for calculation
Less complex additions
Less complex multiplications
Increased efficiency

18.State any two properties of Discrete Fourier Transform. [CO1-L1-Nov/Dec 2011]

1. Linaerity:

If DFT{x(n)} = X(k),
then **DFT $\{a_1x_1(n)+a_2 x_2(n)\} = a_1X_1(k)+a_2X_2(k)$;** where a_1 and a_2 are constants.

2. Time shifting property:

If DFT{x(n)} = X(k),
then **DFT{x(n-m)} = X(k)$e^{\frac{-j2\pi km}{N}}$**

19.How many stages of decimations are required in the case of a 64point radix 2 DIT FFT Algorithm?[CO1-L1-May/June 2012]

The Number of additions required in the computation of 64-point using FFT is $N \log_2 N$ (i.e.) $64 \log_2 64$ =64 [log64 / log2] =384.
The Number of multiplications required in the computation of 64-point using FFT is $[N/2] \log_2 N$ (i.e.) $[64/2] \log_2 64$ =32 [log64 / log2] =192.

20.What is meant by in-place computation? [CO1-L1-May/June 2013]

An algorithm (DIT/DIF) that uses the same location to store both the input and output sequence is called in-place algorithm. **[OR]**

An "in-place computation "in FFT is simply an FFT that is calculated entirely inside its original sample memory. In other words, calculating an "in place" FFT does not require additional buffer memory.

21.What is meant by bit reversal? [CO1-L1-May/June 2011]

The FFT time domain decomposition is usually carried out by a **bit reversal sorting** algorithm. This involves rearranging the order of the *N* time domain samples by counting in binary with the bits flipped left-for-right.

INPUT	INDEX	BINARY REPRESENTATION	BIT REVERSED BINARY	BIT REVERSED INDEX	BIT REVERSED INDEX
x(0)	0	00	00	0	x(0)
x(1)	1	01	10	2	x(2)
x(2)	2	10	01	1	x(1)
x(3)	3	11	11	3	x(3)

22.What is the relationship between DTFT & DFT? [CO1-L1-May/June 2008]

a.DFT of a discrete time signal can be obtained by sampling the DTFT of the signal.
b.The drawback in DTFT is that the frequency domain representations of a discrete time signal obtained using DTFT will be a Continuous function of n.
c.The DFT has been developed to convert a continuous function of n to a discrete function of n.
d.The sampling of the DTFT is conventionally performed at N equally spaced frequency points, $0 \le w \le 2\prod$.

23. Compare the number of multiplications required to compute the DFT of a 64 point sequence using direct computation and that using FFT.[CO1-L1]

The Number of multiplications required in the computation of 64-pointusing FFT is $[N/2] \log_2 N$ (i.e.) $[64/2] \log_2 64 = 32 [\log 64 / \log 2] = 192$.
The Number of multiplications required in the computation of 64-point using DFT is N^2 (i.e.) $64^2 = 4096$.

24.What are the applications of FFT algorithms? [CO1-L1-May/June 2006]

The applications of FFT algorithm include

i) linear filtering
ii) correlation
iii) Spectrum analysis

Part-B

1.Summarize the properties of DFT. [CO1-L1-Nov/Dec 2014]

Properties of DFT:-

In this section we will study some important properties of DFT. We know that, the DFT of discrete time sequence, $x(n)$ is denoted by $X(K)$. The DFT and IDFT pair is denoted by,

$$x(n) \overset{-}{\underset{N}{\longleftrightarrow}} X(K)$$

Here 'N' indicates N – point DFT.

1. Linearity :-

If $x(n) \overset{:T}{\longleftrightarrow} X(K)$

and

$$x_2(n) \overset{=T}{\longleftrightarrow} X_2(K)$$

Then $ax_1(n) + bx_2(n)$ DFT $\longleftrightarrow$ $aX_1(K) + bX_2(K)$

Proof:

DFT $[x(n)] = X(K) = \sum_{n=0}^{N-1} x(n) W_N^{-K_n}$

Here $x(n) = ax_1(n) + b(x_2(n))$

$$\therefore \quad X(K) = \sum_{n=0}^{N-1} [\, ax_1(n) + bx_2(n) \,] W_N^{K_n}$$

$$= \sum_{n=0}^{N-1} ax_1(n) W_N^{K_n} + \sum_{n=0}^{N-1} bx_2(n)\, W_N^{K_n}$$

$$= a \sum_{n=0}^{N-1} x_1(n) W_N^{K_n} + b \sum_{n=0}^{N-1} x_2(n)\, W_N^{K_n}$$

$$X(K) = aX_1(K) + b(X_2(K))$$

∴ DFT of linear combination of two or more signals is equal to the sum of linear combination of DFT^ of individual signals.

2. Periodicity :-

$$x(n) \overset{-}{\underset{N}{\longleftrightarrow}} X(K)$$

then $x(n+N) = x(n)$ for all n and

$X(K+N) = X(K)$ for al; K.

Proof:

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{K_n} \qquad \rightarrow \text{(a)}$$

Replacing K by $K+N$ we get

$$X(K+N) = \sum_{n=0}^{N-1} x(n)\, W_N^{(K+N)_n}$$

$$\therefore X(K+N) = \sum_{n=0}^{N-1} x(n)\, W_N^{K_n}.W_N^{N_n} \qquad \rightarrow \text{(b)}$$

We know that W_n is a twiddle actor and it is given by $W_n = e^{-j2\pi/N}$

$$\therefore W_N^{K_n} = e^{\left[\frac{-j2\pi}{N}\right]^{N_n}}$$

$$= e^{-j2\pi n}$$

$$\therefore W_N^{n} = (\cos 2\pi - j\sin\, 2\pi)^n$$

Where n is integer

$$\therefore W_N^{K_n} = 1$$

Putting this value in equation (a)

$$X(K+N) = \sum_{n=0}^{N-1} x(n)\, W_N^{N_n} \qquad \rightarrow \text{(c)}$$

Comparing equation (a) and (c)

$$X(K+N) = X(K)$$

∴ DFT of a finite length sequence results in a periodic sequence.

3. Time shifting property:

If $x(n) \longleftrightarrow X(K)$

then $X(n-m) \xleftrightarrow{DFT} W_N^{mK} X(K)$

proof:

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{nK}$$

DFT $[x(n)]$ = $\sum_{n=0}^{N-1} x(n)\, W_N^{nK}$

DFT $[x(n-m)]$ = $\sum_{n=0}^{N-1} x(n-m)\, W_N^{nK}$

Substitute $(n-m) = l$

$n = l + m$

Lower limit	Upper limit
$n = 0$	$n = N-1$
$\Longrightarrow 0 = l + m$	$\Longrightarrow N-1 = l + m$
$\Longrightarrow l = -m$	$= l = N-m-1$

∴ DFT $[x(n-m)] = \sum_{l=-m}^{N-m-1} x(l)\, W_N^{(l+m)K}$

$$= W_N^{mK} \sum_{l=-m}^{N-m-1} x(l)\, W_N^{lK}$$

∴ there is no term 'm' is the summation

$$= W_N^{mK} \sum_{l=0}^{N-1} x(l)\, W_N^{lK}$$

$$= W_N^{mK} X(K)$$

4. Frequency shifting property:

If $x(n) \longleftrightarrow X(K)$

then $e^{\frac{j2\pi nl}{N}} x(n) \xleftrightarrow{DFT} X(K-l)$

Proof:

DFT $[x(n)]$ = $\sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi nK}{N}}$

DFT $\left[e^{\frac{j2\pi ln}{N}} x(n)\right] = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi nK}{N}} . e^{\frac{j2\pi ln}{N}}$

$= \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi nK}{N}} (K-l)$

$= X[K-l]$

5. Time Reversal Property:

If $x(n) \overset{\text{DFT}}{\longleftrightarrow} X(K)$

then $x[(-n)]_N$ DFT $\longleftrightarrow$ $X(N-n)$

and $X[(-K)]_N$ DFT $\longleftrightarrow$ $X[N-K]$

Proof:

DFT $[x(n)] = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi nK}{N}}$

DFT $[x(N-n)] = \sum_{n=0}^{N-1} x(N-n)\, e^{-\frac{j2\pi nK}{N}}$

Put $m = N - n$: The above equation 'n' replaced into N – m.

$= \sum_{n=0}^{N-1} x(m)\, e^{-\frac{j2\pi K}{N}(N-m)}$

$= \sum_{n=0}^{N-1} x(m)\, e^{-\frac{j2\pi KN}{N}} . e^{\frac{j2\pi Km}{N}}$

$= \sum_{n=0}^{N-1} x(m)\, e^{\frac{j2\pi Km}{N}}$

$= \sum_{n=0}^{N-1} x(m)\, e^{-\frac{j2\pi Km}{N}} (N-K)$

$= X(N-K)$

2.Find the DFT of a sequence $x(n) = \{1, 2, 3, 4\}$[CO1-H1-Nov/Dec 2013]

Solution:

The DFT is given by

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi nK}{N}} \qquad K = 0, 1, \dots N-1.$$

Here number of samples N = 4.

$$X(K) = \sum_{n=0}^{3} x(n)\, e^{-\frac{j\pi nK}{2}} \qquad K = 0, 1, 2, 3.$$

K = 0

$$X(0) = \sum_{n=0}^{3} x(n)\, e^{0}$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 2 + 3 + 4.$$

K = 1

$$X(0) = 10$$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n/2}$$

$$= x(0) + x(1)e^{-j\pi/2} + x(2)e^{-j\pi} + x(3)e^{-j3\pi/2}$$

$$= 1 + 2[\cos\pi/2 - j\sin\,\pi/2] + 3[\cos\pi - j\sin\pi] + 4\left[\cos 3\,{}^{\pi}/_{2} - j\sin 3\,{}^{\pi}/_{2}\right]$$

$$= 1 - 2j - 3 + 4j$$

$$= -2 + 2j$$

K = 2

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-\frac{j\pi n(2)}{2}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$

$$= x(0) + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi}$$

$$= 1 + 2[\cos\pi - j\sin\,\pi] + 3[\cos 2\pi - j\sin 2\pi] + 4[\cos 3\pi - j\sin 3\pi]$$

$$= 1 - 2 + 3 - 4 = -2$$

K = 3

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n(3)/2}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j3\pi n/2}$$

$$= x(0) + x(1)e^{-j3\pi/2} + x(2)e^{-j3\pi 2/2} + x(3)e^{-j9\pi/2}$$

$= 1 + 2[\cos 3\pi/2 - j\sin 3\pi/2] + 3[\cos 3\pi - j\sin 3\pi] +$

$4[\cos 9\pi/2 - j\sin 9\pi/2]$

$X(3) = 1 + 2j - 3 - 4j = -2 - 2j$

$\therefore X(K) = \{10, -2 + 2j, -2, -2 - 2j\}$

3. Compute IDFT of the sequence[CO1-H1-Nov/Dec 2010]

$$\boldsymbol{X(K) = \{0, -2j, 0, 2j\}}$$

Solution:

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{\frac{j2\pi nK}{N}}$$

When n = 4

$$x(n) = \frac{1}{4}\sum_{K=0}^{3} X(K)\, e^{\frac{j\pi nK}{2}}$$

Put n = 0

$$x(0) = \frac{1}{4}\sum_{K=0}^{3} X(K) \,.\, e^{0}$$

$$= \frac{1}{4}[0 - 2j + 0 + 2j]$$

$$x(0) = 0.$$

n = 1

$$x(1) = \frac{1}{4}\sum_{K=0}^{3} X(K)\, e^{\frac{j\pi K}{2}}$$

$$= \frac{1}{4}\left[0 + (-2j)\left[e^{j\frac{\pi}{2}}\right] + 0 + (2j)\left[e^{j3\frac{\pi}{2}}\right]\right]$$

$$= \frac{1}{4}[(-2j)(j) + 2j(-j)]$$

$$= \frac{1}{4}(2 + 2)$$

$$= 1.$$

n = 2

$$x(2) = \frac{1}{4}\sum_{K=0}^{3} X(K) \,.\, e^{j\pi K}$$

$$= \frac{1}{4}\left[0 + (-2j).e^{j\pi} + 0 + (2j)e^{j3\pi}\right]$$

$$= \frac{1}{4}[(-2j)(-1) + 2j(-1)]$$

$$= \frac{1}{4}(0) = 0.$$

n = 3

$$x(3) = \frac{1}{4}\sum_{K=0}^{3} X(K) . e^{j\pi 3\frac{K}{2}}$$

$$= \frac{1}{4}\left[0 + (-2j)e^{j\pi\frac{3}{2}} + 0 + (2j)e^{j\pi\frac{9}{2}}\right]$$

$$= \frac{1}{4}[0 + (-2j)(-j) + (2j)(j)]$$

$$= \frac{1}{4}(-2 - 2)$$

$$= \frac{1}{4}(-4)$$

$$x(3) = -1$$

$$x(n) = \{0, 1, 0, -1\}$$

4. Compute IDFT of the sequence $X(K) = \{0, 1, 1, 0\}$.[CO1-H1-May/June 2010]

Solution:

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j2\pi n\frac{K}{N}}$$ n = 0, 1, 2 …..N – 1

N = 4 $$x(n) = \frac{1}{4}\sum_{K=0}^{3} X(K)\, e^{j2\pi n\frac{K}{N}}$$ n = 0, 1, 2, 3

$$x(n) = \frac{1}{4}\sum_{K=0}^{3} X(K)\, e^{j\pi n\frac{K}{N}}$$

n = 0

$$x(0) = \frac{1}{4}\sum_{K=0}^{3} X(K)$$

$$= \frac{1}{4}[0 + 1 + 1 + 0]$$

$$x(0) = \frac{1}{2} = (0.5)$$

n = 1

$$x(1) = \frac{1}{4}\sum_{K=0}^{3} X(K) . e^{j\pi\frac{K}{2}}$$

$$= \frac{1}{4}\left[0 + e^{j\frac{\pi}{2}} + e^{j\pi} + 0\right]$$

$$= \frac{1}{4}[j - 1] = 0.25 + 0.25j$$

n = 2

$$x(2) = \frac{1}{4}\sum_{K=0}^{3} X(K).e^{j\pi K}$$

$$= \frac{1}{4}\left[0 + e^{j\pi} + e^{j2\pi} + 0\right]$$

$$= \frac{1}{4}[-1 + 1]$$

$$= 0$$

n = 3

$$x(3) = \frac{1}{4}\sum_{K=0}^{3} X(K).e^{\frac{j3\pi K}{2}}$$

$$= \frac{1}{4}\left[0 + e^{j3\frac{\pi}{2}} + e^{j3\pi} + 0\right]$$

$$= \frac{1}{4}[-j - 1]$$

$$= -0.25 - 0.25j$$

$$x(n) = \{0.5, -0.25 + j0.25, 0, -0.25 - 0.025\}$$

$$x(n) = \{0.5, -0.25 + 0.25j, 0, -0.25, -0.025\}$$

5.Compute IDFT of sequence $X(K) = \{6, -2 + j6, 2, -2, -j6\}$[CO1-H1]

Solution:

$$x(n) = \frac{1}{4}\sum_{K=0}^{3} X(K).e^{\frac{j\pi nK}{2}}$$

n = 0

$$x(0) = \frac{1}{4}\sum_{K=0}^{3} X(K)$$

$$= \frac{1}{4}[6 - 2 + j6 + 2 - 2 - j6]$$

$$x(0) = 1$$

n = 1

$$x(1) = \frac{1}{4}\sum_{K=0}^{3} X(K).e^{j\pi K/2}$$

$$= \frac{1}{4}\left[6e^{0} + (-2 + j6)\left(e^{j\frac{\pi}{2}}\right) + 2e^{j\pi} + (-2 - 6j)e^{j\pi\frac{3}{2}}\right]$$

$$= \frac{1}{4}[6 + (-2j) - 6 + (-2) + (-2 - 6j)(-j)]$$

$$= \frac{1}{4}[6 - 2j - 6 - 2 + 2j - 6]$$

$$= \frac{1}{4}[-8]$$

$x(1) = -2$

n = 2

$x(2) = \frac{1}{4}\sum_{K=0}^{3} X(K) . e^{j\pi K}$

$= \frac{1}{4}[6 + (-2 + j6)(-1) + 2(1) + (-2 - 6j)(-1)]$

$= \frac{1}{4}[6 + 2 - j6 + 2 + 2 + 6j]$

$= \frac{1}{4}[12]$

$= 3.$

n = 3

$x(3) = \frac{1}{4}\sum_{K=0}^{3} X(K) . e^{j\pi K\frac{3}{2}}$

$= \frac{1}{4}\left[6 + (-2j)(-2 + j6)\left(e^{j\pi\frac{3}{2}}\right) + 2e^{j\pi 3} + (-2 - 6j)e^{j\pi\frac{9}{2}}\right]$

$= \frac{1}{4}[6 + 2j + 6 - 2 - 2j + 6]$

$= \frac{1}{4}(16)$

$= 4$

$\therefore x(n) = \{1, -2, 3, 4\}$

6. Compute IDFT if $X(K) = \{2, 1 - j, -2, 1 + j\}$[CO1-H1-April/May 2010]

Solution:

$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{j2\pi n\frac{K}{N}}$ n = 0, 1, 2, 3

Where N = 4

$x(n) = \frac{1}{4}\sum_{K=0}^{3} X(K)\, e^{j\pi n\frac{K}{2}}$

n = 0

$x(0) = \frac{1}{4}\sum_{K=0}^{3} X(K)$

$= \frac{1}{4}[2 + 1 - j - 2 + 1 + j]$

$= \frac{1}{2}$

$= 0.5$

n = 1

$$x(1) = \frac{1}{4}\sum_{K=0}^{3} X(K)\, e^{j\pi n\frac{K}{2}}$$

$$= \frac{1}{4}\left[2 + (1-j)e^{j\frac{\pi}{2}} + (-2)e^{j\pi} + (1+j).\, e^{j\pi\frac{3}{2}}\right]$$

$$= \frac{1}{4}[2 + j + 1 + 2 - j + 1]$$

$$= 1.5$$

n = 2

$$x(2) = \frac{1}{4}\sum_{K=0}^{3} X(K)\, e^{j\pi\frac{K}{2}}$$

$$= \frac{1}{4}[2 + (1-j)e^{j\pi} + (-2)e^{2j\pi} + (1+j)e^{3j\pi}]$$

$$= \frac{1}{4}[2 - 1 + j - 2 - 1 - j]$$

$$= -0.5$$

n = 3

$$x(3) = \frac{1}{4}\sum_{K=0}^{3} X(K)\, e^{\frac{j\pi 3K}{2}}$$

$$= \frac{1}{4}\left[2 + (1-j)e^{j3\frac{\pi}{2}} + (-2)e^{j\pi 3} + (1+j).\, e^{j\pi\frac{9}{2}}\right]$$

$$= \frac{1}{4}[2 - j - 1 + 2 + j - 1]$$

$$= 0.5$$

$$x(n) = \{0.5, 1.5, -0.5, 0.5\}$$

7. Find DFT of the sequence $x(n) = \{1, 1, 2, 2, 3, 3\}$[CO1-H1-April/May 2010]

Solution:

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi nK}{N}}$$ where K = 0, 1, ……(N – 1)

N = 6.

K = 0

$$X(0) = \sum_{n=0}^{5} x(n)\, e^{\frac{-j\pi n\,(0)}{6}}$$

$$= \sum_{n=0}^{5} x(n)$$

$$= 1 + 1 + 2 + 2 + 3 + 3 = 12$$

K = 1

$$X(1) = \sum_{n=0}^{5} x(n)\, e^{\frac{-jn\pi}{3}}$$

$$= 1 + e^{\frac{-j\pi}{3}} + 2e^{-j\pi} + 3e^{\frac{-j4\pi}{3}} + 3e^{\frac{-j5\pi}{3}}$$

$$= 1 + (0.5 - j0.866) + 2(-0.5 - 0.866) + 2(-1) + 3(-0.5 + j0.866)$$

$$+3(0.5 + j0.866)$$

$$= -1.5 + j2.6\ .$$

K = 2

$$X(2) = \sum_{n=0}^{5} x(n)\, e^{\frac{-j2\pi n}{3}}$$

$$= 1 + e^{\frac{-j2\pi}{3}} + 2e^{\frac{-j4\pi}{3}} + 2e^{-j2\pi} + 3e^{\frac{-j8\pi}{3}} + 3e^{\frac{-j10\pi}{3}}$$

$$= 1 + (-0.5 - 0.866j) + 2(-0.5 + j0.866) + 2(1) + 3(-0.5 - j0.866)$$

$$+3(-0.5 + j0.866)$$

$$= -1.5 + j0.87\ .$$

K = 3

$$X(3) = \sum_{n=0}^{5} x(n)\, e^{-j\pi n}$$

$$= 1 + e^{-j\pi} + 2e^{-j2\pi} + 2e^{-j3\pi} + 3e^{-j4\pi} + 3e^{-j5\pi}$$

$$= 1 - 1 + 2(1) + 2(-1) + 3(1) + 3(-1)$$

$$= 0.$$

K = 4

$$X(4) = \sum_{n=0}^{5} x(n)\, e^{\frac{-j4\pi n}{3}}$$

$$= 1 + e^{\frac{-j4\pi}{3}} + 2e^{\frac{-j8\pi}{3}} + 2e^{-j4\pi} + 3e^{\frac{-j16\pi}{3}} + 3e^{\frac{-j20\pi}{3}}$$

$$= 1 + (-0.5 + j0.866) + 2(-0.5 - j0.866) + 2(1) + 3(-0.5 + j0.866)$$

$$+3(-0.5 - j0.866)$$

$$= -1.5 - j0.87.$$

K = 5

$$X(5) = \sum_{n=0}^{5} x(n)\, e^{\frac{-j5\pi n}{3}}$$

$$= 1 + e^{\frac{-j5\pi}{3}} + 2e^{\frac{-j10\pi}{3}} + 2e^{-j5\pi} + 3e^{\frac{-j20\pi}{3}} + 3e^{\frac{-j25\pi}{3}}$$

$$= 1 + (-0.5 + j0.866) + 2(-0.5 + j0.866) + 2(-1) + 3(-0.5 - j0.866)$$

$$+3(0.5 - j0.866)$$

$$= -1.5 - j2.6$$

$$X(K) = \{12, -1.5 + j2.6, -1.5 + j0.87, 0, -1.5 - j0.87, -1.5 - j2.6\}$$

8. Find 4 point DFT of the sequence $x(n) = \cos\frac{n\pi}{4}$.[CO1-H1-April/May 2011]

Solution:

4 point DFT N = 4

$$x(n) = \left\{\cos(0), \cos\left[\frac{\pi}{4}\right], \cos\left[\frac{\pi}{2}\right], \cos\left[\frac{3\pi}{4}\right]\right\}$$

$$= \{1, 0.707, 0, -0.707\}$$

Generally $X(K) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi nK}{N}}$ for K = 0, 1, ... (N – 1)

$$X(K) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi nK}{2}}$$

K = 0

$$X(0) = \sum_{n=0}^{3} x(n) = 1 + 0.707 + 0 - 0.707 = 1$$

K = 1

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi n}{2}}$$

$$= 1 + 0.707 e^{\frac{-j\pi}{4}} + 0 + (-0.707) e^{\frac{-j3\pi}{2}}$$

$$= 1 + 0.707(-j) + 0 - (0.707)j$$

$$= 1 + j1.414.$$

K = 2

$$x(2) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$

$$= 1 + 0.707 e^{-j\pi} + 0 + (-0.707) e^{-j3\pi}$$

$$= 1 + 0.707(-1) + 0 - (0.707)(-1)$$

$$= 1.$$

K = 3

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j3\pi n}{2}}$$

$$= 1 + (0.707) e^{\frac{-j3\pi}{4}} + 0 + (-0.707) e^{\frac{-j9\pi}{2}}$$

$$= 1 + (0.707)j + 0(-0.707)(-j)$$

$= 1 + j1.414$

$X(3) = \{1, 1 - j1.414, 1 + j1.414\}$

9. Find the DFT of a sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using Decimation In Time algorithm. [CO1-H1-April/May 2009]

Solution:

$W_8^0 = W_4^0 = W_2^0 = 1$

$W_8^1 = e^{-j2\pi/8} = e^{-j\pi/4} = cos\pi/4 - jsin\pi/4$

$= \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$\therefore W_8^1 = 0.707 - j0.707$

$\therefore W_8^2 = W_4^1 = e^{-j\pi/2}$

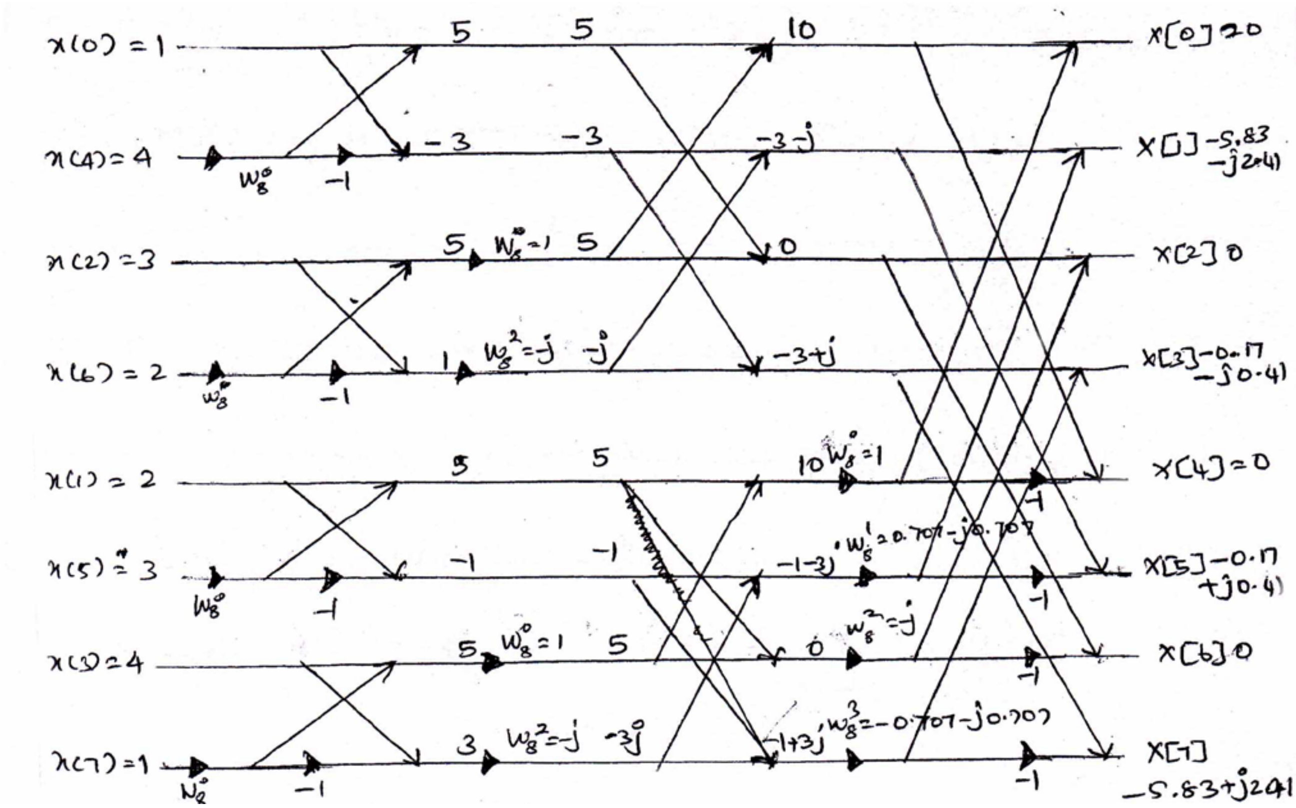
$= -j$

$\therefore W_8^3 = e^{\frac{-j2\pi}{8}.3}$

$= e^{-3\pi/4}$

$= \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$\therefore W_8^3 = -0.707 - j0.707.$



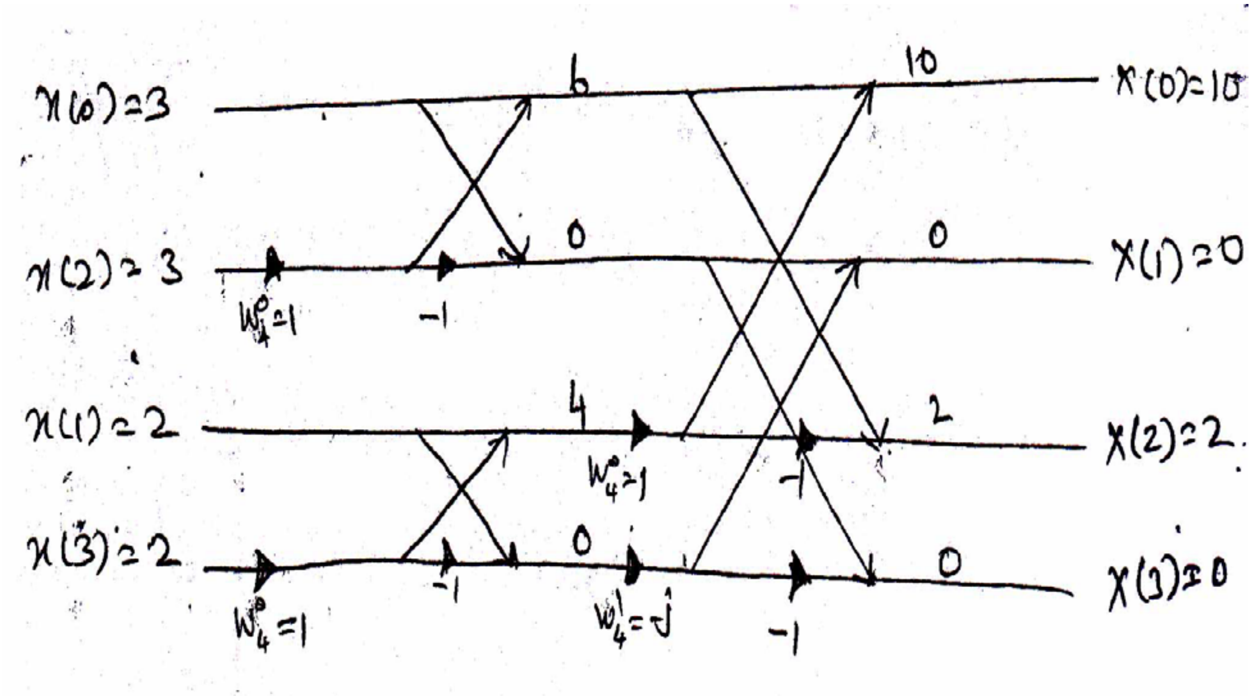
Stage 1	Stage 2	Stage 3
1 + 4 = 5	5 + 5 = 10	10 + 10 (1) = 20
1 – 4 = – 3	– 3 – j = – 3 – j	(– 3 – j) + (– 1 – 3j) (0.707 – j0.707) = – 5.83 – j2.41
3 + 2 = 5	5 – 5 = 0	0 + 0 (– j) = 0
3 – 2 = 1	– 3 + j = – 3 + j	(– 3 + j) + (– 1 + 3j) (– 0.707 – j0.707) = – 0.17 – j0.41
2 + 3 = 5	5 + 5 = 10	10 – 10 = 0
2 – 3 = – 1	1 – 3j = – 1 – 3j	(– 3 – j) + (– 1 – 3j) (0.707 – j0.707) = –0.17– j0.41
4 + 1 = 5	5 – 5 = 0	$0 - 0\left(\frac{1}{\sqrt{2}} - J\frac{1}{\sqrt{2}}\right) = 0$
4 – 1 = 3	– 1 + 3j = – 1 + 3j	(– 3 + j) + (– 1 + 3j) (– 0.707 – j0.707) = – 5.83 – j2.41

∴ X (K) = { 20, – 5.83 – j2.41, 0, – 0.17 – j0.41, 0, – 0.17 + j0.41, 0, – 5.83 + j2.41}

10. $x(n) = \begin{Bmatrix} 3, & 2, & 3, & 2 \\ x(0) & x(1) & x(2) & x(3) \end{Bmatrix}$**. Find X (K) using DIT FFT algorithm?**

[CO1-H1-April/May 2010]

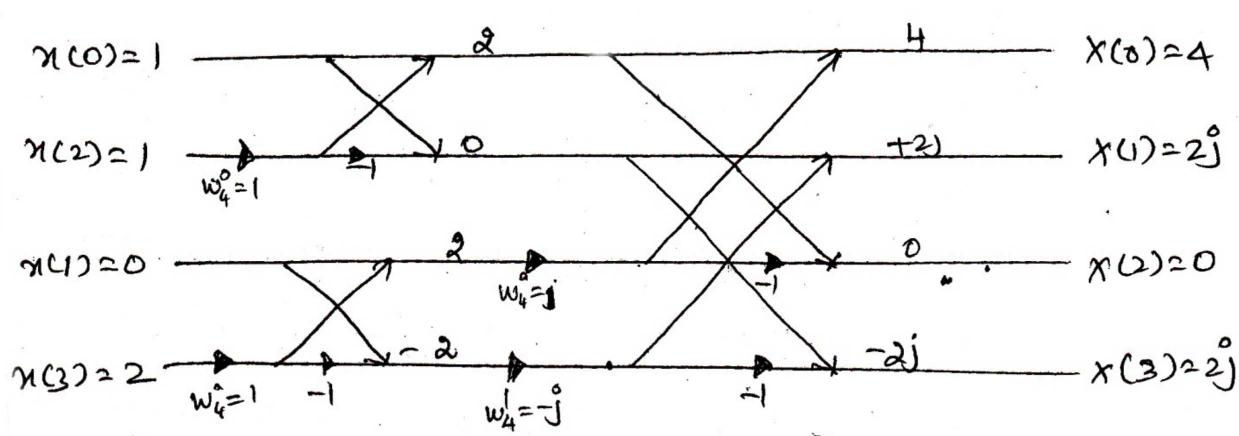
Solution:



X (K) = {10, 0, 2, 0}

11.$x(n) = \{1, 0, 1, 2\}$. **Find X(K) using DIT FFT algorithm. [CO1-H1-April/May 2012]**

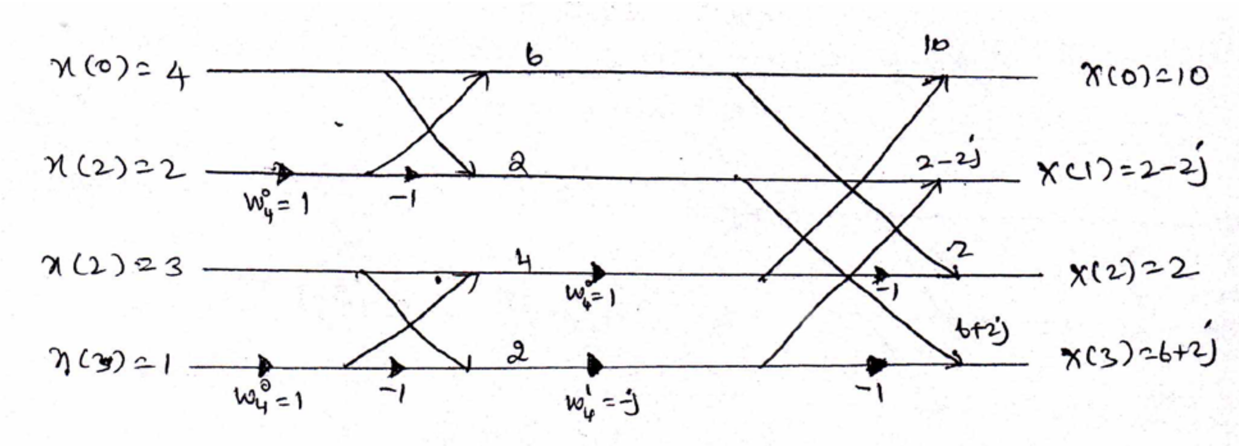
Solution:



X (K) = {4, 2j + 0, – 2j}

12.. $x(n) = \{4, 3, 2, 1\}$ **Find X(K) using DIT FFT algorithm? [CO 1-H1]**

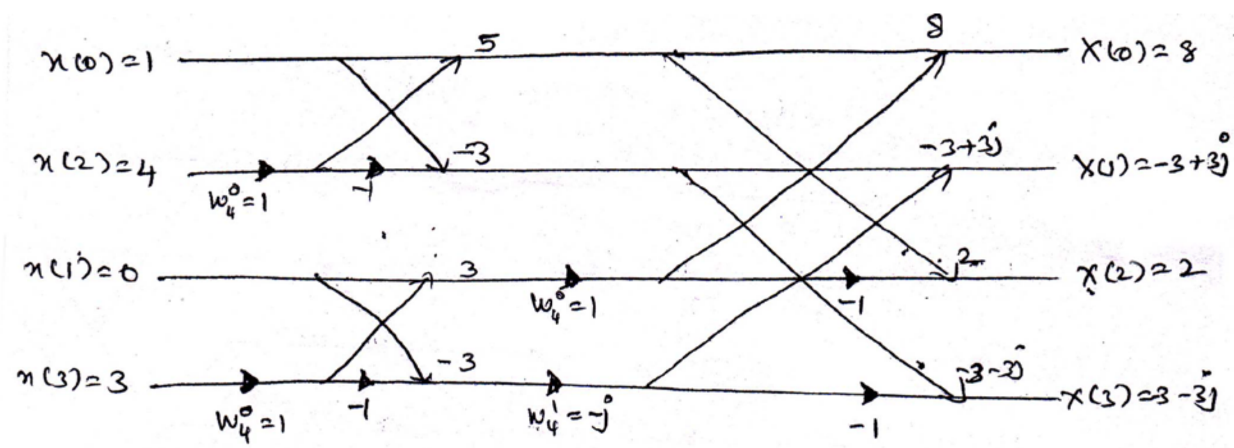
Solution:



X (K) = {10, 2 – 2j, 2, 6 + 2j}

13. $x(n) = \{1, 0, 4, 3\}$ **Find X (K) using DIT – FFT algorithm.? [CO1-H1-Nov/Dec 2010]**

Solution:



X (K) = {8, – 3 + 3j, 2, – 3 – 3j}

14. $x(n) = sin\frac{n\pi}{4}$ at N = 8. Find X (K) DIT – FFT algorithm. [CO1-H1-Nov/Dec 2010]

Solution:

$x(0) = sin\ 0$ = 0

$x(1) = sin\frac{\pi}{4}$ = 0.707

$x(2) = sin\frac{\pi}{2}$ = 1

$x(3) = sin\frac{3\pi}{4}$ = 0.707

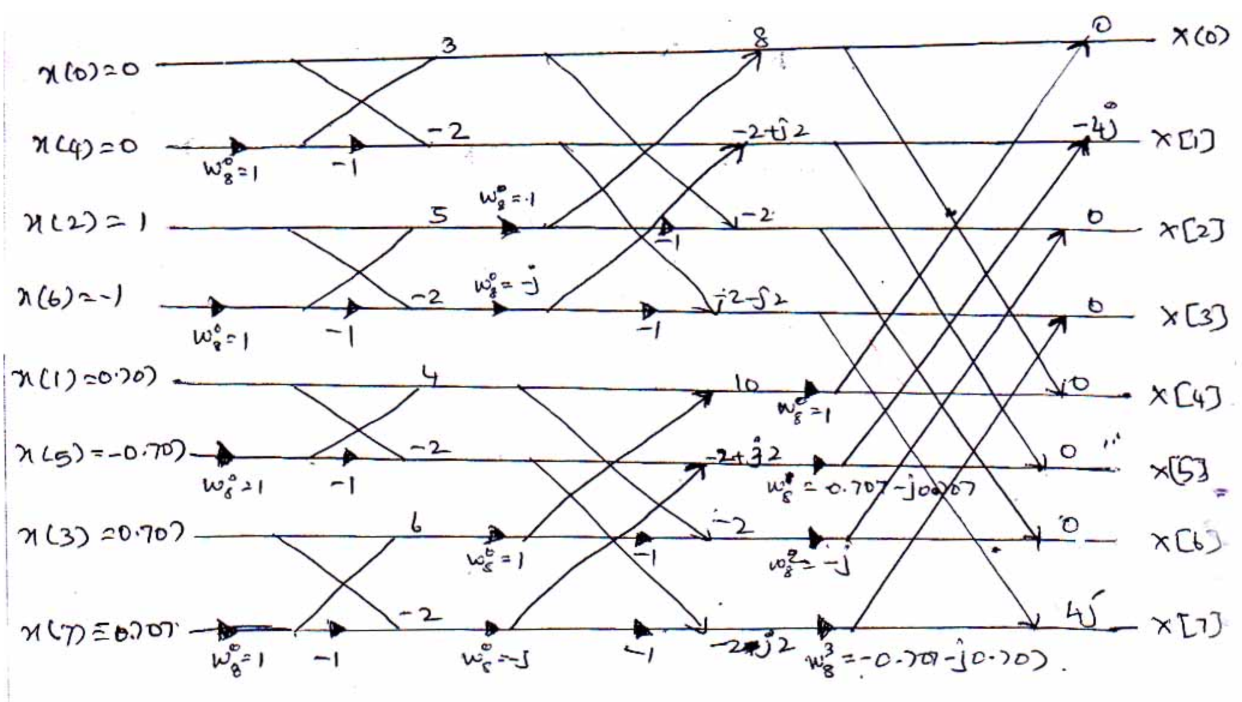
$x(4) = sin\ \pi$ = 0

$x(5) = sin\frac{5\pi}{4}$ = – 0.707

$x(6) = sin\frac{6\pi}{4}$ = – 1

$x(7) = sin\frac{7\pi}{4}$ = – 0.707.

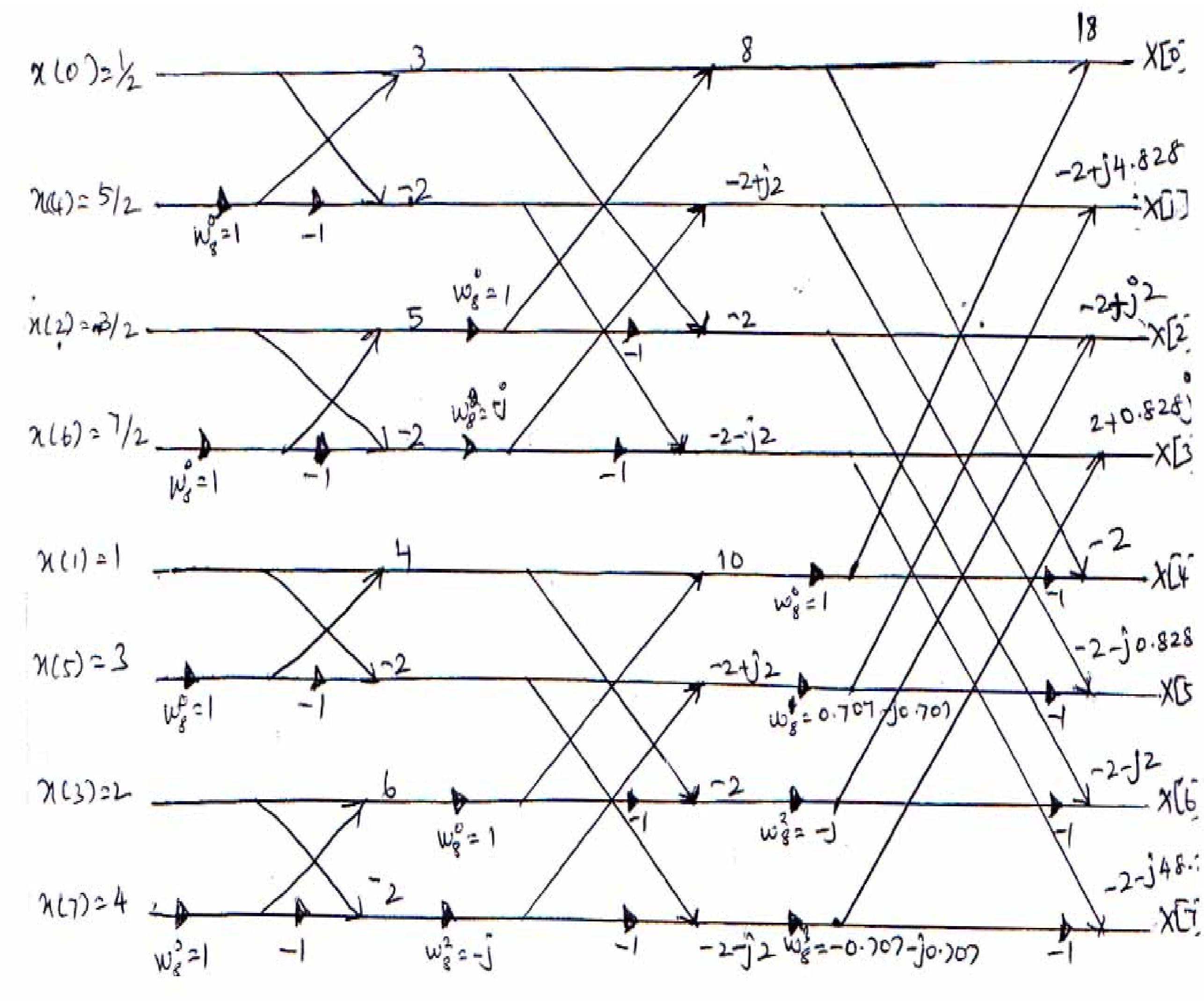
$x(n)$ = {0, 0.707, 1, 0.707, 0 – 0.707, – 1, – 0.207}



∴ X (K) = {0, – 4j, 0, 0, 0, 0, 0, 4j}

15. $x(n) = \frac{n+1}{2}$**. Find X (K) using DIT – FFT algorithm. [CO1-H1-Nov/Dec 2010]**

Solution: $x(n) = \left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4\right\}$

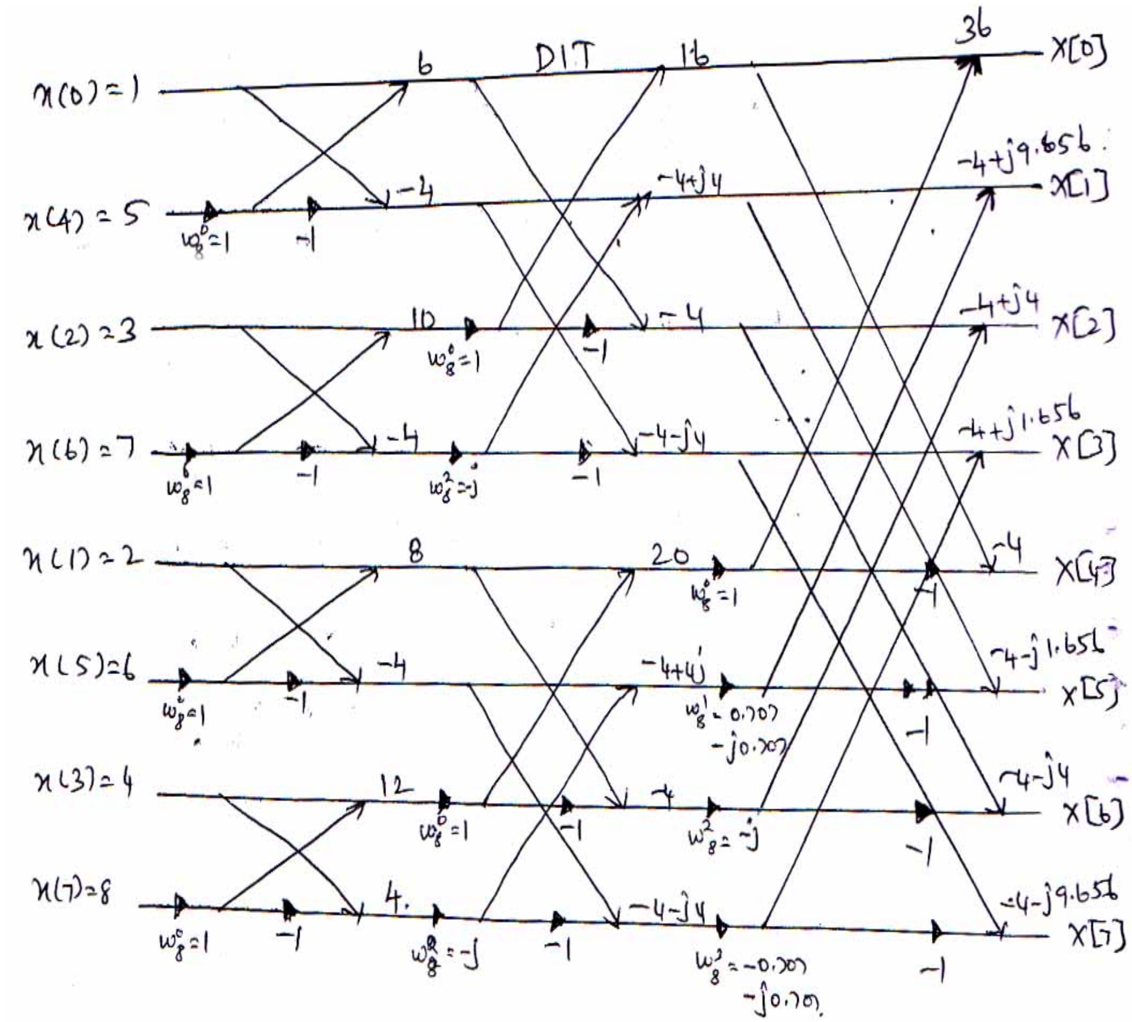


X (K) = {18, – 2 + 4.828j, – 2 + 2j, +2 – 0.828j, – 2, – 2 – 0.828j, – 2 – 2j, – 2 – 48.28j}

16. $x(n) = n + 1$**. Find X (K) using DIT – FFT Algorithm.** $x(n) =$ **{1, 2, 3, 4, 5, 6, 7, 8}**

[CO1-H1-Nov/Dec 2011]

Solution:



X (K) = {36, – 4 + j9.656, – 4 + j4, – 4 + j1.656, – 4, – 4 – j1.656, – 4 – j4, – 4 – j9.656}.

17. $x(n) = \sin n\pi + \cos n\pi$ at N = 8. Find X (K) using DIT FFT algorithm. [CO1-H1]

Solution:

$x(0) = \sin 0 + \cos 0 = 1$

$x(1) = \sin \pi + \cos \pi = -1$

$x(2) = \sin 2\pi + \cos 2\pi = +1$

$x(3) = \sin 3\pi + \cos 3\pi = -1$

$x(4) = \sin 4\pi + \cos 4\pi = 1$

$x(5) = \sin 5\pi + \cos 5\pi = -1$

$x(6) = \sin 6\pi + \cos 6\pi = 1$

$x(7) = \sin 7\pi + \cos 7\pi = -1$

$x(n) = \{1, -1, 1, -1, 1, -1, 1, -1\}$

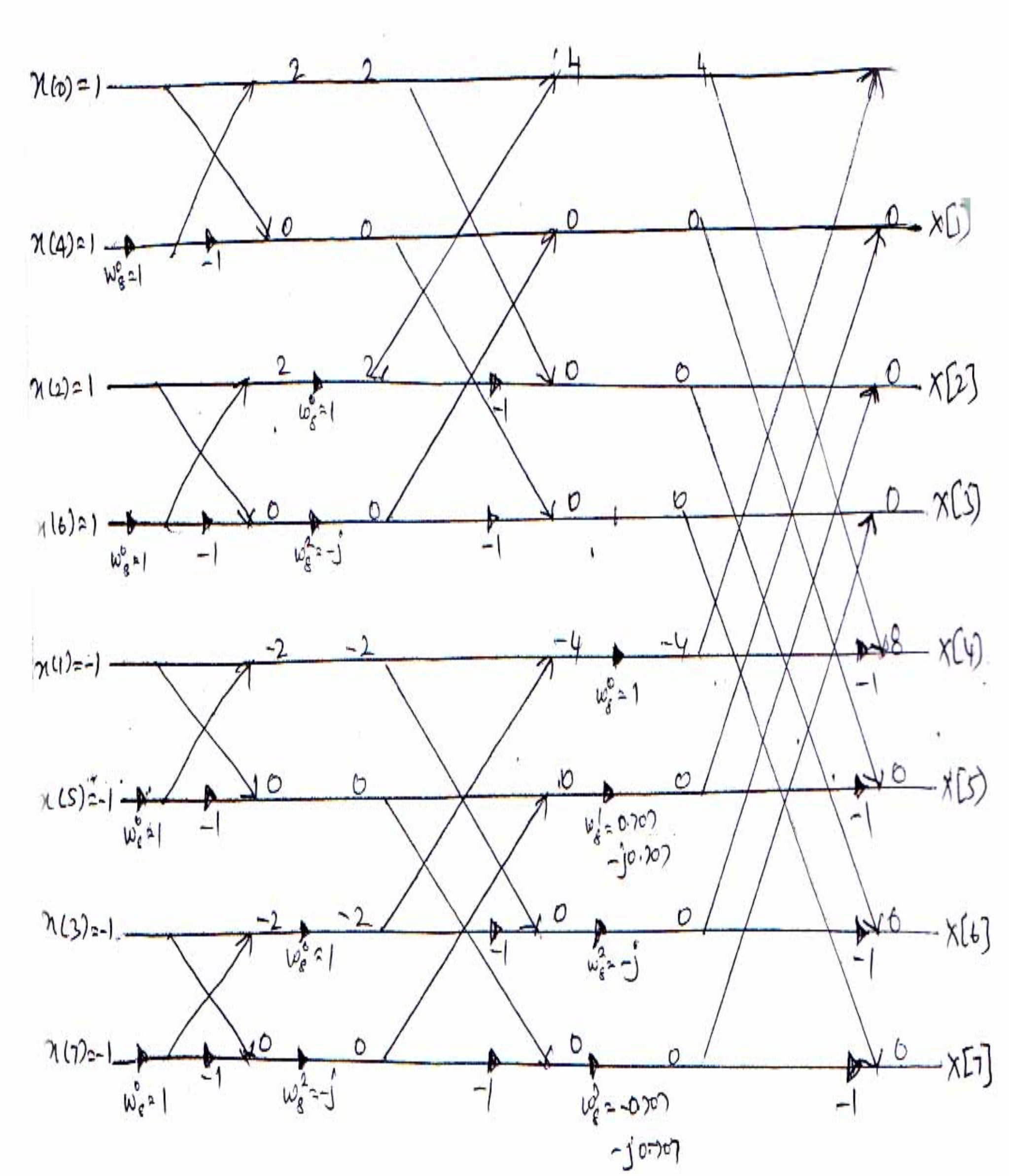
The twiddle factor values are

$W_8^0 = 1$

$W_8^1 = 0.707 - 0.707j$

$W_8^2 = -j$

$W_8^3 = -0.707 - 0.707j$.



X (K) = {0, 0, 0, 0, 8, 0, 0, 0}.

18. $x(n) = cos\ n\ \pi/4$ **at N = 8. Find X (K) using DIT – FFT Algorithm. [CO1-H1]**

Solution:

$$x(0) = cos\ 0$$
$$= 1$$
$$x(1) = cos\ \pi/4$$
$$= 0.707$$

$x(2) = cos\ 2\pi/4$

$= 0$

$x(3) = cos\ 3\pi/4$

$= -0.707$

$x(4) = cos\ 4\pi/4$

$= -1$

$x(5) = cos\ 5\pi/4$

$= -0.707$

$x(6) = cos\ 6\pi/4$

$= 0$

$x(7) = cos\ 7\pi/4$

$= 0.707$

$x(n) = \{1, 0.707, 0, -0.707, -1, -0.707, 0, 0.707\}$

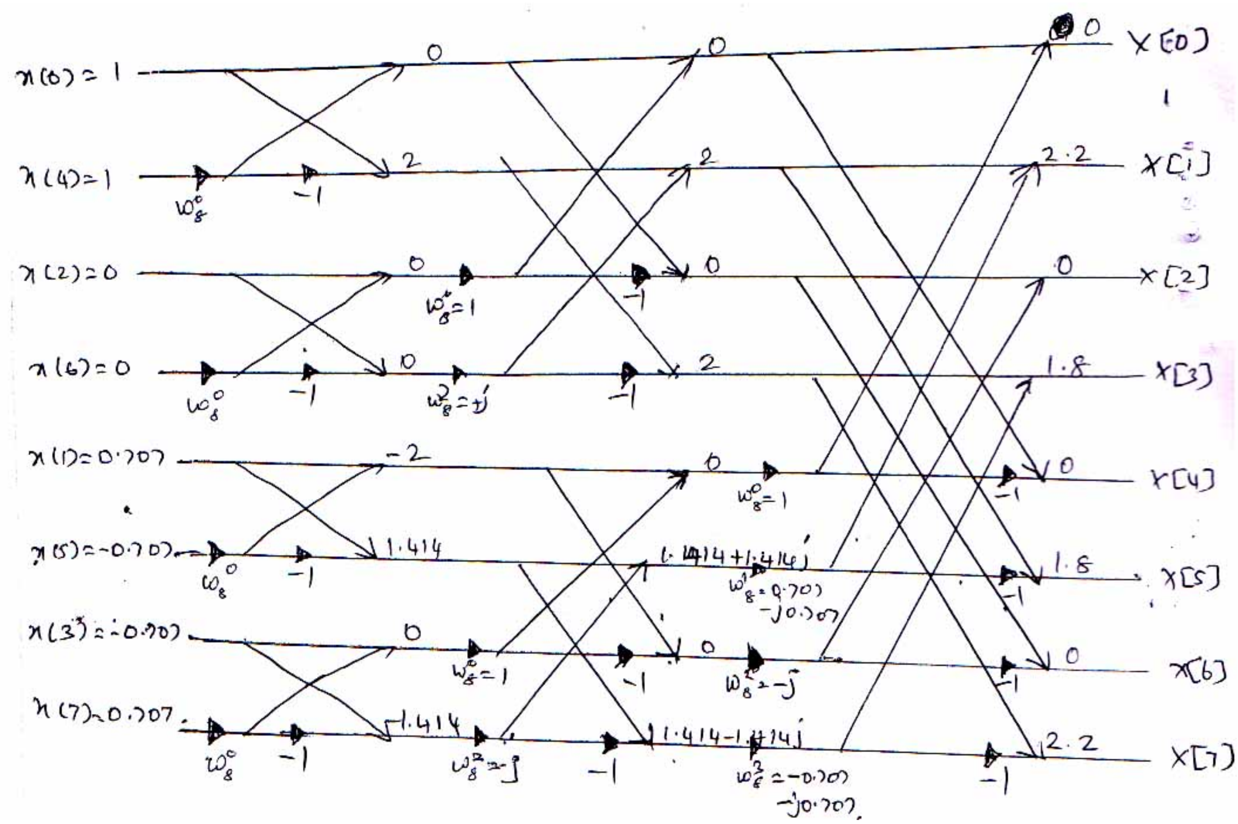
The Twiddle factor values are

$W_8^0 = 1$

$W_8^1 = 0.707 – 0.707j$

$W_8^2 = – j$

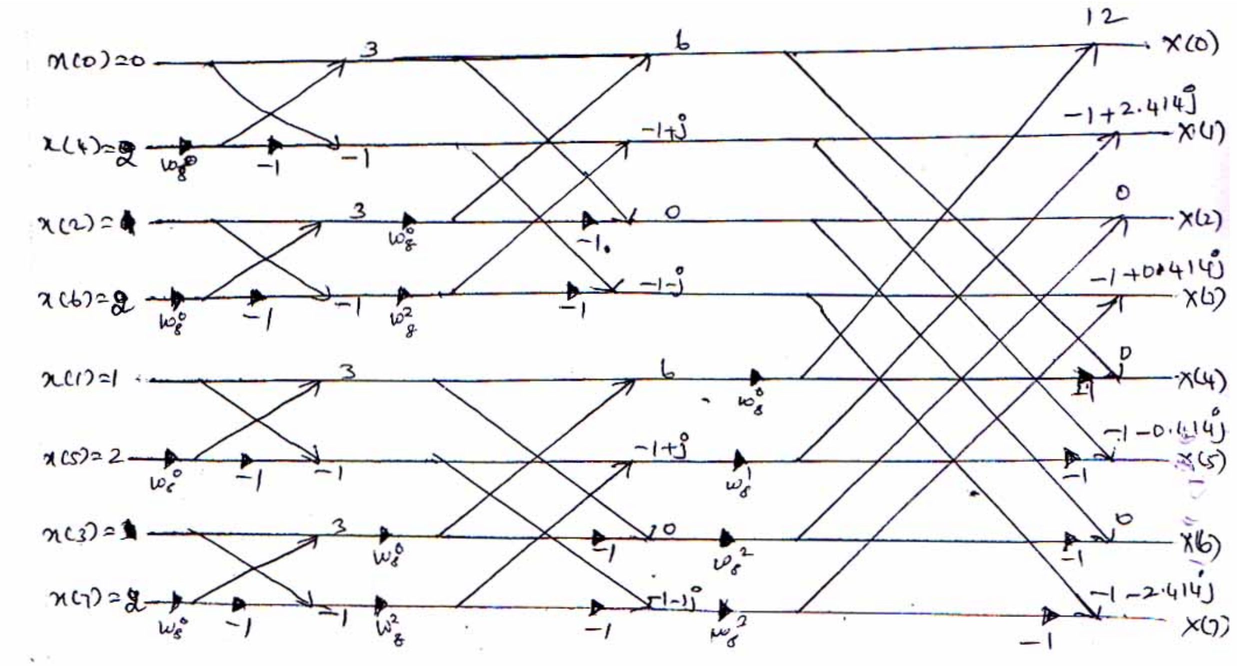
$W_8^3 = – 0.707 – 0.707\ j.$



X (K) = {0, 2.2, 0, 1.8, 0, 1.8, 0, 2.2}

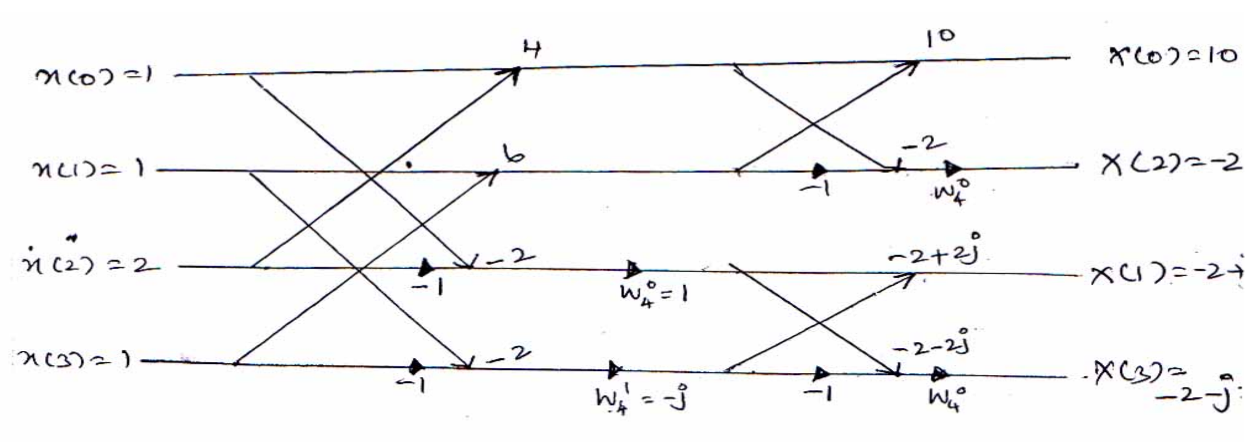
19. Find X (K) for $x(n)$ = {1, 1, 1, 1, 2, 2, 2, 2} using DIT FFT Algorithm. [CO1-H1-Nov/Dec 2006]

Solution:



X (K) = {12, – 1 + 2.414 j, 0, – 1 + 0.414 j, 0, – 1 – 0.414 j, 0, – 1 – 2.414 j}

20. $x(n) =$ {1, 2, 3, 4} . Find X (K) using DIF Algorithm. [CO1-H1-May/June 2010]



X(K) = {10, −2 + 2j, −2, −2 – 2j}

Unit – II

IIR Filter Design

Part – A

1. What is frequency warping in Bilinear transformation? [CO2-L1-Nov/Dec 2011
The mapping of frequency from Ω to ω is approximately linear for small value of Ω & ω. For the higher frequencies, however the relation between Ω & ω becomes highly non-linear. This introduces
the distortion in the frequency scale of digital filter relative to analog filter. This effect is known as warping effect.

2. What are methods used to convert analog to digital filter? [CO2-L1]
Approximation of derivatives, Impulse invariant method & Bilinear transformation method.

3. Write the pole mapping rule in Impulse invariant method? [CO2-L1]
A pole located at $s = s_p$ in the s plane is transferred into a pole in the z plane located at $Z = e^{s_pT_s}$. Each
strip of width 2π/T on left half of s-plane should be mapped to region inside the unit circle in z-plane.
The imaginary axis of s-plane is mapped to unit circle in z-plane. Left half of s-plane is mapped to
outer region of unit circle.

4. What are the disadvantages of Impulse invariant method? [CO2-L1]
It provides many to one pole mapping from s-plane to z-plane. aliasing will occur in IIT.

5. What are the advantages of Bilinear transformation method? [CO2-L1]
The Bilinear transform method provides non linear one to one mapping of the frequency points on the
jw axis in the S plane to those on the unit circle in the Z plane.i.e Entire jw axis for $-\infty < w < \infty$ maps
uniquely on to a unit circle $-\pi/T < \omega/T < -\pi/T$. This procedure allows us to implement digital high pass
filters from their analog counter parts. No aliasing effects.

6. Define prewarping or prescaling. [CO2-L1-May/June 2012]
For large frequency values the non linear compression that occurs in the mapping of Ω to w is more apparent .This compression causes the transfer function at high Ω frequency to be highly distorted when it is translate to the w domain. This compression is being compensated by introducing a prescaling or prewarpping to Ω frequency scale. For bilinear transform Ω scale is converted into Ω^* scale (i.e) $\Omega^* = 2/T_s \tan(\Omega T_s/2)$(prewarped frequency)

7. Comparison of analog and digital filters. [CO2-L2-May/June 2015]

A Analog filter	DDigital filter
ınalog filter both input and output ıtinuous time signal	digital filter,both the input and output are ıte time signals.
can be constructed using active assive components.	;an be constructed using adder, multiplier elay units.
ese filters operate in infinite frq. e, theoretically but in practice it is l by finite max. operating freq. nding upon the devices used.	q. range is restricted to half the sampling and it is also restricted by max. utational speed available for particular ation.
; defined by linear differential eqn.	defined by linear difference eqn

8. What are the advantages of digital filter? [CO2-L1]
1. Filter coefficient can be changed any time thus it implements the adaptive future.
2. It does not require impedance matching between input and output.
3. Multiple filtering is possible. 4. Improved accuracy, stability and dynamic range.

9. What are disadvantages of Digital Filter? [CO2-L1]
The bandwidth of the filter is limited by sampling frequency.The performance of the digital filter depends on the hardware used to implement the filter.
The quantization error arises due to finite word length effect in representation of signal and filter coefficient.

10. What is the difference between Chebyshev Filter type I and typeII? [CO2-L2]
Filter TypeI:
It is all pole filter and exhibits equiripples in the pass band and monotonic characteristics in the stop band.
Filter Type II:It contains both poles and zeros and exhibits a monotonic behaviour in the pass band and equiripple in the stop band.

11. What are the properties of chebyshev filter? [CO2-L1]
1. For $\omega \geq 1$ $H(j\omega)$ decreases monotonically towards zero.
2. For $\omega \leq 1$ $H(j\omega)$ it oscillates between 1 and 1\(1+ε^2)

12**. Compare Butterworth filter and chebyshev filter. [CO2-L2-May/June 2011]**
Butterworth filter
1.The Magnitude response of Butterworth filter decreases monotonically as the frequency increases.
2. The Transition width is more
3.The order of butterworth filter is more, thus it requires more elements to construct and is expensive.
4. The Poles of the butterworth filter lies along the circle.
5. Magnitude response is flat at $\omega=0$ thus it is known as maximally flat filter.
Chebyshev Filter
1. The Magnitude response of Chebyshev filter will not decrease monotonically with frequency because it exhibits ripples in pass band or stop band.
2. The Transition width is very small

3. For the same specifications the order of the filter is small and is less complex and inexpensive.
4. The poles of chebyshev filter lies along the ellipse.
5. Magnitude response produces ripples in the pass band or stop band thus it is known as equripple filter.

13. What are the properties of Chebyshev filter? [CO2-L1]
1. The magnitude response of the Chebyshev filter exhibits ripples either in the pass band or in the stop band. 2.The poles of a Chebyshev filter lie on an ellipse.

14. What are the different structures for realization of IIR systems? [CO2-L1]
Direct Form I, Direct Form II, Cascade, Transposed, Parallel, Lattice ladder Structures

15. What is Butterworth approximation? [CO2-L1]
The frequency response characteristic of the low pass butterworth filter is monotonic in both pass band and stop band. The response approximate to the ideal response as the order N of the filter increases (flat characteristics).

16. What is the relation between Analog and digital frequency in IIT? [CO2-L1]
The relation between Analog and digital frequency is given by digital frequency = ΩT
Where Ω = analog frequency and T= sampling period.

17. State the two advantage of bilinear transformation. [CO2-L1]
It avoids aliasing in frequency components.
The transformation of stable analog filter results in a stable digital filter.

18. What are the parameters (specifications)of a Chebyshev filter? [CO2-L1]
Pass band ripple, pass band cut off frequency, stop band cut off frequency, attenuation beyond stop band frequency.

19. What is Chebyshev approximation? [CO2-L1]
In Chebyshev approximation, the error is defined as the difference between the ideal brickwall characteristic and the actual response and this is minimized over a prescribed band of frequencies.

20. Mention the important features of IIR filters. [CO2-L1]
The physically realizable IIR filter does not have linear phase.
The IIR specifications include the desired characteristics for the magnitude response only

21. What is bilinear transformation? What is the main advantages and disadvantages of this technique? [CO2-L1]
It is conformal mapping which utilize prewarping technique used to design IIR filters. It is one to one mapping. The relation between analog and digital frequency is nonlinear, ie $\Omega=2/T \tan(\omega/2)$
Advt.: No aliasing effects Dis-advt.: Due to nonlinear relation between ω and Ω distortion occurs in frequency domain of digital filter.

Part – B

1.Design a Butterworth LPF with 3dB cutoff frequency of 0.2π using Bilinear transformation technique. [CO2-H1]

Soln:

The first order normalized Butterworth filter is H(s) = $\frac{1}{s+1}$

Conversion of normalized Butterworth LPF to analog LPF, S is replaced by $\frac{s}{\Omega_c}$

$\omega = 0.2\pi \quad (given)$

The cut off frequency of analog filter

$$\Omega_c = \frac{2}{T}\tan\left(\frac{\omega_c}{2}\right)$$

$$\Omega_c = \frac{2}{T}\tan(0.1\pi)$$

$$\Omega_c = \frac{0.65}{T}$$

The transfer function

$$H(s) = \frac{\Omega_c}{s+\Omega_c}$$

$$= \frac{0.65/T}{s+\frac{0.65}{T}} = \frac{0.65}{S.T+0.65}$$

The digital filter conversion means substitute

$$S = \frac{2}{T}\left[\frac{Z-1}{Z+1}\right]$$

$$= \frac{0.65}{T\frac{2}{T}\left[\frac{Z-1}{Z+1}\right]+0.65}$$

$$= \frac{0.65 \times (2+1)}{2Z-2+0.65Z+0.65}$$

$$= \frac{0.65\,(Z+1)}{2.65Z-1.35}$$

$$H(z) = \frac{0.245\,(z+1)}{(z-0.509)}$$

2.Using Bilinear transformation convert H(s) = $\frac{1}{(s+2)^2}$ Assume T = 1 sec.

[CO2-H1-Nov/Dec 2007]

For the bilinear transformation

$$H(z) = H(s)\Big|_{S=\frac{2}{T}\left[\frac{Z-1}{Z+1}\right]}$$

Assume T = 1 sec.

$$H(z) = \frac{1}{\left[2\frac{(z-1)}{(z+1)}+2\right]^2} = \frac{(z+1)^2}{(2(z-1)+2(z+1))^2}$$

$$= \frac{(z+1)^2}{(2z-2)^2+(2z+2)^2+2(2z-2)(2z+2)}$$

$$= \frac{(z+1)^2}{(4z^2+4-8z+4z^2+4z+8z+8z^2+8z-8z-8)}$$

$$= \frac{(z+1)^2}{16z^2+4z+4}$$

$$= \frac{(z+1)^2}{4(4z^2+z+1)}$$

3. Convert the analog filter with system function H(s) = $\frac{s+0.1}{(s+0.1)^2+9}$ into digital IIR filter. The resonant frequency $\omega_r = \frac{\pi}{4}$. Apply Bilinear transformation.
[CO2-H1-May/June 2013]

Soln:

$$\Omega_c = \frac{2}{T}\tan\frac{\omega_r}{2}$$

T can be calculated using above equation

$$T = \frac{2}{\Omega_c}\tan\left[\frac{\omega_r}{2}\right] = \frac{2}{3}\tan\frac{\pi}{8} = 0.276\ sec.$$

Using Bilinear transformation

$$H(s) = H(s)\Big|_{S=\frac{2}{T}\left[\frac{z-1}{z+1}\right]}$$

$$H(s) = \frac{\frac{2}{T}\left[\frac{z-1}{z+1}\right]+0.1}{\left[\frac{2}{T}\left[\frac{z-1}{z+1}\right]+0.1\right]^2+9}$$

$$= \frac{\frac{\frac{2}{T}(z-1)+0.1(z+1)}{(z+1)}}{\frac{\left[\frac{2}{T}(z-1)+0.1+(z+1)\right]^2+9(z+1)^2}{(z+1)^2}}$$

$$= \frac{\frac{2}{T}(z-1)(z+1)+0.1(z+1)^2}{\left[\frac{2}{T}(z-1)+0.1+(z+1)\right]^2+9(z+1)^2}$$

Sub T = 0.276 sec.

$$= \frac{\frac{2}{0.276}(z^2-1)+0.1+(z^2+1+2z)}{\left[\frac{2}{0.276}(z-1)+0.1(z+1)\right]^2+9(z^2+1+2z)}$$

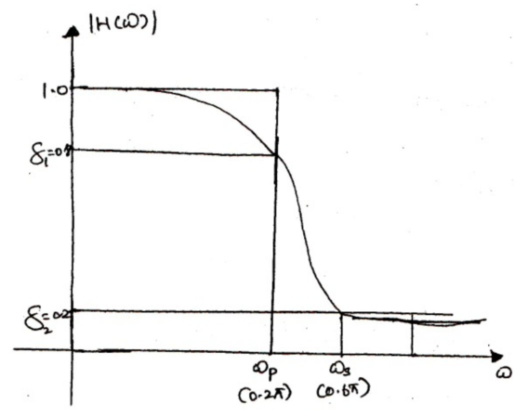
$$H(z) = \frac{z^2+0.0272-0.973}{(8.5722^2-11.84_z+8.177)}$$

4.Design a Butterworth filter using impulse invariant method for the following specifications.

$$\mathbf{0.8 \le \left|H(e^{j\omega})\right| \le 1; \quad 0 \le \omega \le 0.2\pi}$$

$$\mathbf{\left|H(e^{j\omega})\right| \le 0.2; \quad 0.6\pi \le \omega \le \pi}$$

Assume T = 1 sec. **[CO2-H3-May/June 2014]**



Soln:

$\omega_p = 0.2\pi, \qquad \omega_s = 0.6\pi, \qquad \delta_1 = 0.8, \qquad \delta_2 = 0.2$

Step 1:

Determine $\propto_p,\ \propto_s,\ \Omega_s,\ and\ \Omega_p$

$\propto_p = 20\log\delta_1 = -20\log 0.8 = 1.938\ dB$

$\propto_s = -20\log\delta_2 = -20\log 0.2 = 13.98\ dB$

Pass band edge frequency $\Omega_p = \frac{\omega_p}{T} = \frac{0.2\pi}{1} = 0.2\pi$

Stop band edge frequency $\Omega_s = \frac{\omega_p}{T} = 0.6\pi$

Step 2:

Determine the order of the filter.

$$N \ge \frac{log\left[\frac{10^{0.1\propto_s - 1}}{10^{0.1\propto_p - 1}}\right]}{2\log\left[\frac{\Omega_s}{\Omega_p}\right]}$$

$$N \ge 1.7$$

Here N = 2

Step : 3

The normalized transfer function for N = 2

$$H_N(s) = \frac{1}{s^2 + 1.414s + 1}$$

Step : 4

Determine the cutoff frequency

$$\Omega_c = \frac{\Omega_p}{\left(10^{0.1\propto_p} - 1\right)^{1/2N}}$$

$$= \frac{0.2\pi}{(10^{0.1933} - 1)^{1/4}}$$

$$\Omega_c = 0.725$$

Step: 5

Determine the transfer function of analog filter

H(s) = $H_N(s)|s \rightarrow \frac{s}{0.725}$

$$= \frac{1}{\left[\frac{s}{0.725}\right]^2 + 1.414\left[\frac{s}{0.725}\right] + 1}$$

H(s) = $\frac{0.525}{s^2 + 1.025s + 0.526}$

Step:6

$$\frac{0.526}{s^2+1.025s+0.526} = \frac{0.526}{s^2+1.025s+0.263-0.263+0.526}$$

$$= \frac{1.025 \times 0.513}{(s+0.512)^2 + (0.513)^2}$$

Apply $\frac{b}{(s+a)^2+b^2} \rightarrow \frac{e^{-aT} sinb\frac{T}{Z} - 1}{1-2ae^{-aT}\cos(0.513)+e^{-2}\times 0.512z^{-2}}$

$$H(z) = 1.025\left[\frac{e^{-0.512T} SIN(0.513)z^{-1}}{1-2e^{-0.512}\cos(0.513)+e^{-2}\times 0.512z^{-2}}\right]$$

$$= 1.025\left[\frac{0.294z^{-1}}{1-1.49z^{-1}+0.359z^{-2}}\right]$$

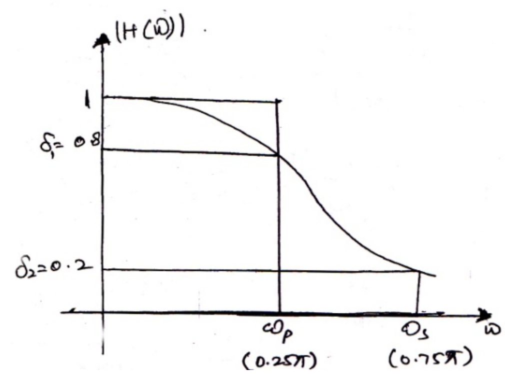
$$H(z) = \frac{0.3014z^{-1}}{1-1.0449z^{-1}+0.359z^{-2}}$$

5. Design a Butterworth filter using impulse invariant method for the following specifications.

$0.8 \leq |H(\omega)| \leq 1 \qquad 0 \leq \omega \leq 0.25\pi$

$|H(\omega)| \leq 0.2 \qquad 0.75\pi \leq \omega \leq \pi$

Soln: **[CO2-H3]**



Step:1

Determine $\propto_p, \propto_s, \Omega_p$ *and* Ω_s

$$\propto_p = -20\log\delta_1 = -20\log(0.8)$$

= 1.94

$$\propto_s = -20\log\delta_2$$

$$= -20\log(0.2) = 13.98$$

$$\Omega_p = \frac{\omega_p}{T} = 0.25\pi \; (\therefore T = 1s)$$

$$\Omega_s = \frac{\omega_s}{T} = 0.75\pi$$

Step 2:

Determine the order of the filter.

$$N \geq \frac{\log\left[\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}\right]}{2\log\left[\frac{\Omega_s}{\Omega_p}\right]}$$

$$N \geq \frac{\log\left[\frac{10^{0.1}\times 13.98-1}{10^{0.1}\times 1.99-1}\right]}{2\log\left[\frac{0.75\pi}{0.25\pi}\right]}$$

$$N \geq 1.71$$

The order of the filter N = 2

Step: 3

Determine the transfer function of the filter is N = 2

$$\therefore\ H_N(s) = \frac{1}{s^2+1.414s+1}$$

Step: 4

Determine the cut off frequency.

$$\Omega_c = \frac{\Omega_p}{\left(10^{0.1\alpha_p}-1\right)^{1/2N}} = \frac{0.25\pi}{(0.5631)^{1/4}}$$

= 0.907

Step: 5

Convert normalized transfer function into transfer function of an analog filter means s $\rightarrow \frac{s}{\Omega_c}$

$$H(s) = \frac{1}{\left[\frac{s}{0.907}\right]^2+1.414\left[\frac{s}{0.907}\right]+1}$$

$$= \frac{0.822}{s^2+1.282s+0.822}$$

Step:6

Using impulse invariant method

$$\frac{1}{S-P_K} \rightarrow \frac{1}{1-e^{P_K T}Z^{-1}}$$

$$= \frac{0.822}{(s+0.64-j0.64)(s+0.64+j0.64)}$$

$$= \frac{A}{(S+0.64-j0.64)} + \frac{B}{(S+0.64+j0.64)}$$

Using partial fraction method A = 0.64j & B = - 0.64j

$$H(z) = \frac{j0.64}{1-e^{-(0.64+j0.64)}Z^{-1}} - \frac{j0.64}{1-e^{-(0.64-j0.64)}Z^{-1}}$$

Note: Use Radian mode in the calculator.

$$= \frac{j0.64}{1-(\cos 0.64 - j\sin 0.64)0.527z^{-1}} - \frac{j0.64}{1-(\cos 0.64 + j\sin 0.64)0.527z^{-1}}$$

$$= \frac{j0.64(1-0.422z^{-1}-j0.315z^{-1})-0.64j(1-0.422z^{-1})+j0.315z^{-1}}{(1-0.422z^{-1})^2+(0.315z^{-1})^2}$$

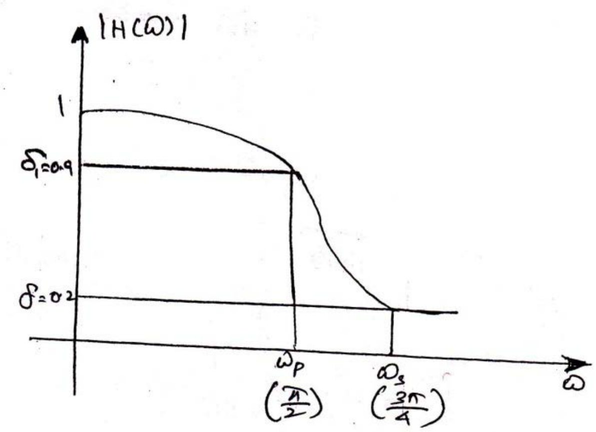
$$H(z) = \frac{0.4032z^{-1}}{1-0.844z^{-1}+0.277z^{-2}}$$

6.Design a digital Butterworth filter that satisfies the following constraints using bilinear transformation

$$\mathbf{0.9} \le |H(\omega)| \le \mathbf{1} \qquad \mathbf{0} \le \omega \le \frac{\pi}{2}$$

$$|H(\omega)| \le \mathbf{0.2}\ \frac{3\pi}{4} \le \omega \le \pi$$

Soln: **[CO2-H3-Nov/Dec 2010]**



Magnitude response

$\delta_1 = 0.9,\ \ \delta_2 = 0.2,\ \ \omega_p = \frac{\pi}{2}\ \ and\ \omega_s = \frac{3\pi}{4}$

Step:1

Determine of $\propto_p,\ \propto_s,\ \Omega_s\ \ and\ \Omega_p$

$$\propto_p = -20\log\delta_1 = -20\log 0.9$$

$$= 0.9515 \text{ dB}$$

$$\propto_s = -20\log\delta_2 = 13.979\ dB$$

$$\Omega_p = \frac{2}{T} = \tan\left[\frac{\omega_p}{T}\right] = 2$$

$$\Omega_s = \frac{2}{T} = \tan\left[\frac{\omega_p}{T}\right] = 4.828$$

Step:2

Determine the order of the filter

$$N \ge \frac{log\left[\frac{10^{0.1\propto_s-1}}{10^{0.1\propto_p-1}}\right]}{2\log\left[\frac{\Omega_s}{\Omega_p}\right]}$$

N = 3

Step:3

The normalized transfer function at N = 3 using table 3.1

Step:4

Determine the cut-off frequency

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\propto_p}-1)^{\frac{1}{2N}}} = \frac{2}{(10^{0.09151}-1)^{1/6}}$$

Step:5

Determine the transfer function of analog filter

$$H(s) = H_N(s)|_{s \to \frac{s}{2.5468}}$$

$$H(s) = \frac{1}{\left[\frac{s}{2.5468}+1\right]\left[\left(\frac{s}{2.5468}\right)^2+\left(\frac{s}{2.5468}\right)+1\right]}$$

$$= \frac{16.519}{(s+2.5468)(s^2+2.5468s+6.486)}$$

Step:6

Transforming H(s) into H(z) (T=1 sec) using bilinear transformation method.

$$H(s) = H_N(s)|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{16.519}{\left[2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+2.5468\right]\left[\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2+2.5468\times 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+6.436\right]}$$

$$= \frac{16.519(1+z^{-1})^3}{(2(1-z^{-1})+2.5468)[4(1+z^{-2}-2z^{-1})+5.0936(1-z^{-2})+6.436(1+z^{-2}+2z^{-1})]}$$

$$= \frac{16.519(1+z^{-1})^3}{70.83+31.12z^{-1}+27.24z^{-2}+2.945z^{-3}}$$

$$H(z) = \frac{0.233(1+z^{-1})^3}{1+0.439z^{-1}+0.384z^{-2}+0.042z^{-3}}$$

7.Design a digital Butterworth LPF filter with following specifications.

$0.8 \le |H(\omega)| \le 1 \quad 0 \le \omega \le 0.25\pi$

$|H(\omega)| \le 0.1 0.5\pi \le \omega \le \pi$ **[CO2-H3]**

Soln:

Step:1

Determine of $\propto_p, \propto_s, \Omega_p$ and Ω_s

$$\propto_p = -20\log\delta_1 = -20\log(0.8) = 1.94$$

$$\propto_s = -20\log\delta_2 = -20\log(0.1) = 20$$

$$\Omega_p = \frac{2}{T} = \tan\left[\frac{\omega_p}{2}\right] = \frac{2}{T} = \tan\left[\frac{0.25\pi}{2}\right] = 0.83$$

$$\Omega_s = \frac{2}{T} = \tan\left[\frac{\omega_s}{T}\right] = 2\tan\left[\frac{0.5\pi}{2}\right] = 2$$

Step:2

Determine the order of the filter

$$N \ge \frac{log\left[\frac{10^{0.1\propto_s-1}}{10^{0.1\propto_p-1}}\right]}{2\log\left[\frac{\Omega_s}{\Omega_p}\right]}$$

$$N \ge 2.94$$

N = 3

Step:3

Normalized transfer function for N = 3 using table 3.1

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

Step:4

The cut off frequency

$$\Omega_c = \frac{\Omega_p}{\left[10^{0.1\alpha_p}-1\right]^{1/2\times 1}}$$

$$= \frac{0.83}{(0.563)^{1/6}}$$

$$= 0.9134$$

Step:5

Convert normalized transfer function $H_N(s)$ into transfer function H(s) by substituting $s = \frac{s}{0.9134}$

$$H(s) = \frac{1}{\left[\frac{s}{0.9134}+1\right]\left[\left(\frac{s}{0.9134}\right)^2+\left(\frac{s}{0.9134}\right)+1\right]}$$

$$= \frac{0.7620}{(s+0.9134)(s^2+0.9134s+0.8342)}$$

Step:6

Convert H(s) to H(z) by substituting $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

$$H(z) = \frac{0.7620}{\left[2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+0.9134\right]\left[4\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2+0.9134\times 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+0.8342\right]}$$

$$= \frac{0.7620(1+z^{-1})^2}{(2.91-1.09z^{-1})(3z^{-2}+6.33z^{-1}+6.66)}$$

$$= \frac{0.7620(1+z^{-1})^2}{12.12+11.52z^{-1}+5.46z^{-2}}$$

$$= \frac{0.0628(1+z^{-1})^3}{1+0.95z^{-1}+0.45}$$

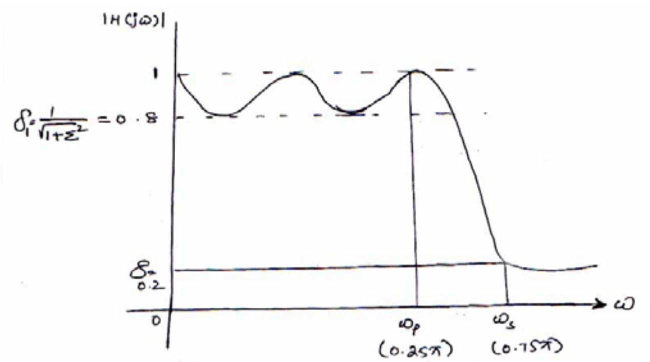
8. Design a chebyshev digital filter to satisfy the following constraints

$$0.8 \le |H(w)| \le 1 \qquad \text{for } 0 \le w \le 0.25\pi$$

$$\le |H(w)| \le 0.2 \qquad \text{for } 0.75\pi \le w \le \pi$$

Using impulse invariant method assume T = 1sec

[CO2-H3-May/June 2014]



Step 1:

Determination of $d_p, d_s, \ \Omega_p and\ \Omega_s$

$$d_p = -20 log\ \delta_1 = -20\ log\ 0.8$$

= 1.938dB

$$d_s = -20 log\ \delta_2 = -20\ log\ 0.2$$

= 13.979dB

Band edge frequencies

$$\Omega_p = \frac{w_p}{T} = 0.25\pi\, rad/sec$$

$$\Omega_s = \frac{w_p}{T} = 0.75\pi\, rad/sec$$

Step 2:

Determining the order of the filter

$$N \le \frac{\cosh^{-1}\sqrt{\frac{10^{0.1ds}-1}{10^{0.1ds}-1}}}{\cosh^{-1}\left[\frac{\Omega_s}{\Omega_p}\right]}$$

$$\le \frac{\cosh^{-1}\sqrt{\frac{23.99}{0.562}}}{\cosh^{-1}\left[\frac{0.75\pi}{0.25\pi}\right]}$$

$$N \le 1.433$$

Step 3:

Calculating ε, μ, a and b

$\varepsilon = \sqrt{10^{0.1ds} - 1}$ = 0.7499

$\mu = \varepsilon^{-1} + \sqrt{1+ \varepsilon^{-2}}$ = 3

a = $\Omega_p + \left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right]$ = 0.4534

b = $\Omega_p + \left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right]$ = 0.906

Pole locations $s_k = a_k \cos \phi_k + jb \sin \phi_k$

Where $\phi_k = (2k + N - 1)\frac{\pi}{2N}$

N = 2

$s_1 = 0.4534 \cos\frac{3\pi}{4} + j0.906 \sin\frac{3\pi}{4}$

$= -0.3206 + j0.6406$

$s_2 = 0.4534 \cos\frac{3\pi}{4} + j0.906 \sin\frac{3\pi}{4}$

$= -0.3206 + j0.6406$

Step 4:

Driving the transfer function H (s) of an log filter

$$H\ (s) = \frac{1}{(s-s_1)\ (s-s_2)} = \frac{1}{s^2 + 0.6412s + 0.5131}$$

Since N is even,

$$\boldsymbol{k} = \frac{0.5132}{\sqrt{1+\varepsilon^2}} = 0.4105$$

Step 5:

Convert H (s) to H (z)

$$H\ (s) = \frac{0.4105}{s^2 + 0.6412s + 0.5131}$$

Applying partial fraction method

$$= \frac{A}{s + 0.3206 + j0.6406} + \frac{B}{s + 0.3206 - j0.6406}$$

A = $j0.32\ B = A* = -j0.32$

$$= \frac{j0.32}{s + 0.3206 + j0.6406} + \frac{-j0.32}{s + 0.3206 - j0.6406}$$

Applying transformation

$$\frac{1}{s - p_k} \rightarrow \frac{1}{1 - e^{PkT}\ z^{-1}}$$

$$H\,(z) = \frac{j0.32}{1-\,e^{-0.3206-j0.6406}\,z^{-1}} - \frac{j0.32}{1-\,e^{-0.3206-j0.6406}\,z^{-1}}$$

(Note: Use radian mode for calculation)

$$= \frac{j0.32}{1-0.581\;e^{-1}+j0.433\,z^{-1}} - \frac{j0.32}{1-\,0.581\,e^{-1}-j0.433\,z^{-1}}$$

$$= \frac{j0.32\,(1-0.581z^{-1}-j0.433z^{-1}) - j0.52(1-0.581z^{-1}+j0.433z^{-1})}{1-\,0.581z^{1})^2+(0.433)^2}$$

$$H\,(z) = \frac{0.277z^{-1}}{1-\,1.163\,z^{-1}-\,0.527\,z^{-2}}$$

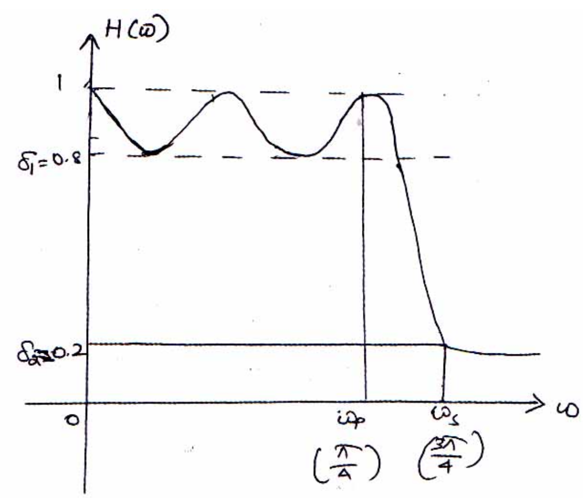
9.Design a chebysher digital filter for the following constraints

$$0.75 \;\le\; |H\,(w)| \;\le 1 \qquad 0 \;\le\; w \;\le \frac{\pi}{4}$$

$$|H\,(w)| \;\le 0.2 \qquad \frac{3\pi}{4} \le\; w \;\le \pi$$

Apply impulse invariant techniques (Assume T = 1 sec) [CO2-H3]

Soln:



Step 1:

Specifying $d_p, d_s,\; \Omega_p and\; \Omega_s$

$$d_p\; = -20log\;\delta_1 = -20\;log\;(0.75)$$

= 2.5

$$d_s\; = -20log\;\delta_2 = -20\;log\;0.2$$

= 13.98

$$\Omega_p\; = \frac{2}{T} = tan\;\left(\frac{w_p}{2}\right) = 2\tan\left[\frac{\pi}{2}\right] = 0.83$$

$$\Omega_s = \frac{2}{T} = tan\left[\frac{w_s}{2}\right] = \frac{2}{T}\tan\left[\frac{3\pi}{8}\right] = 4.83$$

Step 2:

Determine the order of the filter (N)

$$N \le \frac{\cosh^{-1}\sqrt{\frac{10^{0.1ds}-1}{10^{0.1ds}-1}}}{\cosh^{-1}\left[\frac{\Omega_s}{\Omega_p}\right]}$$

$$N \le \frac{\cosh^{-1}\sqrt{\frac{24}{0.78}}}{\cosh^{-1}(5.81)}$$

$$N \le 0.97$$

Step 3:

Calculating ε, μ, a and b

$$\varepsilon = \sqrt{10^{0.1ds} - 1} = \sqrt{0.78} = 0.833$$

$$\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}} = 2.65$$

$$a = \Omega_p + \left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right] = 0.94$$

$$b = \Omega_p + \left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right]$$

$$= 1.26$$

$$\phi_k = (2k + N - 1)\frac{\pi}{2N}$$

N = 2

$$\phi_1 = \pi$$

$$s_k = a\ cos\ \phi_k + b\ sin\ \phi_k$$

$$s_1 = -0.94$$

Step 4:

$$H(s) = \frac{k}{(s+0.94)}$$

Since N is odd, substitute s = 0 in denominator polynomial and numerator k is determined

$$H(s) = \frac{k}{0.94} \quad \therefore k = 0.94$$

$$H(s) = \frac{0.94}{(s+0.94)}$$

Step 5:

Convert H (s) to H (z)

$$\frac{1}{s-p_k} \rightarrow \frac{1}{1-e^{PkT}\, z^{-1}}$$

$$\frac{0.94}{s+0.94} \rightarrow \frac{0.94}{1-e^{-0.94}\, z^{-1}}$$

$$H\,(z) = \frac{0.94}{1-\,0.39\, z^{-1}}$$

10. Design a digital chebysher LPF to satisfy the following constraints

$$0.8 \le |H\,(w)| \le 1 \quad \text{for} \quad 0 \le w \le 0.25\pi$$

$$|H\,(w)| \le 0.2 \quad \text{for} \quad 0.75\pi \le w \le \pi$$

Using bilinear transformation. Assume T = 1 sec [CO2-H3]

Soln:

Step 1:

Determination of $d_p, d_s,\ \Omega_p and\ \Omega_s$

$$d_p = -20log\ 0.8 = 1.938dB$$

$$d_s = -20log\ 0.2 = 13.979dB$$

The band edge frequencies

$$\Omega_p = \frac{2}{T} = tan\left(\frac{0.25\pi}{2}\right) = 0.8284$$

$$\Omega_s = \frac{2}{T} = tan\left[\frac{0.75\pi}{2}\right] = 4.8284$$

Step 2:

Determine the order of the filter (N)

$$N \le \frac{\cosh^{-1}\sqrt{\frac{10^{0.1ds}-1}{10^{0.1ds}-1}}}{\cosh^{-1}\left[\frac{\Omega_s}{\Omega_p}\right]}$$

$$N \le 1.66$$

Step 3:

Calculatingε, μ, a and b and binding the pole locations

$$\varepsilon = \sqrt{10^{0.1ds} - 1} = 0.749$$

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 3$$

$$a = \Omega_p + \left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right] = 0.478$$

$$b = \Omega_p + \left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right] = 0.957$$

The pole locations $s_k = a\cos\phi_k + jb\sin\phi_k$

Where

$$\phi_k = (2k + N - 1)\frac{\pi}{2N}$$

$$s_1 = 0.478\cos\frac{3\pi}{4} + j0.957\sin\frac{3\pi}{4} = -0.338 + j0.677$$

$$s_2 = 0.478\cos\frac{5\pi}{4} + j0.957\sin\frac{5\pi}{4} = -0.338 + j0.677$$

Step 4:

Deriving the analog transfer function H (s)

$$H(s) = \frac{k}{(s - s_1)(s - s_2)}$$

$$= \frac{k}{(s + 0.338 - j0.697)(s + 0.338 + j0.677)}$$

$$H(s) = \frac{k}{s^2 + 0.67605 + 0.572}$$

Since N is even, the value of k is determined by

$$K = \frac{0.572}{\sqrt{1 + \varepsilon^{-2}}} = 0.4576$$

Analog transfer function H (s) = $\frac{0.4576}{s^2 + 0.67605 + 0.572}$

Step 5:

Convert H (s) to H (z)

$$s \rightarrow \frac{2}{T}\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right]$$ using bilinear transformation

$$H(z) = \frac{0.4576}{\frac{4(1 - z^{-1})^2}{(1 + z^{-1})^2} + 0.672\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right] + 0.572}$$

$$H(z) = \frac{0.072\,(1 - z^{-1})^2}{1 - 1.157z^{-1} + 0.544z^{-2}}$$

11. Design a digital chebycher LPF with following specification

$$0.75 \le |H(w)| \le 1 \quad \text{for } 0 \le w \le \frac{\pi}{4}$$

$$|H(w)| \le 0.2 \quad \text{for } \frac{3\pi}{4} \le w \le \pi$$ **[CO2-H3]**

Soln:

Step 1:

Determination of d_p, d_s $in\ dBs$

$$d_p = -20 log\ \delta_1 = -20\ log\ 0.75$$

= 2.5

$$d_s = -20 log\ \delta_2 = -20\ log\ (0.2)$$

= 13.98

$$\Omega_p = \frac{2}{T} = tan\ \frac{w_p}{2} = 2 \tan\left[\frac{\pi}{8}\right] = 0.83$$

$$\Omega_s = \frac{2}{T} = tan\ \frac{w_s}{2} = 2 \tan \frac{3\pi}{8} = 4.83$$

Step 2:

Determine the order of the filter

$$N \le \frac{\cosh^{-1}\sqrt{\frac{10^{0.1ds}-1}{10^{0.1ds}-1}}}{\cosh^{-1}\left[\frac{\Omega s}{\Omega p}\right]}$$

$$N \le 0.97$$

St***ep 3:***

Determining ε, μ, a and b. find the pole location s_k

ε = $\sqrt{10^{0.1ds} - 1}$ = 0.883

μ = $\varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}}$ = 2.65

a = $\Omega_p + \left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right]$ = 0.94

b = $\Omega_p + \left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right]$

= 1.26

$$\phi_k = \frac{\pi}{2N}\ (2k + N - 1)$$

N = 1

$$\phi_1 = \pi$$

$$s_k = a\ cos\ \phi_k + jb\ sin\ \phi_k$$

$$s_1 = -0.94$$

Step 4:

Determining the transfer function H (s)

$$H\ (s) = \frac{k}{(s+0.94)}$$

Since N is odd, sub k = 0 in the denominator polynomial and numerator k is determined

$$H\ (s) = \frac{k}{0.94}\ \therefore\ k = 0.94$$

$$H\ (s) = \frac{0.94}{s+0.94}$$

Step 5:

$$H\ (z) = H\,(s)|s \rightarrow \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

$$H\ (z) = \frac{0.94}{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 0.94}$$

$$= \frac{0.94}{2\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 0.94}$$

$$= \frac{0.94\ (1-z^{-1})}{2\ (1-z^{-1}) + 0.94\ (1-z^{-1})}$$

$$H\ (z) = \frac{0.32\ (1-z^{-1})}{1-0.36\ z^{-1}}$$

12. Draw the direct form – I realization for the given difference equation

$$\boldsymbol{y\ (n) + \ 6y\ (n-1) + 7y(n-2) = \ x\ (n) + \ 3x\ (n-1) + \ 4x\ (n-2)}$$

Soln: [CO3-H1-Nov/Dec 2011]

$$y\ (n) + \ 6y\ (n-1) + 7y(n-2) = \ x\ (n) + \ 3x\ (n-1) + \ 4x\ (n-2)$$

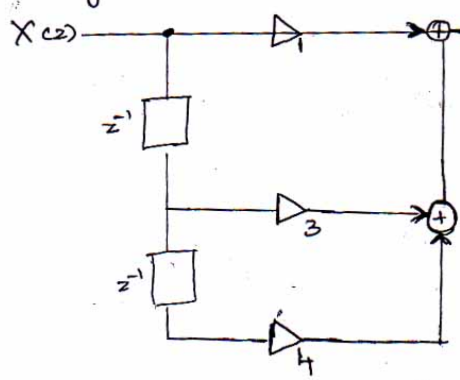
Taking z – transform on both sides and assuming initial conditions are zero

$$y\ (n) + \ 6z^{-1}y\ (z) + 7z^{-2}y(z) = \ x\ (z) + \ 3z^{-1}x\ (z) + \ 4z^{-2}x\ (z)$$

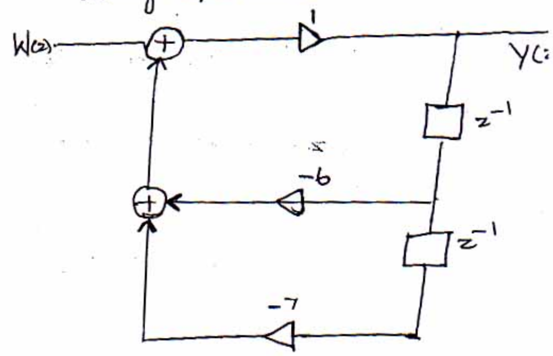
Where $w\ (z) = x\ (z) + \ 3z^{-1}x\ (z) + \ 4z^{-2}x\ (z)$ → (a)

$$y\ (z) + \ 6z^{-1}y\ (z) + 7z^{-2}y(z) = \ x\ (z)$$

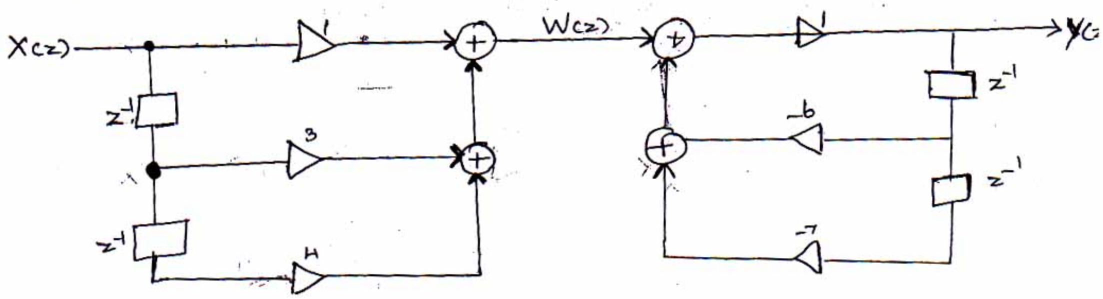
$y\ (z) = w\ (z) - \ 6z^{-1}y\ (z) - \ 7z^{-2}y\ (z)$ → (b) using equation (a)



using equation (b)



Combined the above two diagrams, we get direct form I realization



Unit – III

FIR Filter Design

Part – A

1. What is the basic difference between cascade form and direct form structures for FIR systems? [CO3-L2]
Basis occurs in the usage of memory space in bats coses. Cascade form is basically in need of series memory. No of memory space required less in case of direct-2 form of FIR w.r.t. cascade form start use of FIR systems.

2. Which is more sensitive network to finite word length?
(a) Direct form-II (b) Cascade form Justify your answer. [CO3-L1]
The direct form II realization requires only the layer of M or N storage elements. When compared to direct form I realisation the direct form II uses minimum number of storage elements and hence said to be a Canonic striictur JJrweves wJ' the Jc is performed sequentially, the direct form II needs two adders instead of one adder required for the direct form I.

Though the direct form I and II are commonly employed, they have two drawbacks viz (i) they lock hardware flexibility and (ii) due to finite precision arithmetic, the sensitivity of the co-efficients to quantisation effects increases with the order of the filter. This sensitivity may change the co-efficient values and hence the frequency response, thereby causing the filter to become unstable. To overcome these effects, the cascade and parallel realizations can be implemented.

3. Compare different form structures of filter realization from the point of view of speed and memory requirement. [CO3-L2]
The structural representation provides the relations between some pertinent internal variable with the input and output that in turn provide the keys to implementations. There are various form of structural representations of a digital filter.In digital implementations, the delay operation can be implemented by providing stronger register for each unit delay that is required. In case of direct I form structure realization separate delay for both input and output signal samples. So more memory is utilized by this form. In case of direct-II form structure realization only one delay is required for both input and output signal samples. Therefore it is more efficient in term of memory requirements.

4**.What is the importance of Windowing? [CO3-L1]**
1. The infinite duration impulse response can be converted to a finite duration impulse response by trucating the infinite series at $n = \pm N$. But this results in undesirable oscillations in the pass-band and step-band of the digital filter. This is due to slow convergence of the Fourier series near the point of discontinuity. These undesirable oscillations can be reduced by using a set of time limited weighing functions z e referred as windowing function.
2 The windowmg function consists of main lobe which contains most of the energy of window function and side lobes which decay rapidly
3 A major effect of windowing is that the discontinuities is $H(e^{j\omega})$are converted into transition bands between values on either side of the discontinuity
4 Window function have side lobes that decrease in energy rapidly as tends to π

5. What will happen if length of windows is increased in design of FIR filters? [CO3-L1]
If length of window is increased in design of FIR filter more coefficients need to calculated
A more memory space used for it. More lengths of window means more accuracy in transation process.

6. What are the essential features of a good window for FIR filters? [CO3-L1]
Features of a good window for F/R filters:
1. Side lobe level should be small.
2. Broaden middle section.
3. Attenuation should be more.
4. Smoother magnitude response.
5. The trade off between main lobe widths and side lobe level can be adjusted.
6. Smoother ends.
7. If cosine term is used then side lobes are duced further.

7. Define Ripple ratio[CO3-L1]
The Ripple ratio is defined as , the ratio of maximum sidelobes amplitude to the mainlobe amplitude. i.e. %RR=(maximum side lobe amplitude/main lobe amplitude)x100

8.What is Gibb's Oscillation? (or) State the effect of having abrupt discontinuity in frequency response of FIR filters. [CO3-L1]
The truncation of Fourier series is known to introduce the unwanted ripples in the frequency response characteristics H(w) due to non uniform convergence of Fourier series at a discontinuity These ripples or oscillatory behaviour near the band edge of the filter is known as "Gibb's phenomenon or Gibb's oscillation ".

9.What are the methods used to reduce Gibb's phenomenon? [CO3-L1]
There are two methods to reduce Gibb's phenomenon
1.The discontinuity between pass band and stop band in the frequency response is avoided by introducing the transition between the pass band and stop band.
2.Another technique used for the reduction of Gibb's phenomenon is by using window function that contains a taper which decays towards zero gradually instead abruptly.

10.What are FIR filters? [CO3-L1]
The specifications of the desired filter will be given in terms of ideal frequecy response $H_d(\omega)$.The impulse response $h_d(n)$ of desired filter can be obtained by inverse fourier transform of $H_d(\omega)$ which consists of infinite samples.The filters designed by selecting finite no of samples of impulse response are called FIR filters.

11. What are the disadvantages of FIR filter? [CO3-L1]
The duration of impulse response should be large to realize sharp cut off filters.
The non-integral delay can lead to problems in some signal processing applications.

12. What is the necessary and sufficient conditions for linear phase characteristics of a FIR filter? [CO3-L1-May/June 2008]
The necessary and sufficient conditions for linear phase characteristics of a FIR filter is that the phase function should be a linear function of ω,which in turn requires constant phase delay or constant phase and group delay.

13.What are thepossible types of impulse response for linear phase FIR filter? [CO3-L1]
i.Symmetric impulse response when N is odd
ii. Symmetric impulse response when N is even
iii. Antisymmetric impulse response when N is odd
iv. Antisymmetric impulse response when N is even.

14.List well known design techniques for linear phase FIR filter? [CO3-L1]
i.Fourier series method and window method. ii.Freq sampling method iii.Optimal filter design methods.

15.List the factors that are to be specified in the filter design problem. [CO3-L1]
i.The maximum tolerable passband ripple. ii.The max tolerable stopband ripple.
iii.The passband edge freq ωp iv.The stopband edge freq ωs.

16.What are the conditions that are to be satisfied for const phase delay in linear phase FIR filter? [CO3-L1]
The conditions for const phase delay are,
Phase delay, $\alpha = (N-1)/2$ (i.e phase delay is const)
Impulse response $h(n)=h(N-1-n)$ (i.e. impulse response is symmetric).

17.Characteristic features of rectangular window. [CO3-L1]
i.The mainlobe width is equal to $4\pi/N$.
ii.The max sidelobe magnitude is -13 dB.
iii.The sidelobe magnitude does not decreases significantly with increasing ω.

18.List features of hanning window spectrum. [CO3-L1]
i.The mainlobe width is equal to $8\pi/N$.
ii.The max sidelobe magnitude is -31dB.
iii.The sidelobe magnitude decreases with increasing ω.

19. List features of hamming window spectrum. [CO3-L1]
i.The mainlobe width is equal to $8\pi/N$.
ii.The max sidelobe magnitude is -41dB.
iii.The sidelobe magnitude remains constant for increasing ω.

20.What are the advantages of Kaiser window? [CO3-L1]
1.It provides flexibility for the designer to select side lobe level and N
2.It has the attractive property that the side level can be varied continuously from the value in the Blackman window to the high value in the rectangular window.

Part – B

1. A low pass filter is required to be designed with the desired frequency response.

$$H_d(w) = \begin{cases} e^{-j2w} & , \quad -0.25\pi < w < 0.25\pi \\ 0 & , \quad 0.25\pi < w < \pi \end{cases}$$

$$w|n| = \begin{cases} 1 & , \quad 0 \le n \le 4 \\ 0 & , \quad Otherious\ M = 5 \end{cases}$$

Step 1: Draw the graph **[CO3-H3-Nov/Dec 2015]**

Step 2: Find hd (n)

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d\left(e^{jw}\right)e^{jw}.dw$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{-j2w}.e^{jwn}.dw$$

$$= \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{jw(n-2)}.dw$$

$$= \frac{1}{\pi(n-2)}\left[\frac{e^{j(n-2)/4} - e^{j(n-2)/4}}{2j}\right]$$

$$= \frac{1}{\pi(n-2)}\ sin\frac{\pi}{4}(n-2) \text{ for } n \neq 2$$

$$h_d(2) = \frac{1}{4}$$

$$h_d(0) = \frac{1}{2\pi} = h_d(4)$$

$$h_d(1) = \frac{1}{\sqrt{2\pi}} = h_d(3)$$

$$h(n) = h_d(n).w(n)$$

Step 3: Find h (n)

$$h(n) = h_d(n).W_R(n)$$

$$h(o) = \frac{1}{2\pi} = h(4)$$

$$h(1) = \frac{1}{\sqrt{2\pi}} = h_d(3) \text{ and}$$

$$h(o) = \frac{1}{2\pi},\ h(1) = \frac{1}{\sqrt{2\pi}},\ h(2) = \frac{1}{4},\ h(3) = \frac{1}{\sqrt{2\pi}},\ h(4) = \frac{1}{2\pi}$$

Step 4: Find H (z)

$$H(z) = \sum_{n=0}^{4} h(n)z^{-n} = \frac{1}{2\pi}z^0 + \frac{1}{\sqrt{2\pi}}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{\sqrt{2\pi}}z^{-3} + \frac{1}{2\pi}z^{-4}$$

$$H(z) = \frac{1}{2\pi}(1 + z^{-4}) + \frac{1}{\sqrt{2\pi}},(z^{-1} + z^{-3}) + \frac{1}{4}z^{-2}$$

Step 5: Find magnitude of H (w)

M = N = 5

$|H\ (w)| = h\left[\frac{5-1}{2}\right] + 2\sum_{n=0}^{\frac{5-3}{2}} h\ (n).\cos w\left[n - \frac{5-1}{2}\right]$

$= h\ (2) + 2\sum_{n=0}^{1} h\ (n).\cos w\ (n-2)$

$= h\ (2) + 2\ [h\ (0).\cos w\ (0-2) + h\ (1).\cos w\ (1-2)]$

$|H\ (w)| = \frac{1}{4} + 2\left[\frac{1}{2\pi}\cos(-2w) + \frac{1}{\sqrt{2\pi}}\cos w\ (-1)\right]$

Find H (w) at w = 0, 180, 360 and plot the graph

2. The desired response of a low pass filter is.

$$H_d|e^{jw}|\begin{cases} e^{jw}, & -3\frac{3\pi}{4} \le w \le 3\frac{\pi}{4} \\ 0\ , & 3\frac{3\pi}{4} \le w \le \pi \end{cases}$$ **[CO3-H3]**

Determine the frequency response of the filter for M = 7 using a hamming condor

Step 1: Computer $h_d\ (n)$

In dPF $h_d\ (n) = \frac{\sin w\, t\ (n-d)}{\pi\,(n-d)}$ for n ≠ d

$h_d\ (n) = \frac{w_c}{\pi}$ for n = d

d = $\frac{M-1}{2} = \frac{7-1}{2} = \frac{6}{2} = 3$

$h_d\ (0) = h_d\ (6) = \frac{\sin\frac{3\pi}{4}(-3)}{\pi\,(-3)} = \frac{1}{3\sqrt{2\pi}}$ = 0.075

$h_d\ (1) = h_d\ (5) = \frac{\sin\frac{3\pi}{4}(-2)}{\pi\,(-2)} = \frac{-1}{\sqrt{2\pi}}$ = -0.159

$h_d\ (2) = h_d\ (4) = \frac{\sin\frac{3\pi}{4}(-1)}{\pi\,(-1)} = \frac{1}{\sqrt{2\pi}}$ = 0.225

$h_d\ (3) = \frac{w_c}{\pi} = \frac{3}{4} = 0.75$

$h_d\ (n) = h_d\ (n).w\ (n)$

Step 2:

$$W_{H_m}\ (n)\begin{cases} 0.54 + 0.46\cos\frac{2n\pi}{N-1} & for\ 0 \le n \le N-1 \\ 0 & Otherwise \end{cases}$$

n = 0 to 6

W (0) = W (6) = 1

W (1) = W (5) = 0.77

W (2) = W (4) = 0.31

W (3) = 0.08

Step 3:

FIR filter coefficients are given by using

$$h\ (n) = h_d\ (n)\ W\ (n)$$

$$h\ (0) = h\ (6) = h_d\ (0)\ W\ (0) = 0.075$$

$$h\ (1) = h\ (5) = h_d\ (1)\ W\ (1) = -0.12243$$

$$h\ (2) = h\ (4) = h_d\ (2)\ W\ (2) = 0.06975$$

$$h\ (3) = h_d\ (3)\,.W\ (3) = 0.06$$

FIR filter coefficients are

$h\ (n)$ = {0.075, -0.12243, 0.06975, 0.06, 0.075, -0.12243, 0.06975}

3. Design an ideal differentiator with frequency response
$H_d|e^{jw}| = jw \qquad -\pi \leq w \leq \pi$ **[CO3-H3-May/June 2013]**

Using rectangular and hamming window with N = 7

Step 1: Draw the graph

From the graph, it is shown that, this is anti symmetric filter

Step 2:

Find $h_d\ (n)$

$$h_d\ (n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d\left(e^{jw}\right) e^{jw}\ .w$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(e^{jw}\right) e^{jw}\ .w$$

$$= \frac{j}{2\pi} \int_{-\pi}^{\pi} we^{jwn}\ .dw$$

Step 3: Find H (z)

Transfer function $= \sum_{n=0}^{\frac{M-3}{2}} h\ (n).\cos w \left[n - \frac{M-1}{2}\right]$

Step 4: Magnitude of H (w)

$$)| = h\left[\frac{M-1}{2}\right] + 2\sum_{n=0}^{\frac{M-3}{2}} h\ (n).\cos w \left[n - \frac{M-1}{2}\right]$$

Find

$$|H(w)| = h(3) + 2\sum_{n=0}^{2} h(n).\cos w(n-3)$$

= 0.06 + 2 [0.075 cos w (-3) + (-0.12243 cos w (-2)
+ 0.06975 cos w (-1)]

$$)| = 0.06 + 0.15\cos w(-3) - 0.245\, w(-2) + 0.1395\cos(-w)$$

$$= \frac{j}{2\pi}\left[\left[\frac{we^{jwn}}{jn}\right]_{-\pi}^{\pi} - \frac{1}{n}\int_{-\pi}^{\pi} e^{jwn}.dw\right]$$

$$= \frac{j}{2\pi}\left[\frac{we^{jwn}}{jn}\right]_{-\pi}^{\pi} - \frac{1}{n}\left[\frac{we^{jwn}}{jn}\right]_{-\pi}^{\pi}$$

$$= \frac{j}{2\pi}\left[\frac{\pi e^{j\pi n}}{jn} - \left[\frac{-\pi e^{-j\pi n}}{jn}\right] - \frac{1}{n}\left[\frac{e^{j\pi n}}{jn} - \frac{e^{-j\pi n}}{jn}\right]\right]$$

$$= \frac{j}{2\pi}\left[\frac{\pi e^{-j\pi n} + \pi e^{-j\pi n}}{jn} - \frac{e^{j\pi n}}{jn^2} + \frac{e^{-j\pi n}}{jn^2}\right]$$

$$= \frac{j}{2\pi n}\left[\pi e^{j\pi n} + \pi e^{-j\pi n} + \frac{\pi e^{-j\pi n} - e^{j\pi n}}{n}\right]$$

$$= \frac{j}{2\pi n}\left[\pi e^{j\pi n} + \pi e^{-j\pi n}\right] - \frac{1}{2\pi n^2}\left[e^{j\pi n} - e^{-j\pi n}\right]$$

$$= \frac{1}{n}\cos\pi n - \frac{j}{\pi n^2}\sin n\pi$$

$$) = \frac{\cos\pi n}{n}$$

$$) = 0\ for\ n = 0$$

This is anti symmetric filter, so

$$h(n) = -h(N-1-n)$$

(Or)

$$h(n) = -h(-n)$$

So,

$$h_d(1) = -1 => -h(-1) = 1$$

$$h_d(2) = 0.5 => -h(-2) = -0.5$$

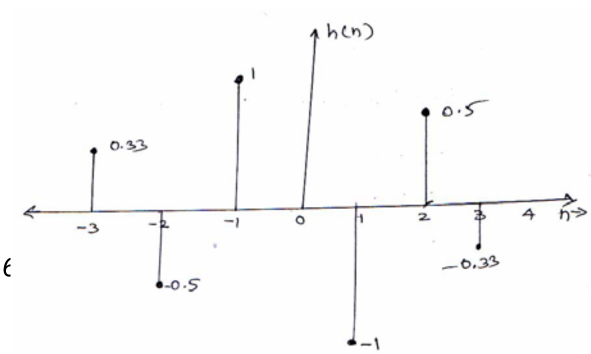
$$h_d(3) = -0.33 => -h(-3) = 0.33$$

Step 5: Find W_R (n)

Rectangular window:

$$W_R(n) = \begin{cases} 1 & -3 < n < 3 \\ 0 & Elsewhere \end{cases}$$

Here, rectangular window is used



Step 6: Find h (n)

$$h\ (n) = h_d\ (n) \times W_R\ (n)$$
$$h\ (0) = 0$$
$$h\ (1) = -1$$
$$h\ (2) = 0.5$$
$$h\ (3) = -0.33$$

Step 7: Find H (z)

$$H\ (z) = \sum_{n=3}^{3} h\ (n) z^{-n}$$
$$= \sum_{n=3}^{3} h\ (n) z^{-n}$$
$$= h\ (3)\ z^{3} + h\ (-2)\ z^{2} + h\ (-1)\ z^{1} + h\ (0) + h\ (1)\ z^{-1} + h\ (2)\ z^{-2} + h\ (3)\ z^{-3}$$
$$H\ (z) = 0.33\ z^{3} + 0.5\ z^{2} + z + 0 + z^{1}\ (-1) + 0.5\ z^{-2} - 0.3\ z^{-3}$$
$$H\ (z) = 0.3\ (z^{3} - z^{-3}) + 0.5\ (z^{-2} - z^{-2}) + (z - z^{-1})$$

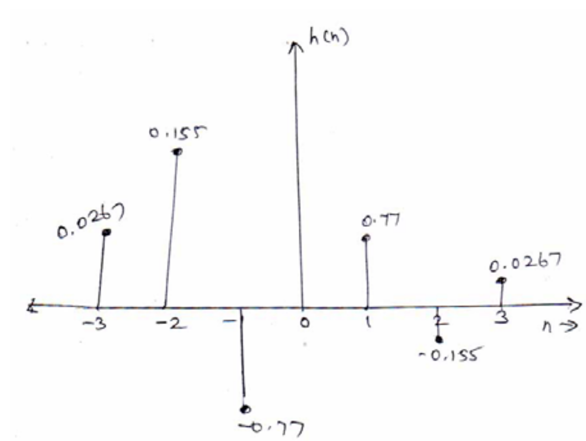
Case 1:

If hamming window is mused

$$W_H\ (n) = 0.54 + 0.46 \cos\frac{2\pi n}{N-1}\ for - \left[\frac{N-1}{2}\right] \leq n \leq \frac{N-1}{2}$$

This is anti symmetric filter, so, find W_H (0), W_H (1), W_H (2), W_H (-1), W_H (-2)

$$W_H\ (0) = 1$$
$$W_H\ (1) = 0.77 = W_H\ (-1)$$
$$W_H\ (2) = 0.31 = W_H\ (-2)$$
$$W_H\ (-3) = 0.08 = W_H\ (3)$$



Find h (n):

$$h\ (n) = h_d\ (n) \times W_H\ (n) - 3 < n < 3$$
$$h\ (0) = h_d\ (0) \times W_H\ (0) = 0$$
$$h\ (1) = -h\ (-1) = 0.77$$
$$h\ (2) = -0.155$$
$$h\ (3) = 0.0267$$

4. Design band reject filter whose specification is given below

$$H_d\left|e^{jw}\right| = \begin{cases} e^{-jdw} & 0 \le |w| \le wc_1, wc_2 \le w \le \pi \\ 0 & Otherwise \end{cases}$$

Step 1: Draw graph **[CO3-H3-Nov/Dec 2014]**

Given values wc_1 = 1 rad/sec

wc_2 = 2 rad/sec

This is symmetric filter

Step 2: Find d

N = 11

$d = \frac{11-1}{2}$

= 5

Step 3: Find $h_d\ (n)$

$h_d\ (n)$ =

$$\frac{1}{2\pi}\int_{-\pi}^{-wc_2} e^{j(n-d)w}.dw + \int_{-wc_1}^{wc_1} e^{j(n-d)w}.dw + \int_{wc_2}^{\pi} e^{j(n-d)w}.dw$$

$$h_d\ (n) = 1 - \left[\frac{wc_2 - wc_1}{\pi}\right]\ for\ n = d$$

$$h_d\ (n) = \frac{1}{\pi(n-d)}\left[\sin wc_1(n-d) - \sin wc_2(n-d) + \sin(n-d)\pi\right]$$ for n $\ne$ d

Put n = 0

$$h_d\ (0) = \frac{1}{\pi(-5)}\left[\sin 1\ (-5) - \sin 2\ (-5) + \sin(-5)\pi\right]$$

$h_d\ (0) = \frac{0.4149}{-5\pi}$ = -0.0264

=>) = -0.0264

Put n = 1,

$$h_d\ (1) = \frac{1}{\pi(1-5)}\left[\sin 1\ (1-5) - \sin 2\ (1-5) + \sin(1-5)\pi\right]$$

$h_d\ (1) = \frac{1.74616}{\pi(-4)}$ = -0.1389

=>) = -0.1389

Put n = 2,

$$h_d\ (2) = \frac{1}{\pi(2-5)}\left[\sin 1\ (2-5) - \sin 2\ (2-5) + \sin(2-5)\pi\right]$$

$h_d\ (2) = \frac{-0.42053}{\pi(-3)}$

=>) = -0.04462

\ Put n = 3,

$$h_d\ (3) = \frac{1}{\pi(3-5)}\left[\sin 1\ (3-5) - \sin 2\ (3-5) + \sin(3-5)\pi\right]$$

$h_d\ (3) = \frac{-1.666}{\pi(-2)}$

=>) = -0.26517

Put n = 4,

$$h_d\ (4) = \frac{1}{\pi(4-5)}\ [\sin 1\ (4-5) - \sin 2\ (4-5) + \sin(4-5)\pi]$$

$$h_d\ (4) = \frac{0.0678}{-\pi}$$

=>) = -0.02158

Put n = 5, Here n = d

So, use formula

$$h_d\ (n) = 1 - \left[\frac{wc_2 - wc_1}{\pi}\right]$$

$$h_d\ (5) = 1 - \left[\frac{2-1}{\pi}\right] = 1 - \frac{1}{\pi}$$

$$h_d\ (5) = \frac{\pi-1}{\pi} = 0.68169$$

=>) = -0.02158

=> $h_d\ (6) = \ h_d\ (4)$

$$[h_d\ (n) = \ h_d\ (N-1-n)h_d\ (n) = \ h_d\ (11-1-4)h_d\ (4) = h_d\ (6)]$$

) = -0.02158

) = $h_d\ (3)$ = -0.26517

) = $h_d\ (2)$ = 0.04462

) = $h_d\ (1)$ = -0.1389

)) = $h_d\ (0)$ = -0.0264

Step 4:

$$h\ (n) = h_d\ (n) \text{ x } W_R\ (n)$$

$$h\ (0) = h_d\ (0)$$

$$h\ (10) = h_d\ (10)$$

$$h\ (2) = -0.155$$

$$h\ (3) = 0.0267$$

FIR filter coefficients are

$$h(0) = 0.0264$$
$$h(1) = 0.1389$$
$$h(2) = 0.04462$$
$$h(3) = 0.26517$$
$$h(4) = 0.02158$$
$$h(5) = 0.68169$$
$$h(6) = -0.02158$$
$$h(7) = 0.26517$$
$$h(8) = 0.04462$$
$$h(9) = -0.1389$$
$$h(10) = 0.0264$$

Step 5:

Write magnitude of H (w)

Find $|H(w)|$

$$|H(w)| = h\left[\frac{N-1}{2}\right] + 2\sum_{n=0}^{\frac{M-3}{2}} h(n).\cos w\left[N - \frac{N-1}{2}\right]$$

$$= h(5) + 2\sum_{n=0}^{4} h(n).\cos w\,(n-5)$$

$$)| = 0.68169 + 2\,[-0.0264\cos w\,(-5) - 0.1389\cos w\,(-4) + 0.04462\cos w\,(-3) + 0.26517\cos w\,(-2) - 0.02158\cos w\,(-1)]$$

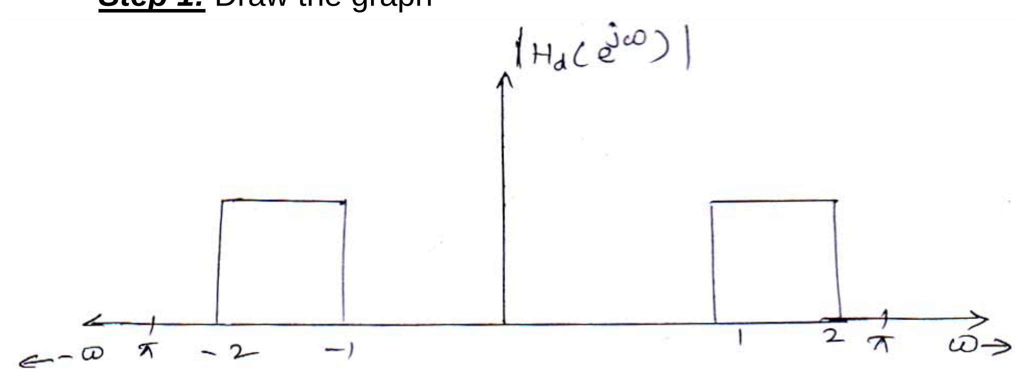
5. Design band pass filter with following specification

$$H_d\left|e^{jw}\right| = \begin{cases} e^{-jdw} & wc_1 \le |w| \le wc_2 \le \pi \\ 0 & Otherwise \end{cases}$$

Choose N = odd (Here take N = 11) **[CO3-H3]**

wc_1 = 1, wc_2 = 2 rad/sec

Step 1: Draw the graph



Step 2: Find d

N = 11

$d = \frac{11-1}{2}$

$d = \frac{10}{2} = 50$

Step 3: Find $h_d\ (n)$

$$h_d\ (n) = \frac{1}{2\pi} \int_{-wc_2}^{-wc_1} e^{j(n-d)w}.dw + \int_{-wc_1}^{wc_2} e^{j(n-d)w}.dw$$

$d = 5$

$$h_d\ (n) = \frac{1}{\pi}\ [\sin wc_2(n-d) - \sin wc_1(n-d)]$$ for n ≠ d

Put n = 0

$$h_d\ (0) = \frac{1}{\pi(-5)}\ [\sin 2\ (-5) - \sin 1\ (-5)]$$

$h_d\ (0) = \frac{0.4149}{-5\pi}$ = +0.0264

$) = 0.02641$

$) = h_d\ (10)$

Put n = 1, $h_d\ (1) = \frac{1}{\pi(-5)}\ [\sin 2\ (-4) - \sin 1\ (-4)]$

$h_d\ (1) = \frac{-1.7461}{-4\pi}$ = 0.13896

$) = 0.13896$

$) = h_d\ (9)$ =0.13896

Put n = 2,

$$h_d\ (2) = \frac{1}{\pi(-3)}\ [\sin 2\ (-3) - \sin 1\ (-3)]$$

$h_d\ (2) = \frac{0.42054}{-3\pi}$

$) = -0.04462$

$) = h_d\ (8)$ = -0.04462

Put n = 3,

$$h_d\ (3) = \frac{1}{\pi(-2)}\ [\sin 2\ (-2) - \sin 1\ (-2)]$$

$h_d\ (3) = \frac{1.6661}{-2\pi}$

$) = -0.2652$

) = h_d (7) = -0.2652

Put n = 4,

$$h_d\ (4) = \frac{1}{\pi(-1)}\ [\sin 2\ (-1) - \sin 1\ (-1)]$$

$$h_d\ (4) = \frac{-0.06783}{-\pi}$$

) = +0.021589

) = h_d (6) = 0.021589

Put n = 5, (n = d)

$$h_d\ (5) = \frac{2-1}{\pi}$$

$$h_d\ (5) = 0.3183$$

) = 0.3183

Step 4: Using rectangular window

$$W_R\ (n) = \begin{cases} 1 & for\ 0 \le n\ \le 10 \\ 0 & Otherwise \end{cases}$$

$W_R\ (0) = 1$

$W_R\ (1) = 1$

$W_R\ (2) = 1$

$W_R\ (3) = 1$

$W_R\ (4) = 1$

$W_R\ (5) = 1$

$W_R\ (6) = 1$

$W_R\ (7) = 1$

$W_R\ (8) = 1$

$W_R\ (9) = 1$

$W_R\ (10) = 1$

Step 5:

Find $h\ (n) = h_d\ (n) \times W_R\ (n)$

$h\ (0) = h_d\ (0)\ .W_R\ (0) = 0.02641$

$h\ (1) = h_d\ (1)\ .W_R\ (1) = 0.13896$

$h\ (2) = h_d\ (2)\ .W_R\ (2) = -0.04462$

$h\ (3) = h_d\ (3)\ \ W_R\ (3) = -0.2652$

$h\ (4) = h_d\ (4)\ \ W_R\ (4) = 0.021589$

$h\ (5) = h_d\ (5)\ \ W_R\ (5) = 0.3183$

$h\ (6) = h_d\ (6)\ \ W_R\ (6) = 0.021589$

$$h\ (7) = h_d\ (7)\ W_R\ (7) = -0.2652$$
$$h\ (8) = h_d\ (8)\ W_R\ (8) = -0.04462$$
$$h\ (9) = h_d\ (9)\ W_R\ (9) = 0.13896$$
$$h\ (10) = h_d\ (10)\ W_R\ (10) = 0.02641$$

Step 6:

Magnitude of H (w)

$$|H\ (w)| = h\left[\frac{N-1}{2}\right] + 2\sum_{n=0}^{\frac{M-3}{2}} h\ (n).\cos w\left[N - \frac{N-1}{2}\right]$$

1

$$= h\ (5) + 2\sum_{n=0}^{4} h\ (n).\cos w\left[n - \frac{11-1}{2}\right]$$

$$)| = h\ (5) + 2\sum_{n=0}^{4} h\ (n)\cos w\ (n-5)$$

$$)| = 0.3183 + 0.043178\cos 1\,w + 0.53034\cos 2\,w$$
$$).08924\cos 3w + 0.27792\cos 4w$$
$$+ 0.05282\cos 5w$$

6.Design FIR – HPR (High pass filter) with cut – off frequency = w_c and having the window function.

$$W\ (n) = \begin{cases} 1 & for\ 0 \le n \le 6 \\ 0 & Otherwise \end{cases}$$ **[CO3-H3-May/June 2010]**

Soln:

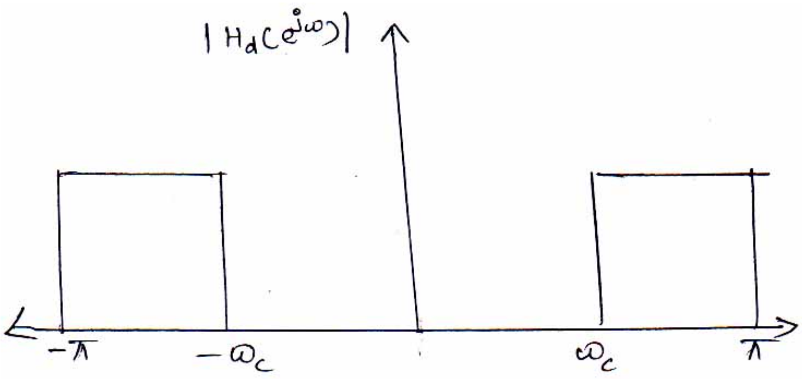
Here, in the specification rectangular window is given $(W_R\ (n))$ and N = 7

Step 1: Find d

$$d = \frac{N-1}{2} = \frac{7-1}{2} = \frac{6}{2} = 3$$

Step 2: Draw the graph and find $h_d\ (n)$

$$H_d\ |e^{jw}| = \begin{cases} 1 & for\ w_c \le |w| \le \pi, -\pi \le w \le -w_c \\ 0 & for - w_c \le w \le w_c \end{cases}$$



This is the HPF graph

Now, $h_d\ (n)$ can be writhen as,

$$h_d\ (n) = \frac{1}{\pi} \int_{-\pi}^{\pi} H_d\ (e^{jw}) e^{jwn}.dw$$

$$) = \begin{cases} 1 - \frac{w_c}{\pi} & for\ n = d \\ \frac{1}{\pi(n-d)} & [\sin(n-d)\pi - \sin(n-d)w_c]\ for\ n \neq d \end{cases}$$

Step 3: Find $h_d\ (n)$ values

Here w_c value is not given, so, we can directly write $h_d\ (n)$ as given below

$$) = \begin{cases} 1 - \frac{w_c}{\pi} & for\ n = d \\ \frac{1}{\pi(n-3)} & [\sin(n-3)\pi - \sin(n-3)w_c]\ for\ n \neq 3 \end{cases}$$

This is final equation

7.Design an ideal high pass filter with a frequency responses.

$$\boldsymbol{H_d\ |e^{jw}| = \begin{cases} 1 & \frac{\pi}{4} \leq |w|\ \leq \pi \\ 0 & |w|\ \leq \frac{\pi}{4} \end{cases}}$$ **[CO3-H3]**

Find $h_d\ (n)$ for N = 11

Use rectangular window

We know that

Here, d = 5 $\left[\because d = \frac{N-1}{2} = \frac{11-1}{2} = 5\right]$

$$h_d\ (n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} h_d\ (e^{jw})\ e^{jwn}.dw$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\frac{-\pi}{4}} e^{-jwd}\ e^{jwn}.dw + \int_{-\pi}^{\frac{-\pi}{4}} e^{-jwd}\ e^{jwn}.dw$$

$$= \frac{1}{2\pi} \left\{ \left[\frac{e^{jw\ (n-d)}}{j\ (n-d)}\right]_{\pi}^{\frac{-\pi}{4}} + \left[\frac{e^{jw\ (n-d)}}{j\ (n-d)}\right]_{\frac{\pi}{4}}^{\pi} \right\}$$

) = $\frac{1}{\pi(n-d)}$ $[\sin(n-d)\pi - \sin(n-d)w_c]$ $for\ n \neq a$

) = $1 - \frac{w_c}{\pi}$ $for\ n - d$

If rectangular window is used

) = $h_d\ (n).W_R\ (n)$

) = $h_d\ (n)$

This is symmetrical odd sequence, so

) = $h\ (N-1-n)$

$h\ (0) =$ 0.045

$h\ (1) =$ 0

$h\ (2) =$ -0.075

$h\ (3) =$ -0.159

$h\ (4) =$ -0.225

$h\ (5) =$ 0.75

$h\ (6) = h\ (4) = -0.225$

$h\ (7) = h\ (3) = -0.159$

$h\ (8) = h\ (2) = 0.075$

$h\ (9) = h\ (1) = 0$

$h\ (10) = h\ (0) = 0.045$

These are FIR filter coefficients the transfer function is given by

$$H\ (z) = h(0) + \sum_{n=1}^{\frac{(N-1)}{2}} [h\ (n)\ (z^n + z^{-n})]$$

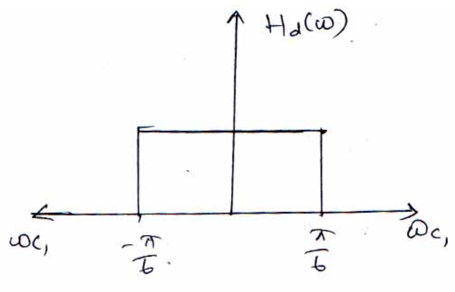
Put the values of $h\ (n)$ and expand this equation

8.Design a linear phase FIR digital filter for given specifications using hamming window of length M = 7.

$$H_d\ (w) = \begin{cases} e^{-j3w}, & for|w| \leq \frac{\pi}{6} \\ 0\ , & for\ \frac{\pi}{6} < |w| \leq \pi \end{cases}$$ **[CO3-H3]**

Soln:

Step 1: Draw the graph



Step 2: Find d

$$d = \frac{M-1}{2}; M = 7$$

Step 3: Find $h_d\ (w)$

$$h_d\ (n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-jwd}\ e^{jwn}.\ d\ (w)$$

But we are using the formula method

$$h_d\ (n) = \frac{w_c}{\pi}\ if\ n = d$$

$$h_d\ (n) = \frac{\sin w_c\ (n-d)}{\pi\ (n-d)}\ if\ n \neq d$$

$$h_d\ (0) = \frac{\sin\frac{\pi}{6}(-3)}{-3\pi} = 0.106$$

$$h_d\ (1) = \frac{\sin\frac{\pi}{6}(-2)}{-2\pi} = 0.1378$$

$$h_d\ (2) = \frac{\sin\frac{\pi}{6}(-3)}{-\pi} = 0.106$$

$$h_d\ (3) = \frac{\frac{\pi}{6}}{\pi} = 0.1666$$

$$h_d\ (4) = \frac{\sin\frac{\pi}{6}(1)}{\pi} = 0.15915$$

$$h_d\ (5) = \frac{\sin\frac{\pi}{6}(2)}{2\pi} = 0.1378$$

$$h_d\ (6) = 0.106$$

Step 4: Find the hamming window coefficients

$$W_h\ (n) = 0.54 + 0.46\ cos\left(\frac{2\pi n}{N-1}\right)$$

$$W_h\ (0) = 0.54 + 0.46\ cos(0) = 1$$

$$W_h\ (1) = 0.54 + 0.46\ cos\frac{2\pi}{6} = 0.77$$

$$W_h\ (2) = 0.54 + 0.46\ cos\frac{4\pi}{6} = 0.31$$

$$W_h\ (3) = 0.54 + 0.46\ cos\frac{6\pi}{6} = 0.08$$

$$W_h\ (4) = 0.54 + 0.46\ cos\frac{8\pi}{6} = 0.31$$

$$W_h\ (5) = 0.54 + 0.46\ cos\frac{10\pi}{6} = 0.77$$

Step 5: Find $h\ (n)$

$$h\ (n) = W_h\ (n)\ .h_d\ (n)$$

$$h\ (0) = 1 \text{ x } 0.106 = 0.106$$

$$h\ (1) = 0.77 \text{ x } 0.1378 = 0.106106$$

$$h\ (2) = 0.31 \text{ x } 0.15915 = 0.04933$$

$$h\ (3) = 0.08 \text{ x } 0.1666 = 0.01332$$

$$h\ (4) = 0.31 \text{ x } 0.15915 = 0.04933$$

$$h\ (5) = 0.77 \text{ x } 0.1378 = 0.106106$$

$$h\ (6) = 0.106$$

Step 6:

$$H\ (z) = \sum_{n=0}^{N-1} h\ (n)\ z^{-n}$$

$$= 0.0508 + 0.04004z^{-1} + 0.04933z^{-2} + 0.01332z^{-3} + 0.04933z^{-4} + 0.04004z^{-5}$$

9.Design a high pass filter using hamming window, with a cut off frequency of 1.2 radian/second and N = 9.

$$\left|H_d\ [e^{jw}]\right| = \begin{cases} 1 & w_c \le |w| \le \pi \\ 0 & Otherwise \end{cases}$$

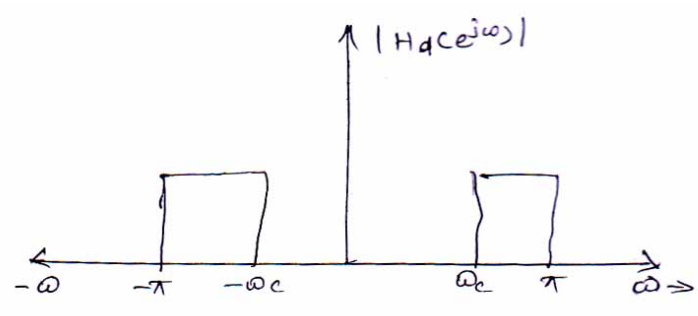
[CO3-H3]

H_{P_F} is to be designed

w_c = 1.2 rad/sec

N = a

Step 1: Draw the graph



Step 2: Find d

$$d = \frac{N-1}{2} = \frac{9-1}{2} = N = 9$$

Step 3: Find $h_d\ (n)$

$$h_d\ (n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-jwd}\ e^{jwn}.\, dw$$

For HPF,

$$h_d\ (n) = 1 - \left[\frac{w_c}{\pi}\right]\ if\ n = d$$

$$h_d\ (n) = \frac{1}{\pi\,(n-d)}\ [\sin(n-d)\,\pi - \sin(n-d)\, w_c]$$

$$h_d\ (4) = 1 - \frac{1.2}{\pi} = 0.618$$

$$h_d\ (0) = \frac{1}{\pi\,(0-4)}\ [\sin(0-4)\,\pi - \sin(0-4)\,1.2]$$

$$h_d\ (0) = \frac{-0.996}{-4\pi} = 0.07926$$

$$h_d\ (1) = \frac{1}{\pi\,(1-4)}\ [\sin(-3\pi) - \sin(-3)\,1.2]$$

$$= \frac{-0.4425}{-3\pi} = 0.04695$$

$$h_d\ (5) = \frac{1}{\pi\,(2-4)}\ [\sin(2-4)\,\pi - \sin(2-4)\,1.2]$$

$$= \frac{0.675}{-2\pi} = -0.1074$$

$$h_d\ (3) = \frac{0.932}{-\pi} = -0.29667$$

$$h_d\ (0) = 0.07926$$

$$h_d\ (1) = 0.04695$$

$$h_d\ (2) = -0.1074$$

$$h_d\ (3) = -0.29667$$

$$h_d\ (4) = 0.618$$

$$h_d\ (5) = -0.29667$$

$h_d\ (6) = -0.1074$

$h_d\ (7) = 0.04695$

$h_d\ (8) = 0.07926$

$h_d\ (n) = h_d\ (N-1-n)$

$h_d\ (5) = h_d\ (9-1-5) = h_d\ (3)$

$h_d\ (6) = h_d\ (9-1-6) = \ h_d\ (2)$

$h_d\ (7) = h_d\ (1)$

Step 4: Find window coefficients

N = 9 and hamming window is used

$W_H\ (n) = 0.54 + 0.46\ cos\left[\frac{2\pi n}{N-1}\right] 0 \leq n \leq N-1$

$W_H\ (0) = 1 = W_H\ (8)$

$W_H\ (1) = 0.8653 = W_H\ (7)$

$W_H\ (2) = 0.54 = W_H\ (6)$

$W_H\ (3) = 0.2147 = W_H\ (5)$

$W_H\ (4) = 0.08$

Step 5: Find $h\ (n)$

$h\ (n) = h_d\ (n)\ \mathrm{x}\ W_{H_n}\ (n)$

$h\ (0) = 0.181\ \mathrm{x}\ 1 = 0.181$

$h\ (1) = 0.159\ \mathrm{x}\ 0.75 = -0.11925$

$h\ (2) = 0.092\ \mathrm{x}\ 0.25 = 0.023$

$h\ (3) = 0.25\ \mathrm{x}\ 0 = 0$

$h\ (4) = 0.023$

$h\ (5) = -0.11925$

$h\ (6) = 0.181$

Step 6:

$H\ (z) = \sum_{n=0}^{6} h\ (n).z^{-n}$

$= -0.181 + (-0.11925)(z^{-1} + z^{-5}) + 0.023(z^{-2} + \ z^{-4})$

Unit – I V

Finite Word Length Effects

Part – A

1. What do you understand by a fixed point number? [CO4-L1]

in fixed point arithmetic the position of the binary point is fixed. The bits to the right represent the fractional part and those to the left represent the integer part. For eg. The binary number 01.1100 has the value 1.75 in decimal.

2. Brief on coefficient inaccuracy. [CO4-L1]

The filter coefficients are computed to infinite precision in the design. But in digital computation the filter coefficients are represented in binary and are stored in registers. The filter coefficients must be rounded or truncated to 'b' bits which produces an error. Due to quantization of coefficients the frequency response of a filter may differ appreciably form the desired response and sometimes the filter may fail to meet the desired specification. If the poles of the filter are close to the unit circle then those of the filter quantized coefficients may be just outside the unit circle leading to instability.

3.What is meant by (zero input) limit cycle oscillation? [CO4-L1]

For an IIR filter implemented with infinite precision arithmetic the output should approach zero in the steady state if the input is zero and it should approach a constant value if the input is a constant. However , with an implementation using a finite length register an output can occur even with zero input. The output may be a fixed value or it may oscillate between finite positive and negative values. This effect is referred to as (zero input) limit cycle oscillation.

4.What are the assumptions made concerning the statistical independence of various noise sources that occur in realizing the filter? [CO4-L1]

Assumptions

1. for any n , the error sequence e(n) is uniformly distributed over the range
2. (-q/2) and (q/2). This implies that the mean value of e(n) is zero and its variance is
3. The error sequence e(n) is a stationary white noise source.
4. The error sequence e(n) is uncorrelated with the signal sequence x(n).

5.What is the difference between fixed point arithmetic and floating point arithmetic? [CO4-L1]

Fixed point arithmetic	Floating point arithmetic
1. fast operation	slow operation
2. small dynamic range	increased dynamic range
3 relatively economical	more expensive due to costlier hardware
4. round-off errors occur only in addition	round-off errors can occur with both multiplication and addition
5. overflow occurs in addition	overflow does not arise
6. used in small computers	used in larger general purpose computers.

6. What are the 3 quantization errors due to finite word length register in digital filters?. [CO4-L1]

1. Input quantization error 2. Coefficient quantization error 3. Product quantization error

7. Explain briefly the need for scaling in the digital filter implementation. [CO4-L1]

To prevent overflow , the signal level at certain points in the digital filter must be scaled so that no overflow occurs in the adder.

8. What is limit cycles due to overflow?

Or

What is overflow oscillations? [CO4-L1-May/June 2011]

The addition of two fixed point arithmetic numbers cause overflow when the sum exceeds the word size available to store the sum. This overflow caused by adder make the filter output to oscillate between maximum amplitude limits. Such limit cycles have been referred to as overflow oscillations.

9. Define 'dead band' of the filter [CO4-L1]

The limit cycles occur as a result of quantization effect in multiplication. The amplitudes of the output during a limit cycle are confined to a range of values called the dead band of the filter.

10. Express the fraction (7/8) and (-7/8) in sign magnitude, 2's complement and 1's complement. [CO4-L1]

fraction (7/8) = (0.111) in sign magnitude , 1's complement and 2's complement

Fraction (-7/8) = (1.111) in sign magnitude

= (1.000) in 1's complement

= (1.001) in 2's complement

11. The filter coefficient H = -0.673 is represented by sign magnitude fixed point arithmetic. I the word length is 6 bits , compute the quantization error due to truncation. [CO4-L1]

(0.673) = (0.1010110…)

(-0.673) = (1.1010110…)

after truncating to 6 bits we get

(1.101011) = -0.671875

Quantization error = $x_q - x$

= (-0.671875)-(-0.673)

= 0.001125

12. Give the expression for the signal to quantization noise ratio and calculate the improvement with an increase of 2 bits to the existing bit. [CO4-L1]

SNR = 6b – 1.24dB where b = Number of bits representation

With an increase of 2 bits , increase in SNR is approximately 12dB.

13. Why rounding is preferred over truncation in realizing digital filters? [CO4-L1]

1.The quantization error due to rounding is independent of the type of arithmetic.2.The mean of rounding error is Zero. 3. The variance of rounding error signal is low.

14.What is product quantization eror? [CO4-L1-Nov/Dec 2010]

Or

What is round-off noise error?

Product quantization error arise at the output of a multiplier. Multiplication of a ‘b’ bit data with a ‘b’ bit coefficient results in a product having 2b bits. Since a ‘b’ bit register is used , the multiplier output must be rounded or truncated to ‘b’ bits which produces an error. This error is known as product quantization error.

15.Why the limit cycle problem does not exist when FIR filter is realized in direct form or cascade form? [CO4-L1]

In FIR filters there are no limit cycle oscillations if the filter is realized in direct form or cascade form since these structures have no feedback .

16.What do you understand by input quantization error? [CO4-L1]

In DSP the continuos time input signals are converted into digital using a ‘b’ bit ADC. The representation of continuos signal amplitude by a produces an error known as input quantization error.

17.What are the assumptions made concerning the statistical independence of various noise sources that occur in realizing the filter? [CO4-L1]

Assumptions for any n , the error sequence e(n) is uniformly distributed over the range (-q/2) and (q/2). This implies that the mean value of e(n) is zero and its variance is The error sequence e(n) is a stationary white noise source. The error sequence e(n) is uncorrelated with the signal sequence x(n).

18. State the method to prevent overflow[CO4-L1]

1. Saturation Arithmetic 2. Scaling

19. What are the two types of Quantization? [CO4-L1]

1. Truncation and 2. Rounding

20. State the need for scaling in filter implementation [CO4-L1]

With fixed-point arithmetic it is possible for filter calculations to overflow. This happens when two numbers of the same sign add to give a value having magnitude greater than one. Since numbers with magnitude greater than one are not representable, the result overflows. It is used to eliminate overflow limit cycle in FIR filters.

Part – B

1.Explain the detail the 3 types of quantization error that occur due to the finite word length of register. [CO4-L2]

➢ DSP algorithms are realized with special purpose digital hardware or as programs. In both the cases the numbers and co-efficient are stored in finite length registers.

➢ Therefore the co-efficient and number are quantized by truncating or rounding when they are stored. This creates error in the output.

➢ These type of effect due to finite precision representation of numbers in digital system are called finite word length effects.

➢ The following errors arises due to quantization of numbers.

* **Input quantization error.**
* **Product quantization error.**
* **Co-efficient quantization error.**

Input quantization error:

The conversion of a continuous time **input signal** into digital value produces an error, which is known as input quantization error. This error arises due to the representation of the input signal by a fixed number of digits in A/D conversion process.

Product quantization error:

Product quantization error arise at the **output of a multiplier**. Multiplication of a b-bit data with a b-bit coefficient results a product having 2b bits.

Since a b-bit register is used, the multiplier output must be rounded or truncated to b-bits which produced an error.

Coefficient quantization error:

The filter coefficients are computed to infinite precision in theory. If they are quantized, the frequency response of the resulting filter may differ from the desired response.. If the poles of the desired filter are close to the unit circle, then those of the filter with quantized coefficient may we outside the unit circle leading to instability.

The other errors arising from quantization are round-off noise and limit cycle oscillations.

2.Explain in detail the input quantization error (or)

Derive the equation for steady state input noise power and steady state output noise power (Or) quantization noise power. [CO4-H1-May/June 2011]

➢ The conversion of a continuous time **input signal** into digital value produces an error, which is known as input quantization error. This error arises due to the representation of the input signal by a fixed number of digits in A/D conversion process.

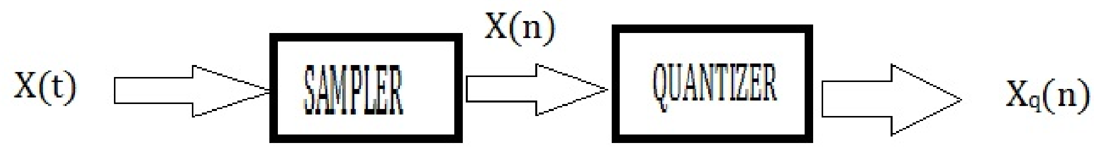
➢ The quantization error occurs whenever a continuous signal is converted into digital signal.

➢ Thus, the quantization error is given as

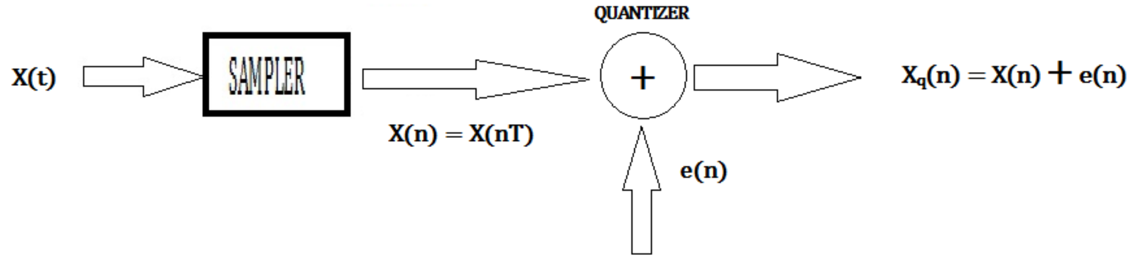
$e(n) = x_q(n) – x(n)$

here, $x_q(n)$ = sampled quantized value of signal and

$x(n)$= sampled unquantized value of signal.



Block Diagram of A/D Converter



Quantization noise model

➢ If rounding of a number is used to obtain $X_q(n)$, then the error signal satisfies the following relation.

$$-\frac{q}{2} \leq e(n) \leq \frac{q}{2}$$

Since the quantized signal may be greater than or less than the actual signal.

➢ The probability density function P(e) for roundoff error and quantization characteristics with rounding have been shown in fig.4.3 a and 4.3 b

➢ In truncation , the signal is reperesented by the highest quantization level which is not greater than the signal. Here, in two's complement truncation, the error e(n) is always negative and satisfies the inequality:$-q \leq e(n) < 0$

The quantizer characteristics for truncation and probability density function P(e) for two's complement truncation has been shown in fig.

STEADY STATE INPUT NOISE POWER

➢ In signal processing, the quantization error is commonly viewed as additive noise signal.

$X_q(n) = X(n) + e(n)$

➢ If rounding is used for quantization, then the quantization error

$e(n) = x_q(n) – x(n)$

and it is bounded by

$$-\frac{q}{2} \leq e(n) \leq \frac{q}{2}$$

➢ Let us assume the A/D conversion error e(n) has the following properties:

* The error sequence e(n) is a sample sequence of a stationary random process.
* The error sequence is uncorrelated with x(n) and other signals in the system.

∗ The error is a white noise process with amplitude probability distribution over the range of quantization error.

➢ **In case of rounding**, the e(n) lies between –q/2 and q/2 with equal probability. The variance of e(n) is given as under:

$$\sigma_e^2 = E[e^2(n)] - E^2[e(n)]$$

here $E[e^2(n)]$ is the average of $e^2(n)$ and $E[e(n)]$ is mean value of e(n).

Therefore, for rounding, we have

$$\sigma_e^2 = \frac{1}{q}\int_{-\frac{q}{2}}^{\frac{q}{2}} e^2(n)de - (0)^2$$

$$\boxed{\sigma_e^2 = \frac{q^2}{12}}$$

We know that

$$q = \frac{R}{2^{b+1}}$$

Let us assume R=2,

$q = 2^{-b}$

we get,

$$\sigma_e^2 = \frac{(2^{-b})^2}{12} = \frac{2^{-2b}}{12}$$

➢ **In case of two's complement truncation**, the e(n) lies between 0 and –q having mean value of –q/2 . The variance or power of the error signal e(n) is given under:

$$\sigma_e^2 = \frac{1}{q}\int_{-q}^{0} e^2(n)de - (-\frac{q}{2})^2$$

$$\sigma_e^2 = \frac{q^2}{3} - \frac{q^2}{4}$$

$$\sigma_e^2 = q^2/12$$

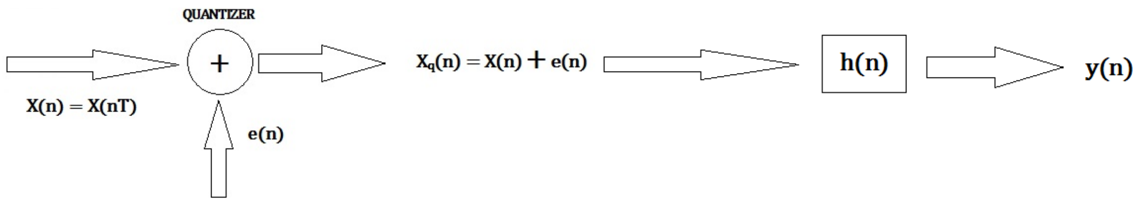
we get,

$$\boxed{\sigma_e^2 = \frac{(2^{-b})^2}{12} = \frac{2^{-2b}}{12}}$$

Here, it may be noticed that in both the cases, the value of error signal is given by $\sigma_e^2 = \frac{(2^{-b})^2}{12} = \frac{2^{-2b}}{12}$ Which is also known as **the steady state noise power due to input quantization (or) steady state input noise power.**

<u>THE STEADY STATE OUTPUT NOISE POWER</u>

Because of A/D conversion noise , we can represent the quantized input to a digital system with impulse response h(n) as described in fig.



Let y(n) be the output noise due to the quantization of input.

Then, we have

$$y(n) = e(n) \otimes h(n)$$

$$y(n) = \sum_{k=0}^{n} h(k)e(n-k)$$

The variance of any term in the above sum is equal to $\sigma_e^2 h^2(n)$.

The variance of the sum of independent random variable is the sum of their variances. If the quantization errors are assumed to be independent at different sampling instances, then the variance of output will be

$$\sigma_y^2 = \sigma_e^2 \sum_{n=0}^{k} h^2(n)$$

To determine the steady state variance, let us extend the limit k upto infinity.

Thus we write, $\sigma_y^2 = \sigma_e^2 \sum_{n=0}^{\alpha} h^2(n)$

Using Parseval's theorem, the steady state output noise variance due to the quantization error can be expressed as

$$\sigma_y^2 = \sigma_{eo}^2 = \frac{\sigma_e^2}{2\pi j} \oint_c H(z)H(z^{-1})z^{-1}\, dz$$

where the closed contour of integration is around the unit circle $|z| = 1$

3.The output of an A/D converter is applied to a digital filter with the system function, $\mathbf{H(z) = \frac{0.5z}{z-0.5}}$ find the output noise power from the digital filter when the input signal is quantized to have 8 bits.

Given data: **[CO4-H1-Nov/Dec 2012]**

$$H(z) = \frac{0.5z}{z - 0.5}$$

Find the **output noisepower**from the digital filters, when the input signal is quantized to have **8-bit.**

Solution:

The **input quantization noise power**(σe^2) is given by,

$$\sigma e^2 = \frac{2^{-2b}}{12}$$

Given b=8, therefore, $\sigma e^2 = \frac{2^{-2(8)}}{12} = 1.27X10^{-6}$

The output noise power is given by,

$$\sigma e0^2 = \frac{\sigma e^2}{2\pi j} \oint H(z)\ H(z^{-1})\ z^{-1} dz$$

$$\sigma e0^2 = \frac{\sigma e^2}{2\pi j} \oint \left(\frac{0.5z}{z - 0.5}\right)\left(\frac{0.5z^{-1}}{z^{-1} - 0.5}\right) z^{-1} dz$$

$$\sigma e0^2 = \frac{\sigma e^2}{2\pi j} \oint \frac{0.25\ z\ z^{-1}}{(z - 0.5)(1 - 0.5z)}\ dz$$

$$\sigma e0^2 = \frac{\sigma e^2}{2\pi j} \oint \frac{0.25}{(z - 0.5)(1 - 0.5z)}\ dz$$

The above integral can be evaluated by the method of residues.

I=sum of residues at the poles within the unit circle within $|z| < 1$

The poles are **z=0.5 (inside the unit circle, so it is stable pole.)**

$$\boldsymbol{z = \frac{1}{0.5} = 2\ (outside\ the\ unit\ circle, so\ it\ is\ unstable\ pole.)}$$

NOTE: Only consider the stable poles.i.e. Poles lie inside the unit circle.

$$I = (z - 0.5)\frac{0.25}{(z - 0.5)(1 - 0.5z)}\quad |\, z = 0.5$$

I=0.33

The output noise power ($\boldsymbol{\sigma e0^2}$)is given by,

$$\boldsymbol{\sigma e0^2 = \sigma e^2 . I}$$

$$\sigma e0^2 = 0.333\sigma e^2$$

$$= 0.333(1.27X\,10^{-6})$$

$$\boxed{\sigma e0^2 = 0.423X10^{-6}}$$

4.In the IIR system given below the products are rounded to 4 bits (including sign bits). The system function is

$$H(Z) = \frac{1}{(1-0.35z^{-1})(1-0.62z^{-1})}$$

Find the output roundoff noise power in *(a)* direct form realization and *(b)* cascade form realization. [CO4-H3]

Solution

(a) **Direct Form Realization**

$$H(Z) = \frac{1}{(1-0.97z^{-1}+0.217z^{-2})}$$

Direct form realization of *H (z)* is shown in Figure

The variance of the error signal is,

Here *R* is not given. So take *R* = 2 *V* and *b* = 4 bits

$$q = \frac{R}{2^b} = \frac{2}{2^4} = \frac{1}{2^3} = \frac{1}{8}$$

$$\sigma_e^2 = \frac{(\frac{1}{8})^2}{12} = \frac{q^2}{12}$$

$$\sigma_e^2 = 1.3021X\,10^{-3}$$

Output noise power due to the noise signale_k *(n) is*

$$\sigma_{0k}^2 = \sigma_e^2 \frac{1}{2\pi j}\oint_c H_k(z)H_k(z^{-1})z^{-1}dz$$

Output noise power due to the noise signal e_l *(n)* is,

$$\sigma_{e01}^2 = \frac{\sigma e^2}{2\pi j}\oint H(z)\ H(z^{-1})\,z^{-1}dz$$

Here,

$$H(Z) = \frac{1}{(1-0.35z^{-1})(1-0.62z^{-1})}$$

$$H(Z) = \frac{z^2}{(z-0.35)(z-0.62)}$$

Therefore,

$$\sigma_{e01}^2 = \frac{\sigma e^2}{2\pi j}\oint\left(\frac{z^2}{(z-0.35)(z-0.62)}\right)\left(\frac{z^{-2}}{(z^{-1}-0.35)\,(z^{-1}-0.62)}\right)z^{-1}dz$$

$$\sigma_{e01}^2 = \frac{\sigma e^2}{2\pi j}\oint \left(\frac{z^{-1}}{(z-0.35)(z-0.62)\,(z^{-1}-0.35)\,(z^{-1}-0.62)}\right)dz$$

The <u>stable</u> poles of *H(z)* are P_l = 0.35 and P_2= 0.62 and <u>unstable</u> poles of H(z) are P3 = 2.86 and P4 = 1.62. For taking residue only consider the stable poles.

$$\mathrm{Res}[\mathrm{H(z)H(z^{-1})z^{-1}}]|(z=0.35) =$$

$$= (z-0.35)\frac{z^{-1}}{(z-0.35)(z-0.62)\,(z^{-1}-0.35)\,(z^{-1}-0.62)}\Bigg| z = 0.35$$

= **-1.8867.**

$$\mathrm{Res}[\mathrm{H(z)H(z^{-1})z^{-1}}]|(z=0.62) =$$

$$= (z-0.62)\frac{z^{-1}}{(z-0.35)(z-0.62)\,(z^{-1}-0.35)\,(z^{-1}-0.62)}\Bigg| z = 0.62$$

=4.7640.

Total = $\mathrm{Res}[\mathrm{H(z)H(z^{-1})z^{-1}}]|(z=0.35) + \mathrm{Res}[\mathrm{H(z)H(z^{-1})z^{-1}}]|(z=0.62)$

= **-1.8867+ 4.7640.**

= 2.8773.

Therefore,

$$\sigma_{e01}^2 = \frac{\sigma e^2}{2\pi j}\oint H(z)\ H(z^{-1})\ z^{-1}dz$$

$= 1.3021 \times 10^{-3} \times 2.8733$

$= 3.7465 \times 10^{-3}$

Here the output noise due to error source $e_2(n)$ is same as that of $e_1(n)$, *i.e.,*

e_2 (n)* noise power = noise power of *e_1(n)

$$\sigma_{e01}^2 = \sigma_{e02}^2$$

$= 3.7465 \times 10^{-3}$

Total output noise power due to all the noise sources is,

$$\boldsymbol{\sigma_{e0}^2 = \sigma_{e01}^2 + \sigma_{e02}^2}$$

$$\sigma_{e0}^2 = 7.493\ X\ 10^{-3}$$

(b) **<u>Cascade Realization</u>**

Given

$$\boldsymbol{H(Z) = \frac{1}{(1-0.35z^{-1})(1-0.62z^{-1})}}$$

Let H(z) =H_1(Z)H_2(Z), i.e.,

$$\boldsymbol{H_1(z) = \frac{1}{1-0.35z^{-1}}\ and\ H_2(z) = \frac{1}{1-0.62z^{-1}}}$$

Case *(i)* <u>*H(z) = H₁(Z)H₂(Z)*</u>

The cascade form realization of *H(z)* is shown in Figure

Output noise power due to the noise signal e_k *(n) is*

$$\sigma_{0k}^2 = \sigma_e^2 \frac{1}{2\pi j}\oint_c H_k(z)H_k(z^{-1})z^{-1}dz$$

The order of cascading is $H_1(Z)H_2(Z)$. Output noise power due to errorsignal$e_1(n)$ is

$$\sigma_{e01}^2 = \frac{\sigma e^2}{2\pi j}\oint H(z)\ H(z^{-1})\ z^{-1}dz$$

$= 3.7465 \times 10^{-3}$[refer direct form]

Output noise power due to the error, signal *e2(n)* is

$$\sigma_{e02}^2 = \frac{\sigma e^2}{2\pi j}\oint H_2(z)H_2(z^{-1})z^{-1}\ dz$$

$$H_2(z)H_2(z^{-1})z^{-1} = \frac{z^{-1}}{(z-0.62)(z^{-1}-0.62)}$$

$$\text{Res}[H_2(z)H_2(z^{-1})z^{-1}]|(z=0.62) = (z-0.62)\frac{z^{-1}}{(z-0.62)\ (z^{-1}-0.62)}\Bigg|\ z=0.62$$

=1.6244

$$\sigma_{e02}^2 = \frac{\sigma e^2}{2\pi j}\oint H_2(z)H_2(z^{-1})z^{-1}\ dz$$

$=1.3021 \times 10^{-3} \times 1.6244$

$= 2.1151 \times 10^{-3}$

Total Output noise power

$$\sigma_{e0}^2 = \sigma_{e01}^2 + \sigma_{e02}^2$$

$= 3.7465 \times 10^{-3} + 2.1151 \times 10^{-3}$

$$\boxed{\sigma_{e0}^2 = 5.8616 \times 10^{-3}}$$

Case *(ii)* The order of cascading is $H(z) = H_2(Z)H_1(z)$ and is shown in Figure

The output noise power due to error source $e_1$is,

$$\sigma_{e01}^2 = 3.7465 \times 10^{-3}$$

The output noise power due to error source $e_1(n)$ is,

$$\sigma_{e02}^2 = \frac{\sigma e^2}{2\pi j}\oint H_1(z)H_1(z^{-1})z^{-1}\ dz$$

$$H_1(z)H_1(z^{-1})z^{-1} = \frac{z^{-1}}{(z-0.35)(z^{-1}-0.35)}$$

$$\text{Res}[H_1(z)H_1(z^{-1})z^{-1}]|(z=0.35) = (z-0.35)\frac{z^{-1}}{(z-0.35)\,(z^{-1}-0.35)}\Bigg|\, z=0.35$$

= 1.1396

$$\sigma_{e02}^2 = 1.1396\,X\,1.3021\,X\,10^{-3}$$

= 1.4839 X 10^{-3}

Total output noise power

$$\sigma_{e0}^2 = \sigma_{e01+}^2\sigma_{e02}^2$$

=3.7465 10^{-3}+ 1.4839 X 10^{-3}

$$\boxed{\sigma_{e0}^2 = 5.2304\,X\,10^{-3}}$$

Conclusion: **Thus, in cascade form realization, the product noise round off power is less in case *(ii)* when compared to case (i) and also direct form realization.**

5.Consider the transfer function where *H(z)* = H$_1$*(z)H$_2$(z)*
Let H(z) =H$_1$(Z)H$_2$(Z), i.e.,

$$\boldsymbol{H_1(z) = \frac{1}{1-0.5z^{-1}}\,and\,H_2(z) = \frac{1}{1-0.6z^{-1}}}$$ **[CO4-H1-Nov/Dec 2014]**

Find the output roundoff noise power.

The roundoff noise model for *H(z) = HI (Z)H2(Z)* is shown in Figure . From the realization, the noise transfer function seen by noise source *e* $_1$*(n)* is written as,

$$\boldsymbol{H(Z) = \frac{1}{(1-0.5z^{-1})(1-0.6z^{-1})}}$$

The noise transfer function seen by *e2(n)* is written as

$$\boldsymbol{H_2(z) = \frac{1}{1-0.6z^{-1}}}$$

Output noise power due to the noise signal e_l *(n)* is,

$$\sigma_{e01}^2 = \frac{\sigma e^2}{2\pi j}\oint H(z)\;H(z^{-1})\,z^{-1}dz$$

Here,

$$\oint H(z)\;H(z^{-1})\,z^{-1}dz = \frac{z^{-1}}{(1-0.5z^{-1})(1-0.6z^{-1})(1-0.5z)(1-0.6z)}$$

Here, the stable poles are 0.5 and 0.6 and the unstable pole are (1/0.5) and (1/0.6). Only consider the stable pole.

$$\mathrm{Res}[H(z)H(z^{-1})z^{-1}]|(z=0.5)=$$
$$=(z-0.5)\frac{z^{-1}}{(z-0.5)(z-0.6)\,(z^{-1}-0.5)\,(z^{-1}-0.6)}\Bigg|\, z=0.5$$

= -9.5238

$$\mathrm{Res}[H(z)H(z^{-1})z^{-1}]|(z=0.6)=$$
$$=(z-0.6)\frac{z^{-1}}{(z-0.5)(z-0.6)\,(z^{-1}-0.5)\,(z^{-1}-0.6)}\Bigg|\, z=0.6$$

= 13.3928

$$\sigma_{e01}^2=\sigma_e^2[-9.5238+13.3928]$$

$$\sigma_{e01}^2=\sigma_e^2[3.8690]$$

The steady state output noise power due to $e_2(n)$ is,

$$\sigma_{e02}^2=\frac{\sigma e^2}{2\pi j}\oint H_2(z)H_2(z^{-1})z^{-1}\,dz$$

$$H_2(z)H_2(z^{-1})z^{-1}=\frac{z^{-1}}{(z-0.6)(z^{-1}-0.6)}$$

Here the stable poles are 0.6 and the unstable poles are (1/0.6). Only consider the stable pole

$$\mathrm{Res}[H_2(z)H_2(z^{-1})z^{-1}]|(z=0.6)=(z-0.6)\frac{z^{-1}}{(z-0.6)\,(z^{-1}-0.6)}\Bigg|\, z=0.6$$

=1.5626

$$\sigma_{e01}^2=\frac{\sigma e^2}{2\pi j}\oint H(z)\;H(z^{-1})\,z^{-1}dz$$

$$\sigma_{e01}^2=\sigma_e^2[1.5626]$$

Total Output noise power

$$\sigma_{e0}^2=\sigma_{e01+}^2\sigma_{e02}^2$$

$$=\sigma_e^2[3.8690+1.5626]$$

$$\boxed{\sigma_{e0}^2=\sigma_e^2[5.4315]}$$

For Example b= 4 bits,

$$\sigma_e^2=\frac{q^2}{12}$$

$$q = \frac{R}{2^b} = \frac{2}{2^4} = \frac{1}{2^3} = \frac{1}{8}$$

$$\sigma_e^2 = \frac{(\frac{1}{8})^2}{12} = \frac{q^2}{12}$$

$$\sigma_e^2 = 1.3021X\,10^{-3}$$

$$\sigma_{e0}^2 = \sigma_e^2[5.4315]$$

$$\sigma_{e0}^2 = 1.3021X\,10^{-3} \quad X \quad [5.4315]$$

The total roundoff noise power is given by,

$$\boxed{\sigma_{e0}^2 = 7.0718\,X\,10^{-3}}$$

6.Find the output round off noise power for the system having transfer function

$$\boldsymbol{H \quad (z) = \frac{1}{(1-0.5z^{-1})(1+0.4z^{-1})}}$$

Which is realized in cascade form? Assume the word length is 4 bits[CO4-H1]

The roundoff noise model for**cascade form**is *H(z) = HI (Z)H2(Z)* is shown in Figure . From the realization, the noise transfer function seen by noise source $e_1(n)$ is written as,

$$\boldsymbol{H(Z) = \frac{1}{(1-0.5z^{-1})(1+0.4z^{-1})}}$$

The noise transfer function seen by *e2(n)* is written as

$$\boldsymbol{H_2(z) = \frac{1}{1+0.4z^{-1}}}$$

Output noise power due to the noise signal e_l *(n)* is,

$$\sigma_{e01}^2 = \frac{\sigma e^2}{2\pi j}\oint H(z)\; H(z^{-1})\, z^{-1} dz$$

Here,

$$\oint H(z)\; H(z^{-1})\, z^{-1} dz = \frac{z^{-1}}{(1-0.5z^{-1})(1+0.4z^{-1})(1-0.5z)(1+0.4z)}$$

Here, the stable poles are 0.5 and 0.4 and the unstable pole are (1/0.5) and (1/0.4). Only consider the stable pole.

$$\text{Res}[H(z)H(z^{-1})z^{-1}]|(z=0.5)=$$
$$= (z-0.5)\frac{z^{-1}}{(z-0.5)(z-0.6)\,(z^{-1}-0.5)\,(z^{-1}-0.6)}\Big|\, z=0.5$$

= -9.5238

$$\text{Res}[H(z)H(z^{-1})z^{-1}]|(z=0.4)=$$
$$= (z-0.4)\frac{z^{-1}}{(z-0.5)(z-0.4)\,(z^{-1}-0.5)\,(z^{-1}-0.4)}\Big|\, z=0.4$$

= -5.95

$$\sigma_{e01}^2 = \sigma_e^2[-9.5238 - 5.95]$$

$$\sigma_{e01}^2 = \sigma_e^2[-15.4738]$$

The steady state output noise power due to $e_2(n)$ is,

$$\sigma_{e02}^2 = \frac{\sigma e^2}{2\pi j}\oint H_2(z)H_2(z^{-1})z^{-1}\,dz$$

$$H_2(z)H_2(z^{-1})z^{-1} = \frac{z^{-1}}{(z-0.4)(z^{-1}-0.4)}$$

Here the stable poles are 0.6 and the unstable poles are (1/0.6). Only consider the stable pole

$$\text{Res}[H_2(z)H_2(z^{-1})z^{-1}]|(z=0.4) = (z-0.4)\frac{z^{-1}}{(z-0.4)\,(z^{-1}-0.4)}\Big|\, z=0.4$$

=1.19

$$\sigma_{e01}^2 = \frac{\sigma e^2}{2\pi j}\oint H(z)\;H(z^{-1})\,z^{-1}dz$$

$$\sigma_{e01}^2 = \sigma_e^2[1.19]$$

Total Output noise power

$$\sigma_{e0}^2 = \sigma_{e01+}^2\sigma_{e02}^2$$

= σ_e^2[-15.4738 + 1.19]

$$\boxed{\sigma_{e0}^2 = \sigma_e^2[-14.2838]}$$

Given, b= 4 bits,

$$\sigma_e^2 = \frac{q^2}{12}$$

$$q = \frac{R}{2^b} = \frac{2}{2^4} = \frac{1}{2^3} = \frac{1}{8}$$

$$\sigma_e^2 = \frac{(\frac{1}{8})^2}{12} = \frac{q^2}{12}$$

$$\sigma_e^2 = 1.3021 X\, 10^{-3}$$

$$\sigma_{e0}^2 = \sigma_e^2 [-14.2838]$$

$$\sigma_{e0}^2 = 1.3021 X\, 10^{-3} \quad X \quad [-14.2838]$$

The total roundoff noise power is given by,

$$\boxed{\sigma_{e0}^2 = -18.598\, X\, 10^{-3}}$$

7.For a second order IIR filter $H(z) = \frac{1}{(1-0.9\, z^{-1})(1-0.8z^{-1})}$ find the effect of shift in pole location with 3-bit coefficient presentation in direct from and cascade form. [CO4-H1-May/June 2015]

Given data:

$$H(z) = \frac{1}{(1-0.9\, z^{-1})(1-0.8z^{-1})}$$

$$H(z) = \frac{z^2}{(z-0.9\,)(z-0.8)}$$

The roots of the denominator of H(z) are the original poles

P1=0.9, P2=0.8

Direct form:

$$H(z) = \frac{1}{(1-0.9\, z^{-1})(1-0.8z^{-1})}$$

$$H(z) = \frac{1}{1-0.8z^{-1} - 0.9\, z^{-1} + 0.72\, z^{-2}}$$

$$H(z) = \frac{1}{1-1.7z^{-1} + 0.72\, z^{-2}}$$

Let us quantize the coefficient by **truncation of 3 bit**

$$1.7 \frac{convert}{binary} (1.1011)_2 \frac{convert}{3\ bits} (1.101)_2 \frac{convert}{decimal} (2.625)_{10}$$

1.101=1X2^{1}.1X2^{-1}+0X2^{-2} + 1X2^{-3}

0.70X2=**1**.4

=2 + . 0.5+ 0 + 0.125

↓

0.40X2=**0**.8 $=(2.625)_{10}$

0.80X2=**1**.6

0.60X2=1.2

$$0.72 \frac{convert}{binary}(0.1011)_2 \frac{truncate}{3\ bits}(0.101)_2 \frac{convert}{decimal}(0.62)_{10}$$

0.72X2=**1**.44 $(0.101)_2$ =1X2^{-1}+0X2^{-2}+1X2^{-3}

0.44X2=**0**.88 =0.5+ 0 + 0.125

0.88X2=**1**.76 ↓ $=(0.625)_{10}$

0.76X2=**1**.52

Let $\overline{(H(z))}$ be the transfer function. After quantizing the co-efficient.

$$\overline{H(z)} = \frac{1}{1 - 2.625\, z^{-1} + 0.625 z^{-2}}$$

The new poles are,

Pd_1=2.360

Pd_2=0.264

Compare P_1, Pd_1 and P_2, Pd_2. We can observe that there is a lot of difference in the position of quantize and unquantized poles.

Cascade form:

We know that for cascade form,

$$\boldsymbol{H(z) = H_1(z).H_2(z)}$$

Given,

$$H(z) = \frac{1}{(1 - 0.9\, z^{-1})(1 - 0.8 z^{-1})}$$

$$Therefore, H_1(z) = \frac{1}{1 - 0.9\, z^{-1}}, \qquad H_2(z) = \frac{1}{1 - 0.8\, z^{-1}}$$

P1=0.9, P2=0.8

Let us quantize the co-efficient of $H_1(z)$and $H_2(z)$

$$0.9 \frac{convert}{binary}(0.1110)_2 \frac{truncate}{3\ bits}(0.111)_2 \frac{convert}{decimal}(0.875)_{10}$$

0.9X2=**1**.80 ↓
0.80X2=**1**.60
0.60X2=**1**.20
0.20X2=**0**.40

$(0.111)_2$→1X2[-1]+1X2[-2]+1X2[-3]
=0.5+0.25+0.125
=0.875

$$0.8 \frac{convert}{binary} (0.1100)_2 \frac{truncate}{3\ bits} (0.110)_2 \frac{convert}{decimal} (0.75)_{10}$$

0.80X2=**1**.60 ↓
0.60X2=**1**.20
0.20X2=**0**.40
0.40X2=**0**.80

$(0.110)_2$→1X2[-1]+1X2[-2]+0X2[-3]
=0.5+0.25
=0.75

$$\overline{H_1(z)} = \frac{1}{1-0.875\ z^{-1}} = PC_1 = 0.875$$
$$\overline{H_2(z)} = \frac{1}{1-0.75\ z^{-1}} = PC_2 = 0.75$$

On comparing the poles of cascade system after quantization with the unquantized coefficients P_1, Pc_1 and P_2, Pc_2 are having slight difference in their poles.

Conclusion:

• From direct form, we can see that the quantized poles deviate very much from the original poles .

• From cascade form, we can see that one pole is exactly the same while the other pole is very close to the original pole.

8.For the second order IIR filter, the system function is,

$$H(Z) = \frac{1}{(1-0.5z^{-1})(1-0.45z^{-1})}$$

Find the effect of shift in pole location with 3 bit coefficient representation in direct and cascade forms. [CO4-H1]

Solution:

$$H(Z) = \frac{1}{(1-0.5z^{-1})(1-0.45z^{-1})} = \frac{z^2}{(z-0.5)(z-0.45)}$$

Original poles of H(Z) is P_1= 0.5 and P_2= 0.45.

CASE 1: DIRECT FORM

$$H(Z) = \frac{1}{(1-0.5z^{-1})(1-0.45z^{-1})} = \frac{1}{(1-0.95z^{-1}+0.225z^{-2})}$$

Quantization of coefficient by truncation

convert to binary truncate to 3 bits convert to decimal
0.95_{10} → 0.1111_2 → 0.111_2 → 0.875_{10}
convert to binary truncate to 3 bits convert to decimal
0.225_{10} → 0.0011_2 → 0.001_2 → 0.125_{10}

$$H(Z) = \frac{1}{(1 - 0.875z^{-1} + 0.125z^{-2})}$$

$$H(Z) = \frac{1}{(1 - 0.695z^{-1})(1 - 0.179z^{-1})}$$

The poles are at P_1 = 0.695 and P_2= 0.179.
Case *(ii)* Cascade Form

Given,

$$H(Z) = \frac{1}{(1 - 0.5z^{-1})(1 - 0.45z^{-1})}$$

Quantization of coefficient by truncation

convert to binary truncate to 3 bits convert to decimal
0.5_{10} 0.1000_2 → 0.100_2 → 0.5_{10} →
convert to binary truncate to 3 bits convert to decimal
0.45_{10} → 0.0111_2 → 0.375_{10} →

$$H(Z) = \frac{1}{(1 - 0.5z^{-1})(1 - 0.375z^{-1})}$$

The poles are P_1 = 0.5 and P_2 = 0.375
Conclusion:
• From direct form, we can see that the quantized poles deviate very much from the original poles .
• From cascade form, we can see that one pole is exactly the same while the other pole is very close to the original pole.

9.Briefly explain Limit Cycle oscillation [CO4-H1-May/June 2011]

(a).ZERO-INPUT LIMIT CYCLE OSCILLATION
In recursive systems, when the input is zero or some non-zero constant value, the non-linearity's due to finite precision arithmetic operation may cause periodic oscillations, in the output. During periodic oscillations, the output *y(n)* of a system will oscillate between a finite positive and negative value for increasing *n* or the output will become constant for increasing n. Such oscillations are called **limit cycles**. If the system output enters a limit cycle, it will continue to

remain in limit cycle even when the input is made zero. Hence, these limit cycles are also called zero input limit cycles.
Consider the following difference equation of first order system with one pole only.
$y(n) = ay(n - 1) + x(n)$
The system has one product $ay(n - 1)$. If the product is quantized to finite word length then the response $y(n)$ will deviate from actual value. Let $y'(n)$ be theresponse of the system when the product is quantized.
$y'(n) = Q[ay'(n - 1)] + x(n)$

Dead Band
In a limit cycle the amplitudes of the output are confined to a range of values, whichis called the dead band of the filter.
For a first order system described by the equation, $y(n) = ay(n - 1) + x(n)$, the dead band is given by
The limit cycles occur as a result of the quantization effects in multiplications. The amplitude of the output during a limit cycle are confined to a range of values that is called the dead band of the filter.
The dead band is given by

$$Dead\ band = \pm\frac{2^{-b}}{1-|a|} = \left[\frac{-2^{-b}}{1-|a|}, \frac{2^{-b}}{1-|a|}\right]$$

whereb=number of bits (including sign bits) used to represent the product. Fora second order system described by the difference equation
$\mathbf{y(n) = a_1y(n - 1) + a_2y(n - 2) + x(n)}$,the dead band is

$$Dead\ band = \pm\frac{2^{-b}}{1-|a_2|} = \left[\frac{-2^{-b}}{1-|a_2|}, \frac{2^{-b}}{1-|a_2|}\right]$$

(b) Overflow limit cycle oscillation .

- In addition to limit cycle oscillations caused by rounding, the result of multiplication, there are several types of limit cycle oscillation caused by addition , that make the filter output oscillate between maximum and minimum amplitudes. Such limit cycles have been referred to as overflow oscillations.
- In fixed point addition the overflow occurs when the sum exceeds the finite word length of the register used to store the sum. The overflow addition may lead to oscillation in the output, termed as overflow limit cycle oscillations
- The oscillation can be eliminated if the saturation arithmetic is performed. The characteristics of saturation order is shown in the figure. In the saturation arithmetic when an overflow is sensed, the output in set equal to the maximum allowable value and when an underflow is sensed, the output is set equal minimum allowable value.
- Let us consider two positive numbers n_1 and n_2 , i.e,

n_1 = 0.111 ------> 7/8
n_2 = 0.110 ------> 6/8
n_1+n_2= 1.101 ----> 5/8 in sign magnitude.

In this example, when two positive numbers are added, the sum is wrongly interpreted as negative number.

10.Explain the characteristics of a limit cycle oscillation with respect to the system described by the equation
y(n) = 0.95y(n* - 1) + *x(n).
Determine the dead band of the filter.(Assume sign magnitude is 5 bit)
[CO4-H1-Nov/Dec 2014]

Given that
$y(n) = 0.95y(n-1) + x(n)$
Let $y'(n)$ be the response of system when the product is quantized by rounding.
.'. $y'(n) = Q[0.95y(n-1)] + x(n)$
Where Q is quantization.Given that 5 bit sign-magnitude binary representation with 4 bit for magnitude and 1 bit for sign.
Let
$y'(n) = 0$ for $n < 0$
and

$$x(n) = \begin{cases} 0.75, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

when n=0,
y'(n)=Q[ay'(n-1)]+ x(n)
$Y'(0) = Q[0.95\, Y(-1)] + x(O)$
$= Q[0.95 \times 0] + 0.75$
$= 0.75$
$Y'(0) = 0.75_{10} = 0.1100_2$
Covert to binary round to 4 bitconvert to decimal
$0.75_{10} \longrightarrow 0.11000_2 \longrightarrow 0.1100_2 \longrightarrow 0.75_{10}$
when*n*=1,
y'(n)=Q[ay'(n-1)]+ x(n)
$Y'(1) = Q[0.95\, Y(0)] + x(1)$
$= Q[0.95 \times 0.75] + 0$
$= Q[0.7125]$
Covert to binary round to 4 bitconvert to decimal
$0.7125_{10} \longrightarrow 0.10110_2 \longrightarrow 0.1011_2 \longrightarrow \mathbf{0.6875_{10}}$
$Y(1) = 0.6875_{10}$
when*n*=2,
y'(n)=Q[ay'(n-1)]+ x(n)
$Y'(2) = Q[0.95\, Y(1)] + x(2)$
$= Q[0.95 \times 0.6875] + 0$
$= Q[0.653125]$
Covert to binary round to 4 bitconvert to decimal
$0.653125_{10} \longrightarrow 0.10100_2 \longrightarrow 0.1010_2 \longrightarrow \mathbf{0.625_{10}}$
$Y(2) = 0.625_{10}$
when*n*=3,
y'(n)=Q[ay'(n-1)]+ x(n)
$Y'(3) = Q[0.95\, Y(2)] + x(3)$
$= Q[0.95 \times 0.625] + 0$

= Q[0.59375]

Covert to binary round to 4 bitconvert to decimal

0.593755_{10} ⟶ 0.10011_2 ⟶ 0.1010_2 ⟶ $\mathbf{0.625_{10}}$

Y(3) = $\mathbf{0.625_{10}}$

Thus, *y'* (2) =*y'* (3), and hence for all values of *n* ≥ 2, *y' (n)* will remain as 0.625.
Therefore, the system enters into the limit cycle when *n* = 2.
For the first order system with only one pole, dead band is given by

$$Dead\ band = \pm\frac{2^{-b}}{1-|a|} = \left[\frac{-2^{-b}}{1-|a|}, \frac{2^{-b}}{1-|a|}\right]$$

where*b*is number of bit in binary representation and lal= $|0.95|$

$$Dead\ band = \pm\frac{2^{-5}}{1-|0.95|} = \left[\frac{-2^{-5}}{1-|0.95|}, \frac{2^{-5}}{1-|0.95|}\right]$$

=[-0.625 , + 0.625]

11. IIR causal filter has the system function

$$H(z) = \frac{z}{z-0.97}$$

Assume that the input signal is zero valued and the computed output signal values are rounded to one decimal place. Show that under those stated conditions, the filter output exhibits dead band effect. What is the dead band range? [CO4-H3]

Solution

Given

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z}{z-0.97}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0.97z^{-1}}$$

$$X(z) = Y(z) - 0.97z^{-1}Y(z)$$

Taking inverse z-transform on both sides we get

y(n) - 0.97y(n - 1) = *x(n)*

y(n) = 0.97y(n - 1) + *x(n)*

Let *y' (n)* be the response of the system when the product is quantized by rounding

y' (n) = Q[O.97y' (n - 1)] + *x(n)*

For a causal filter

y(n) = 0, for *n* <0

let,

$$x(n) = \begin{cases} 11, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

when*n*=0,

y'(n)=Q[ay'(n-1)]+ x(n)

Y'(0) = *Q [0.97 Y* (-1)]+*x(O)*
= *Q [0.97*x 0] + 11
Y'(0) = 11
when *n*=1,
y'(n)=Q[ay'(n-1)]+ x(n)
Y'(1) = *Q [0.97 Y* (0)]+*x(1)*
= *Q [0.97*x 11] + 0
= Q[10.67]
Covert to binary round to 1 decimal convert to decimal
10.67_{10} → 1010.101_2 → 1010.1_2 → $\mathbf{10.5_{10}}$
Y(1) = $\mathbf{10.5_{10}}$
when *n*=2,
y'(n)=Q[ay'(n-1)]+ x(n)
Y'(2) = *Q [0.957Y* (1)]+*x(2)*
= *Q [0.97*x 10.5] + 0
= Q[10.185]
Covert to binary round to 1 decimal convert to decimal
10.185_{10} → 1010.001_2 → 1010_2 → $\mathbf{10_{10}}$
Y(2) = $\mathbf{10_{10}}$
when *n*=3,
y'(n)=Q[ay'(n-1)]+ x(n)
Y'(3) = *Q [0.97 Y* (2)]+*x(3)*
= *Q [0.97*x 10] + 0
= Q[9.7]
Covert to binary round to 1 decimal convert to decimal
9.7_{10} → 1001.101_2 → 1001.1_2 → $\mathbf{9.5_{10}}$
Y(3) = $\mathbf{9.5_{10}}$

when *n*=4,
y'(n)=Q[ay'(n-1)]+ x(n)
Y'(4) = *Q [0.97 Y* (3)]+*x(4)*
= *Q [0.97*x 9.5] + 0
= Q[9.215]
Covert to binary round to 1 decimal convert to decimal
9.215_{10} → 1001.001_2 → 1001_2 → $\mathbf{9_{10}}$
Y(4) = $\mathbf{9_{10}}$
when *n*=5,
y'(n)=Q[ay'(n-1)]+ x(n)
Y'(4) = *Q [0.97 Y* (4)]+*x(5)*
= *Q [0.97*x 9] + 0
= Q[8.73]
Covert to binary round to 1 decimal convert to decimal
8.73_{10} → 1000.101_2 → 1000.1_2 → $\mathbf{8.5_{10}}$
Y(5) = $\mathbf{8.5_{10}}$

when *n*=6,
y'(n)=Q[ay'(n-1)]+ x(n)
Y'(6) = *Q [0.97 Y* (5)]+*x(6)*
= *Q [0.97*x 8.5] + 0

= Q[8.245]

Covert to binary round to 1 decimal convert to decimal

8.245_{10} ⟶ 1000.001_2 ⟶ 1000_2 ⟶ $\mathbf{8_{10}}$

Y(6) = 8_{10}

when *n*=7,

y'(n)=Q[ay'(n-1)]+ x(n)

Y'(7) = *Q [0.97 Y* (6)]+*x(7)*

= *Q [0.97*x 8] + 0

= Q[7.76]

Covert to binary round to 1 decimal convert to decimal

7.76_{10} ⟶ 0111.110_2 ⟶ 1000_2 ⟶ $\mathbf{8_{10}}$

Y(7) = 8_{10}

Thus, *y'* (7) = *y'* (6) and hence for all values of *n*≥ 6, *y' (n)* will remain as 8.
Therefore, the system enters into limit cycle when *n* = 6

$$Dead\ band = \pm\frac{2^{-b}}{1-|a|} = \left[\frac{-2^{-b}}{1-|a|}, \frac{2^{-b}}{1-|a|}\right]$$

$$Dead\ band = \pm\frac{2^{-2}}{1-|0.97|} = \left[\frac{-2^{-2}}{1-|0.97|}, \frac{2^{-2}}{1-|0.97|}\right]$$

=[-8.333 , + 8.333]

Thus, the dead band interval is [-8.333,8.333].

12.Find the study state variance of the noise in the output due to quantization of input for the first order filter[CO4-L1]

Solu:

Y(n)=ay(n-1)+x(n)

Taking Z-transform on both sides. We have

Y(Z)=aZ^{-1}Y(Z)+X(Z)

H(Z)=$\frac{Y(Z)}{X(Z)}=\frac{1}{1-aZ^{-1}}=\frac{Z}{Z-a}$

H(Z^{-1})=$\frac{Z^{-1}}{Z^{-1}-a}$

WE KNOW

$$\sigma_e^2 = \sigma_e^2 \frac{1}{2\pi j} \oint_c H(Z)H(Z^{-1})Z^{-1}dZ$$

$$\sigma_e^2 = \sigma_e^2 \frac{1}{2\pi j} \oint_c \frac{Z^{-1}}{(Z^{-1}-a)} \frac{Z}{(Z-a)} Z^{-1}dZ$$

$$= \sigma_e^2 \left[\left[residue\ of\ \frac{Z^{-1}}{(Z-a)(Z^{-1}-a)} \right]_{Z=a} + \left[residue\ of\ \frac{Z^{-1}}{(Z-a)(Z^{-1}-a)} \right]_{Z=\frac{1}{a}} \right]$$

$$= \sigma_e^2 \left[(Z-a) \frac{Z^{-1}}{(Z-a)(Z^{-1}-a)} \right]_{Z=a}$$

$$= \sigma_e^2 \frac{a^{-1}}{a^{-1}-a}$$

$$= \sigma_e^2 \frac{1}{1-a^2}$$

$$= \frac{2^{-2b}}{12} \left[\frac{1}{1-a^2} \right]$$

The impulse response for the above filter is given by h(n)=$a^n u(n)$

$$\sigma_e^2 = \sigma_e^2 \sum_{n=0}^{\infty} h^2(n) = \sigma_e^2 \sum_{n=0}^{\infty} a^{2n}$$

$= \sigma_e^2[1 + a^2 + a^4 + \cdots + \infty]$u(n)= $\begin{cases} 0 & for\ n < 0 \\ 1 & for\ n \geq 0 \end{cases}$ $= \sigma_e^2 \frac{1}{1-a^2} = \frac{2^{-2b}}{12} \left[\frac{1}{1-a^2} \right]$

13. Realize the first order transfer function H(z)=$\frac{1}{1-aZ^{-1}}$ and draw its quantization noise model. Find the steady state noise power due to product round off.

[CO4-H1]

$$\sigma_0^2 = \sigma_e^2 \frac{1}{2\pi j}\oint_c \mathrm{H}_1(Z)\mathrm{H}_1(Z^{-1})Z^{-1}dZ$$

$$\sigma_0^2 = \sigma_e^2 \frac{1}{2\pi j}\oint_c \frac{1}{1-aZ^{-1}}\frac{1}{1-aZ}Z^{-1}dZ$$

$$= \sigma_e^2 \left|(Z-a)\frac{Z^{-1}}{(1-aZ^{-1})(1-aZ)}\right|_{Z=a}$$

$$\sigma_0^2 = \frac{2^{-2b}}{12}\left(\frac{1}{1-a^2}\right)$$

15.The output signal of A/D converter is passed through a first order LPF, with transfer function given by

$$\mathbf{H(z)} = \frac{(1-a)Z}{Z-a} \qquad for\ 0$$

Find the steady state output noise power due to quantization at the output of digital filter. **[CO4-H1]**

$$\mathrm{H}(Z^{-1}) = \frac{(1-a)Z^{-1}}{Z^{-1}-a}$$

$$\sigma_\epsilon^2 = \sigma_e^2 \frac{1}{2\pi j}\oint_c \mathrm{H}_1(Z)\mathrm{H}_1(Z^{-1})Z^{-1}dZ$$

$$\sigma_\epsilon^2 = \sigma_e^2 \frac{1}{2\pi j}\oint_c \frac{(1-a)^2Z^{-1}}{(Z^{-1}-a)(Z-a)}dZ$$

Pole Z=a, Z=1/a

$$= \sigma_e^2 \left|(z-a)\frac{(1-a)^2Z^{-1}}{(Z^{-1}-a)(Z-a)} + 0\right| = \sigma_e^2 \left|\frac{(1-a)^2}{1-a^2}\right|$$

Unit – V

Dsp Applications

Part – A

1.What is Echo cancellation? [CO5-L1-May/June 2012]
Echo cancellation is the process of removing echo from a voice communication.

2.What are narrowband filters? [CO5-L1]
A common need in electronics and DSP is to isolate narrow band of frequencies from a wider bandwidth signal. For example you want to eliminate 60Hz interference in a instrumentation system or isolate the signaling tones in a telephone network. Two types of frequency responses are available. (i) Narrowband Low pass filter (ii) Narrowband Band pass filter.

3.What is decimator or down sampling? [CO5-L1-Nov/Dec 2013]
The process of reducing the sampling rate by a factor D (down sampling by D) of a signal is called decimation.(i.e sampling rate expansion).

4.What is interpolator or up sampling? [CO5-L1]
The process of increasing the sampling rate by a factor I (up sampling by I)of a signal is interpolation. (i.e sampling rate expansion).

5.What is sampling rate conversion? [CO5-L1-May/June 2013]
The process of converting the signal from given sample rate to a different sample rate is called sampling rate conversion.

6.What are the drawbacks in multi stage implementation? [CO5-L1-Nov/Dec 2013]
(i) While converting digital signal x(n) in analog, D\A converter introduces distortion in signal rebuilding.
(ii) A/D converter gives quantization error

7.What do you mean by sub band coding? (OR) Define sub-band coding. [CO5-L1]
Sub band coding is a method where speech signal is sub-divided into several frequency bands and each band is digitally encoded separately by allocating different bits per sample to the signal of different sub bands.

8.What is the need for multirate signal processing? [CO5-L1-May/June 2010]
In telecommunication systems, different types of signals have to be processed at a different rate. The system that employs multiple sampling rates in the processing of different signal is known as multirate signal processing system.

9.What is meant by adaptive equalization? [CO5-L1-Nov/Dec 2010]
In digital communication system the bandwidth of channel has to be used efficiently. The requirement is to design a reliable system with data to be transmitted at a higher rate. The factors that affect the data in the channel are inter symbol interference and thermal noise. Adaptive equalizer is used for compensating the channel distortion so that the detected signal is reliable.

10.State the basic operations of Multirate signal processing? [CO5-L1]
The two basic operations in multirate signal processing are decimation and interpolation. Decimation reduces that sampling rate, whereas interpolation increases the sampling rate.

11.What is the effect of Downsampling on the spectrum of a signal? [CO5-L1]
Downsampling or decimation is used to avoid aliasing effect.

12.What is an anti-imaging filter? [CO5-L1]
The filter which is used to remove the image spectra is known as anti-imaging filter.

13.What are the methods are sampling rate conversion of a digital signal? (OR) Give the steps in multistage sampling rate converter design. [CO5-L1]
Sampling rate conversion can be done in i) analog domain and ii) digital domain.
In analog domain using DAC the signal is converted into analog and then filtering is applied. Then the analog signal is converted back to digital using ADC. In digital domain all processing is done with signal in digital form. In the first method, the new sampling rate doesn't have any relationship with old sampling rate. But major disadvantage is the signal distortion. So the digital domain sampling rate conversion is preferred even then the new sampling rate depends on the old sampling rate.

14.What is the advantage & disadvantage of multirate signal processing? [CO5-L1]
Advantage: new sampling rate is selected which do not have any special relationship to old sampling rate.
Disadvantage: signal distortion introduced by D/A convertors.

15.What are the applications of sub band coding? [CO5-L1]
- Filter design.
- Synthesis method.
- Data compression in image signal processing.

16.What is speech synthesis? What are its applications? [CO5-L1]
A machine is developed which can accept as input a piece of English text and convert it to natural sounding speech.

Applications:

- Speech output from computers

- Reading machine for visually challenged.
- Accessing medical records stored in central computers.

17.Define speech coding. And its applications.
(OR) What do you mean by speech compression? [CO5-L1]
Speech coding is concerned with the developments of techniques which exploits the redundancy is the speech signal, in order to represent the less bits reduce the number of bits to present it.
Applications:

- Voice mail systems
- Cordless telephone channel
- Narrow band cellular radio
- Military communication

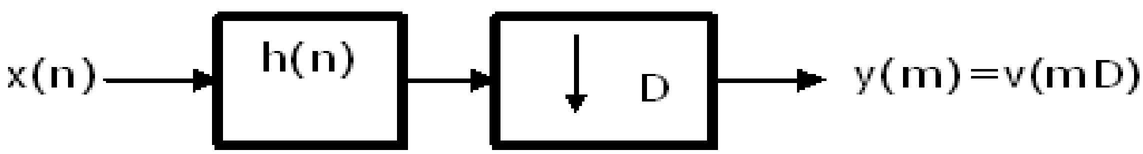
18.What are the classifications of speech sounds? [CO5-L1-Nov/Dec 2010]
- Voiced sounds –vocal card vibrates produces quasi-periodic pulses.
- Fricative or unvoiced sounds – noise excitation.
- Plosive sounds-rapid release of pressure.

19.List the application of adaptive filtering? [CO5-L1]
- Noise cancellation
- Signal prediction
- Adaptive feedback cancellation
- Echo cancellation

Part-B

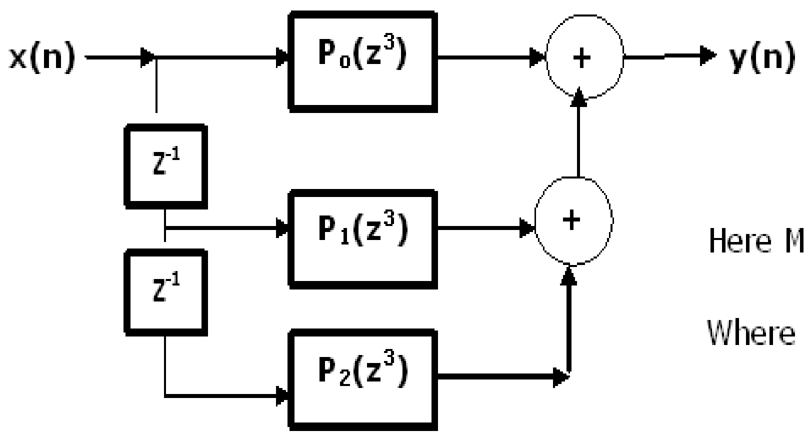
1.Explain the efficient transversal structure of decimator & interpolator?
[CO5-L2-Nov/Dec 2013]

1. Decimation system or decimator process:



Decimated sequence y(m) is obtained by passing the input sequence x(n) through a linear filter h(n) & then down sampling the filtered output by a factor D at a sampling rate (Fx)

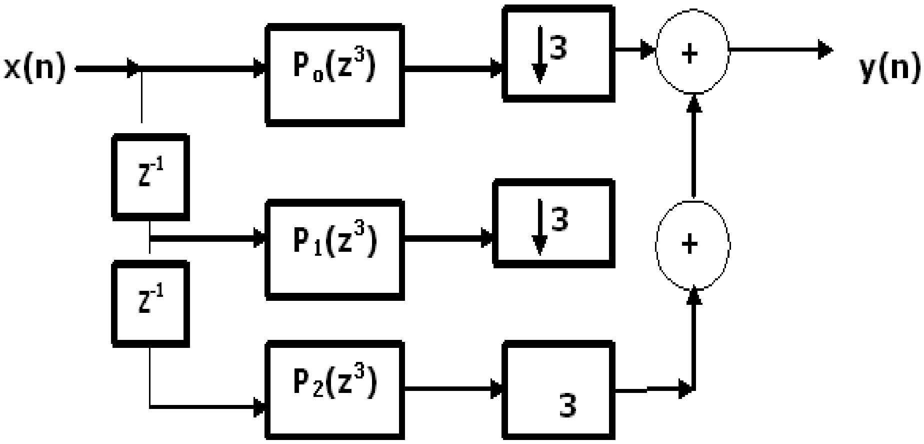
Block diagram of polyphase structure for M=3:



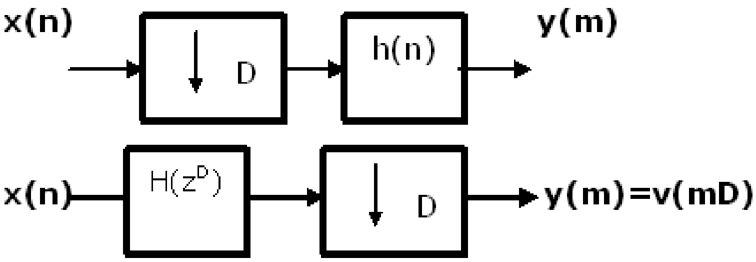
Here M=3,

Where M→Polyphase decomposition factor.

Combining above two block diagrams (structures) we obtain the following structure.

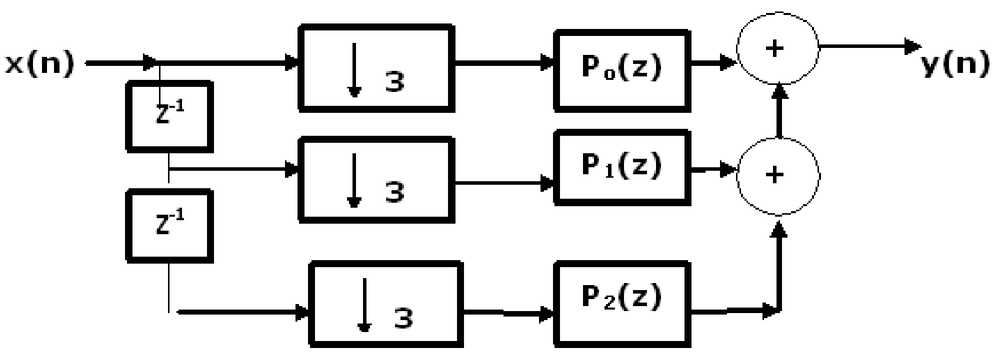


The two equivalent down sampling systems are



If we apply the above identity we obtain the desired implementation of decimation system using a polyphase structure as follows.

If we apply the above identity we obtain the desired implementation of decimation system using a polyphase structure as follows.

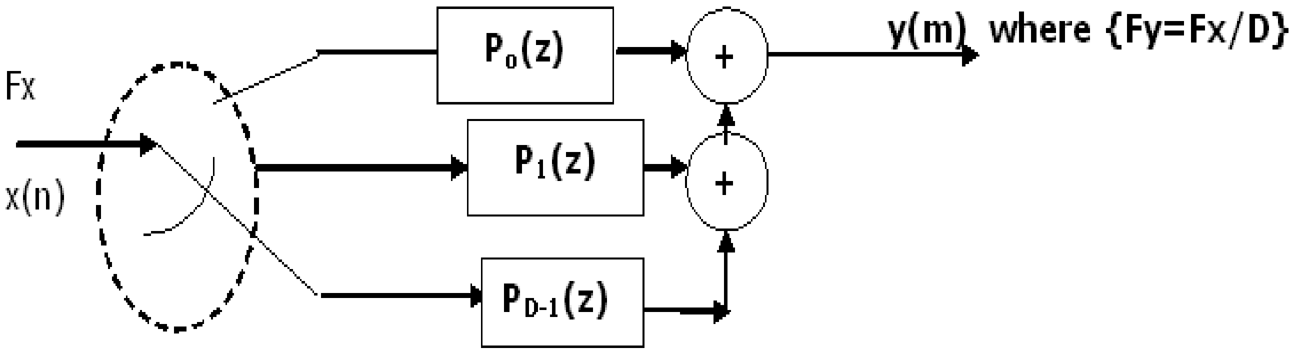


This is the final implementation of decimator using the polyphase structure. In this filtering structure only the needed samples are computed at the sampling rate (Fx/D) and thus we have achieved the desired efficiency. Additional reduction in computation can be achieved by using a FIR linear phase filter & utilizing the property of symmetry of h(n).

Implementation of polyphase decimator using a commutator model:

Commutator rotates counter clockwise starting at n=0 and distributes the block of D input samples to the polyphase filter starting i=D-1 and continuing in reverse order until i=0.

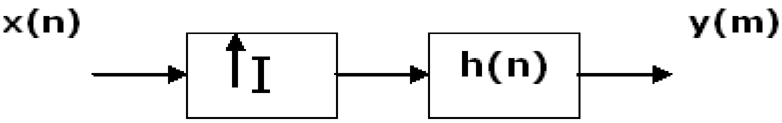
The model of decimation using a polyphase filter and a commutator is as follows,



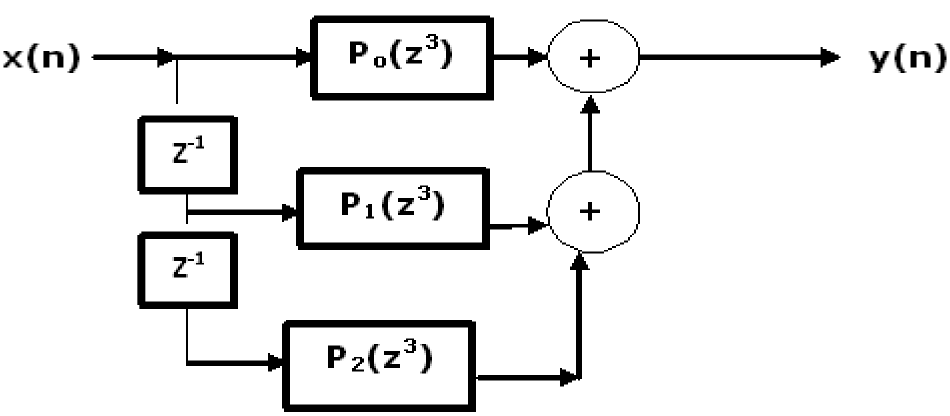
Polyphase implementation of interpolators:

Let us consider the effective implementation of interpolators which is realized by inserting 'i-1' zeros between successive samples of x(n) and then filtering the resultant sequence. The major problems with these filter computations are performed at high sampling rate, high effects.

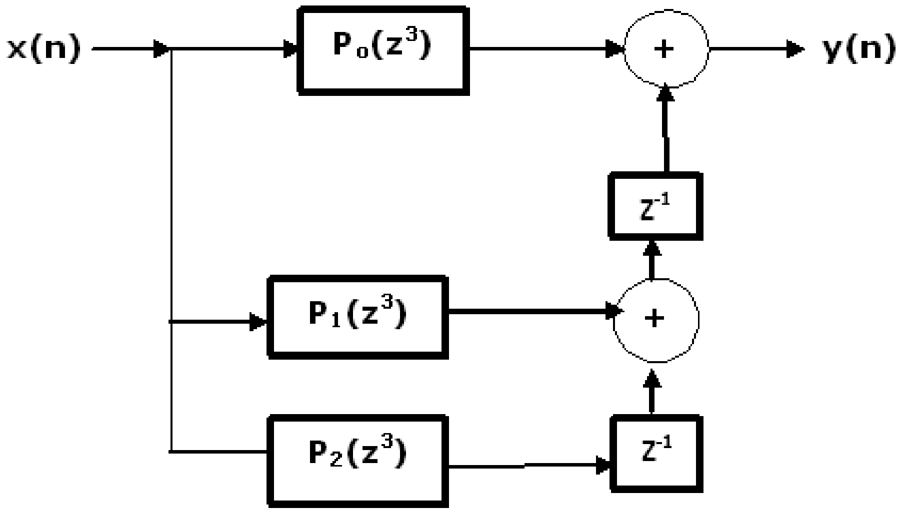
Interpolation system (or) Interpolation process:



Block diagram of polyphase structure for M=3:



Block diagram of transpose polyphase structure for M=3:



The desired simplification is achieved by combining the basic interpolation process with transpose polyphase structure.

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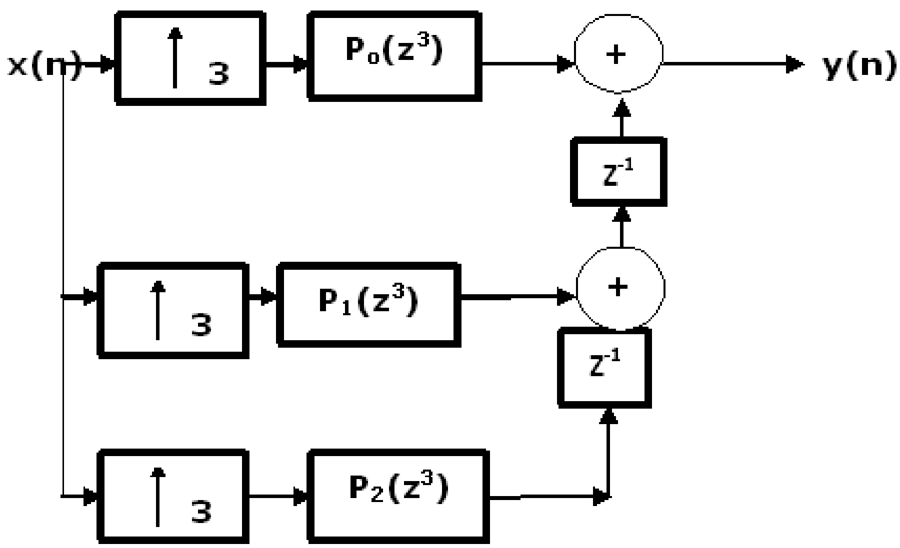
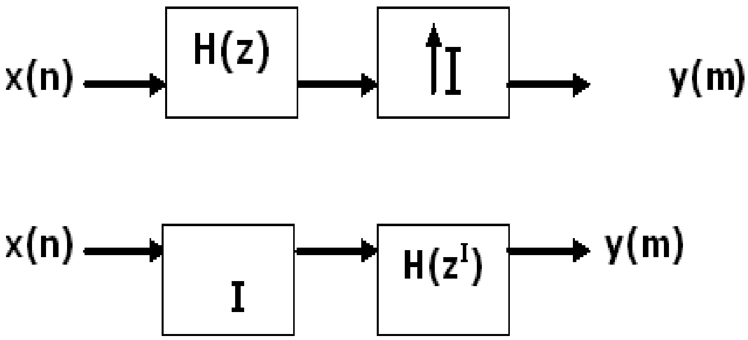
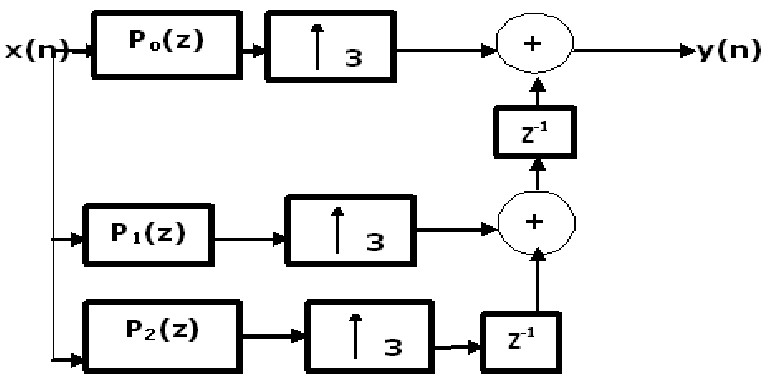


The two equivalent upsampling systems are :



If we apply the above identity we obtain the desired implementation of interpolator system using a polyphase structure as follows.



Thus all the filtering multiplications are performed at sampling rate effects.

2.Explain interpolation by a factor I? [CO5-L2-Nov/Dec 2010]

Interpolation: Sampling rate increase by a factor I

Introduction:

Interpolation is obtained by introducing (I-1) new samples or zeros between successive values of the signal.

Let x(n) be a input signal, after adding (I-1) zeros between successive values of the x(n), we obtain

γ(m)={ x(m/I) , where m=0,±I, ±2I and 0 otherwise }

Taking z-transform of γ(m),

$V(z) = \sum_{m=-\infty}^{\infty} \gamma(m) z^{-m}$

Since γ(m) is obtained from x(m),

Let γ(m)=x(m)

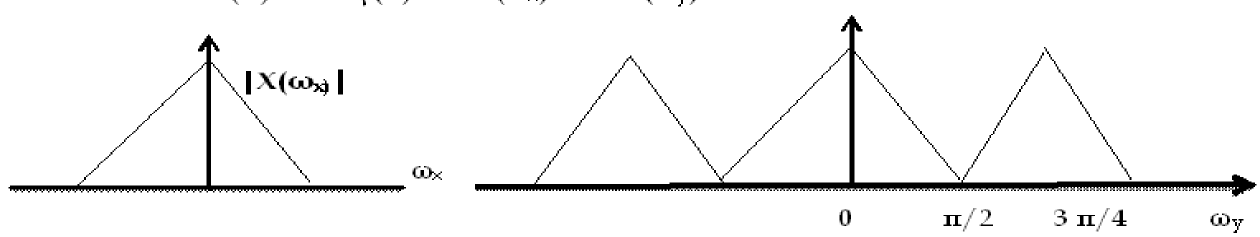
Therefore $V(z) = \sum_{m=-\infty}^{\infty} \gamma(m) z^{-ml}$

➔V(z)=X(z^{l})

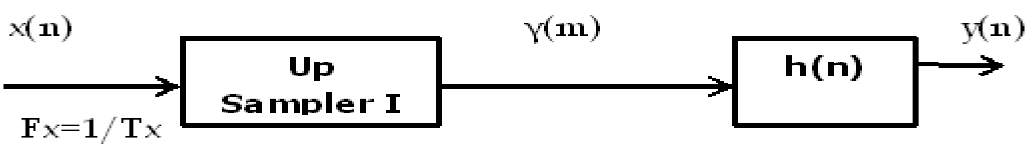
By evaluating V(z) in the unit circle, we obtain $V(\omega_y)$

Therefore $V(\omega_y)=X(\omega_y\, l)$, where $\omega_y = \frac{2\pi F}{Fy}$ and Fy=lFx also $\omega_{y=}\omega_{x/l}$

SPECTRUM OF x(n) AND γ(n) are $X(\omega_x)$ and $V(\omega_y)$



Interpolation Process:



By passing γ(m) through a LPF with frequency response $H_l(\omega_y)$,

Where $H_l(\omega_y)$={ C, $0\leq |\omega_y| \leq \pi/l$ and =0 otherwise}

Here C is a scale factor to normalize output sequence y(m).

Output spectrum

$Y(\omega_y)$={ $CX(\omega_y l)$, $0\leq |\omega_y| \leq \pi/l$ and =0 otherwise}

(i.e) $Y(\omega_y) = CX(\omega_y l)$→equation (A)

Scale factor C is selected so that output

y(m) =x(m/l) ; for m=0,±l,±2l….

Let m=0, Therefore $y(0) = \frac{1}{2\pi}\int_{\pi}^{-\pi} Y(\omega_y)d\omega_y$

From equation (A)

$y(0) = \frac{C}{2\pi}\int_{\pi/l}^{-\pi/l} X(\omega_y l)d\omega_y$

Since $\omega_{y} = \{\omega_x/l\}$ →$\omega_{x} = \{\omega_y l\}$

$y(0) = \frac{C}{2\pi}\int_{\pi/l}^{-\pi/l} X(\omega_x)d\omega_x$

$y(0) = \frac{C}{I2\pi}\int_{\pi/I}^{-\pi/I} X(\omega_x)d\omega_x$

$y(0)=\frac{C}{I}x(0)$,Therefore C=I is the desired normalization factor.

Since y(m)=γ(m)*h(n)

$y(m)=\sum_{K=-\infty}^{\infty} h(m-k)\gamma(m) \rightarrow$ equation (B)

As γ(k)=0 except at multiplies of I, Where γ(kI)=x(k)

Therefore equation B becomes

$y(m)=\sum_{K=-\infty}^{\infty} h(m-kI)x(k)$ this is the final expression of interpolation.

3.Explain multirate implementation of sampling rate conversion? [CO5-L2]

Introduction:

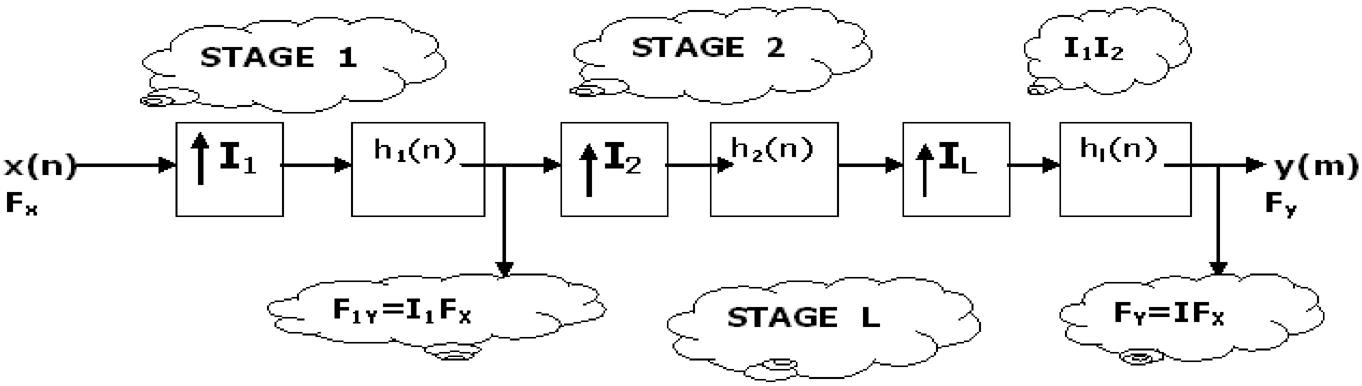
In practical applications of sampling rate conversion, decimation factors and interpolation factors are larger than unity. i.e D>>1 and I>>1.

For example, if sampling rate I/D =130/63, it requires 130 interpolators and 63 decimators i.e. polyphase filters, which makes computationally inefficient.

Let us consider some methods for performing sampling rate conversion for D>>1 and I>>1in multiple stages.

CASE 1: INTERPOLATION BY A FACTOR I>>1 [OR]

MULTISTAGE IMPLEMENTATION OF INTERPOLATION BY 'I'.



Input signal x(n) has a sampling rate F_x

Output signal y(m) has a sampling rate $F_y=I\,F_x$ where L→No.of.Stages

Output of 1st stage sampling rate= $F_x I_1$

Output of 2nd stage sampling rate= $F_x I_1 I_2$

Each stage has Interpolator (I) & Filter (h(n))

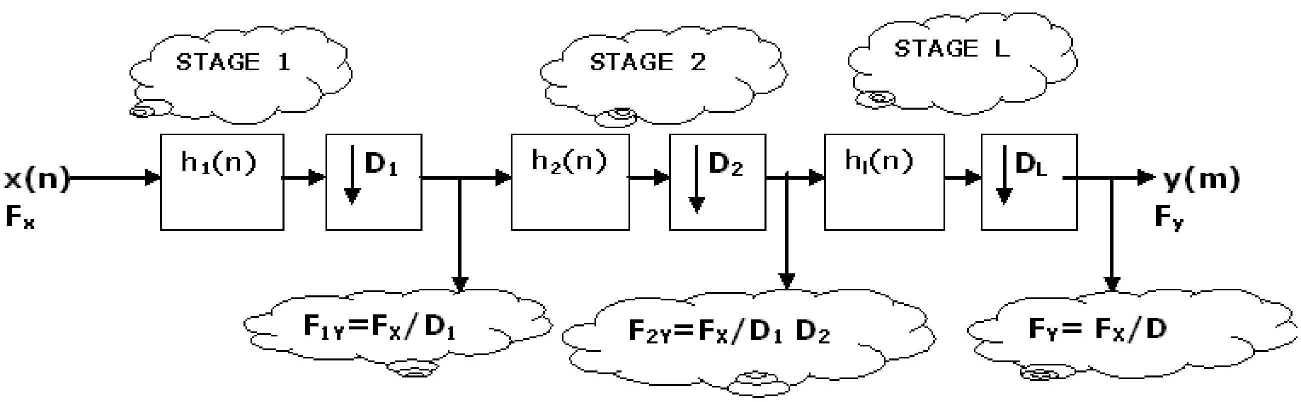
Let I can be factored into a product of positive integers as

$$I = \prod_{i=1}^{L} \mathbf{Ii}$$

Sampling rate at the output of the ith stage is $\mathbf{F_i=F_{i-1}I}$ where i=1,2,…L

Interpolator eliminates the images introduced by upsampling process.

CASE 2: DECIMATION BY A FACTOR D>>1 [OR] MULTISTAGE IMPLEMENTATION OF DECIMATOR BY 'D'.



Input signal x(n) has a sampling rate F_x

Output signal y(m) has a sampling rate $F_y = F_x/D$ where L→No.of.Stages

Output of 1st stage sampling rate= F_x/D_1

Output of 2nd stage sampling rate= F_x/D_1D_2

Each stage has Decimator (D) & Filter (h(n))

Let I can be factored into a product of positive integers as

$$D = \prod_{i=1}^{L} \mathbf{Di}$$

Sampling rate at the output of the ith stage is $\mathbf{F_i=F_{i-1}/D}$ where i=1, 2,…L

Decimator eliminates the images introduced by down sampling process.

4.Discuss About Musical Sound Processing[CO5-L1-May/June 2010]

The musical programs are produced in two stages. In the first stage the sound from each individual singer or instrument is recorded in acoustically inert studio on single track of multitrack tape. In the second stage, the special audio effects are added to these individual sounds. Then these individual sounds are combined in the mix-down system to generate stereo recording. The special audio effects can be generated by DSP as discussed next.

Generation of echo effects

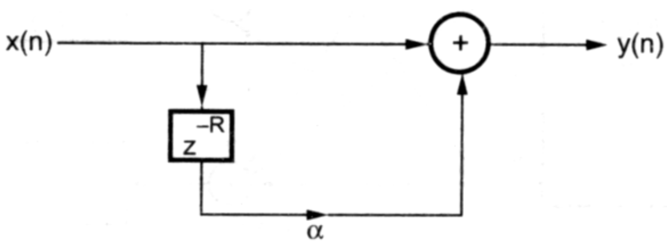
The echoes are generated by delay units. For example direct sound and its single echo can be generated with the help of FIR filter having following difference equation.

$y(n) = x(n) + \alpha x(n-R)$ ...(5.7.1)

Hence $x(n)$ is the direct sound and $x(n-R)$ is the echo delayed by 'R' sampling periods. The amplitude of echo is 'α'. The transfer function of this filter will be,

$H(z) = 1 + \alpha z^{-R}$ ... (5.7.2)

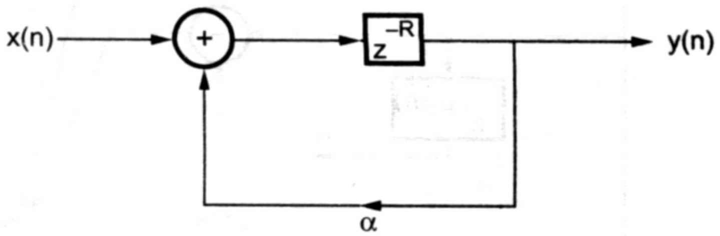
Fig. shows the realization of such filter.



The multiple echoes which are spaced *'R'* sampling periods apart and exponentially decaying amplitudes can be generated with the help of FIR filter having transfer function as follows.

Thus the above filter can be realized by IIR filter also. An infinite number of echoes having exponentially decaying amplitudes and spaced at *'R'* sampling periods can be generated by IIR filter having following transfer function.

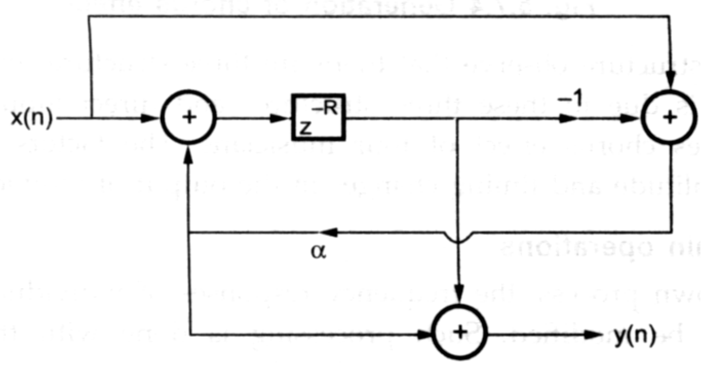
Fig. shows realization of this filter.



Generation of reverberation

The echoes generated by schemes discussed just now do not provide natural sounding. This is because their magnitude is not constant at all frequencies and the output echo density is much lower than that observed in a real room. This causes a fluttering of the composite sound.

Hence reverberation is used that generates flutterfree sounds. The reverberation needs 1000 echoes per second. The allpass filter of Fig. can be used to generate reverberation.



The transfer function of the allpass reverberator shown in above figure is given as,

$$H(z) = \frac{\alpha + z^{-R}}{1+\alpha z^{-R}}, \quad |\alpha| < 1$$

Since this is an allpass filter, the magnitude of all the components remains same. Hence it provides more natural sounding.

Generation of chorus effect

The chorus effect is obtained when several musicians play the same musical piece at the same time. The chorus effect is generated because of amplitude and timing differences of individual musicians. Such effect can be generated with the help of chorus generator. Fig. shows the structure for generating chorus effect.

In the above structure observe that there are three structures connected in parallel. The output *y(n)* is due to these three structures and direct input. Hence the above structure generates chorus effect of four musicians. The factors βi(n)are randomly varied to get amplitude and timing changes in the output of individual structures.

Frequency domain operations

In the mix-down process, the frequency responses of individually recorded pieces of sound are to be modified. Such processing is done with the help of graphic equalizer. The graphic equaliser provides bass-treble adjustments. The graphic equalizer can be implemented with the help of cascaded digital filters.

5.What Are Adaptive Filters? Explain With Its Block Diagram And Its Application? [CO5-L2-May/June 2010]

Principle: The coefficients of the filter are changed automatically according to the changes in input signal. This means the filtering characteristics of the adaptive filter are changed or adapted according to the changes in input signal.

Block diagram

Fig. shows the block diagram of adaptive filter which is used as simple noise canceller. The signal to be filtered is noisy, i.e. *y(n).* It consists of signal *s(n)* plus

noise *n(n).* Only noise from the same signal source is *x(ri)* This noise is given to adaptive filter. The characteristics of the adaptive filter are changed by adaptive algorithm. The adaptive algorithm modifies the filter coefficients depending upon input noise and output signal estimate. The adaptive filter produces an estimate of noise n*(n).* This estimate is subtracted from noisy signal i.e.,

s(n) = *y(n)-n(n)*

= *s(n) + n(n) - n(n)*

Thus output signal s(n) is the estimate of noise free signal. The adaptive filter and adaptive algorithm combinely forms an adaptive filter.

Advantages

i) Adaptive filters extract the signal from noisy signals whose characteristics vary with time.

ii) Adaptive filters extract the signals where signal and noise have the same spectrum.

iii) The filtered signal contains minimum distortion,

iv) Adaptive filters have built in flexibility.

Applications

i) Filtering of artefacts from the human EEC

ii) Filtering of jamming signal in spread spectrum communication.

iii) Adaptive echo cancellation on telephone channels.

iv) Radar signal processing

v) Navigational systems.

vi)Adaptive equalization of communication channels.

Adaptive filtering algorithms

i) Least mean square algorithm

ii) Recursive least squares algorithm.

These algorithms are used to adaptively calculate filter coefficients.

6.Explain in detail about Adaptive noise Cancellation – Principles Adaptive noise Cancellation – Principles[CO5-L2-Nov/Dec 2011]

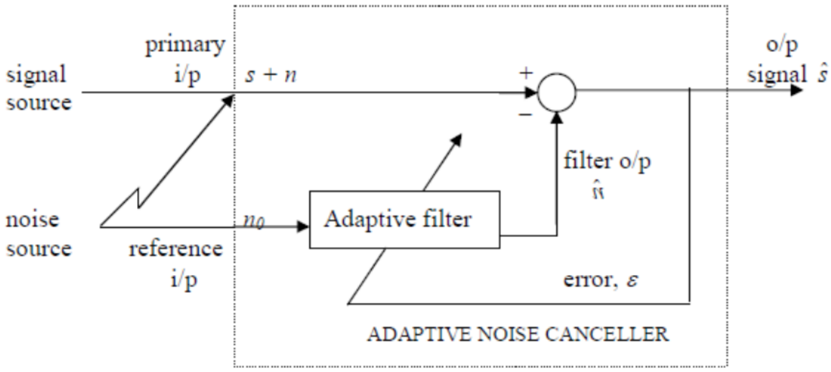


Fig. 1 Adaptive Noise Canceller

As shown in the figure, an Adaptive Noise Canceller (ANC) has two inputs – primary and reference. The primary input receives a signal s from the signal source that is corrupted by the presence of noise n uncorrelated with the signal. The reference input receives a noise n0 uncorrelated with the signal but correlated in some way with the noise n. The noise no passes through a filter to produce an output nˆ that is a close estimate of primary input noise. This noise estimate is subtracted from the corrupted signal to produce an estimate of the signal at sˆ , the ANC system output. In noise canceling systems a practical objective is to produce a system output sˆ = s + n – nˆ that is a best fit in the least squares sense to the signal s. This objective is accomplished by feeding the system output back to the adaptive filter and adjusting the filter through an LMS adaptive algorithm to minimize total system output power.

In other words the system output serves as the error signal for the adaptive process.

Assume that s, n0, n1 and y are statistically stationary and have zero means. The signal s is uncorrelated with n0 and n1, and n1 is correlated with n0.

This is equivalent to causing the output sˆ to be a best least squares estimate of the signal s.

7.Explain the different applications of adaptive filtering. [CO5-L2]

Introduction:

Adaptive Filtering is a specialized branch of Digital Signal Processing, dealing with adaptive filters and system design. They are used in a wide range of applications including system identification, noise cancellation, interference nullities, signal prediction, echo cancellation, beam forming and adaptive channel equalization.

1. System identification

The system identification is an approach to model an unknown system. In this configuration the unknown system is in parallel with an adaptive filter, and both are excited with the same signal. When the output MSE is minimized the filter represents the desired model. The structure used for adaptive system identification is illustrated in figure 1, where P(z) is an unknown system to be identified by an adaptive filter W(z). The signal x(n) excites P(z) and W(z), the desired signal d(n) is the unknown system output, minimizing the difference of output signals y(n) and d(n), the characteristics of P(z) can be determined.

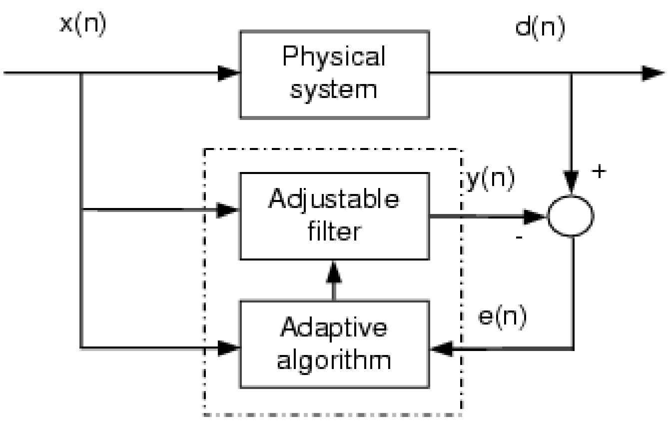


Fig. 1. Adaptive filter for system identification

The estimation error is given as (2)

When the difference between the physical system response d(n) and the adaptive model response y(n) has been minimized, the adaptive model approximates P(z) from the input/output viewpoint. When the plan is time varying, the adaptive algorithm has the task of keeping the modeling error small by continually tracking time variations of the plant dynamics.

Usually, the input signal is a wideband signal, in order to allow the adaptive filter to converge to a good model of the unknown system. If the input signal is a white noise, the best model for the unknown system is a system whose impulse response coincides with the N + 1 first samples of the unknown system impulse response. In the cases where the impulse response of the unknown system is of finite length and the adaptive filter is of sufficient order, the MSE becomes zero if there is no measurement noise (or channel noise).

In practical applications the measurement noise is unavoidable, and if it is uncorrelated with the input signal, the expected value of the adaptive-filter coefficients will coincide with the unknown-system impulse response samples.

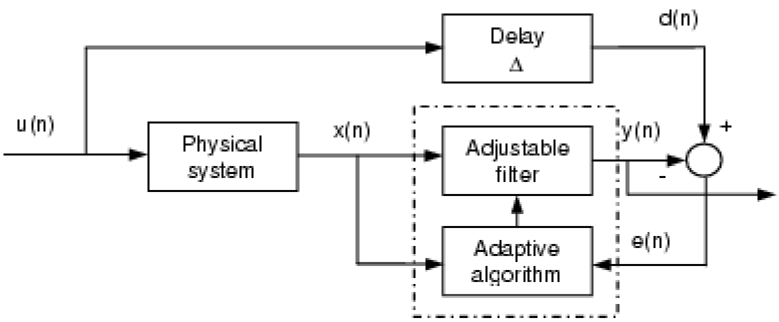
2. Linear predictor

The linear prediction estimates the values of a signal at a future time. This model is wide usually in speech processing applications such as speech coding in cellular telephony, speech enhancement, and speech recognition. In this configuration the desired signal is a forward version of the adaptive filter input signal. When the adaptive algorithm convergences the filter represents a model for the input signal, this model can be used as a prediction model. The linear prediction system is shown in figure 2.

A typical predictor's application is in linear prediction coding of speech signals, where the predictor's task is to estimate the speech parameters. These parameters are part of the coding information that is transmitted or stored along with other information inherent to the speech characteristics, such as pitch period, among others. The adaptive signal predictor is also used for adaptive line enhancement (ALE), where the input signal is a narrowband signal (predictable) added to a wideband signal. After convergence, the predictor output will be an enhanced version of the narrowband signal. Yet another application of the signal predictor is the suppression of narrowband interference in a wideband signal. The input signal, in this case, has the same general characteristics of the ALE.

3. Inverse modeling

The inverse modeling is an application that can be used in the area of channel equalization, for example it is applied in modems to reduce channel distortion resulting from the high speed of data transmission over telephone channels. In order to compensate the channel distortion we need to use an equalizer, which is the inverse of the channel's transfer function. High-speed data transmission through channels with severe distortion can be achieved in several ways, one way is to design the transmit and receive filters so that the combination of filters and channel results in an acceptable error from the combination of intersymbol interference and noise; and the other way is designing an equalizer in the receiver that counteracts the channel distortion. The second method is the most commonly used technology for data transmission applications. Figure 3 shows an adaptive channel equalizer, the received signal y(n) is different from the original signal x(n) because it was distorted by the overall channel transfer function C(z), which includes the transmit filter, the transmission medium, and the receive filter.



[OR]

In practice, the telephone channel is time varying and is unknown in the design stage due to variations in the transmission medium. Thus it is needed an adaptive equalizer that provides precise compensation over the time-varying channel. The adaptive filter requires the desired signal d(n) for computing the error signal e(n) for the LMS algorithm. An adaptive filter requires the desired signal d(n) for computing the error signal e(n) for the LMS algorithm. The delayed version of the transmitted signal x(n - Δ) is the desired response for the adaptive equalizer W(z). Since the adaptive filter is located in the receiver, the desired signal generated by the transmitter is not available at the receiver. The desired signal may be generated locally in the receiver using two methods. During the training stage, the adaptive equalizer coefficients are adjusted by transmitting a short training sequence. This known transmitted sequence is also generated in the receiver and is used as the desired signal d(n) for the LMS algorithm.

After the short training period, the transmitter begins to transmit the data sequence. In the data mode, the output of the equalizer x(n) is used by a decision device to produce binary data. Assuming that the output of the decision device is correct, the binary sequence can be used as the desired signal d(n) to generate the error signal for the LMS algorithm.

4. Jammer suppression

Adaptive filtering can be a powerful tool for the rejection of narrowband interference in a direct sequence spread spectrum receiver. Figure 4 illustrates a jammer suppression system. In this case the output of the filter y(n), is an estimate of the jammer, this signal is subtracted from the received signal x(n), to yield an estimate of the spread spectrum. To enhance the performance of the system a two-stage jammer suppressor is used. The adaptive line enhancer, which is essentially another adaptive filter, counteracts the effects of finite correlation which leads to partial cancellation of the desired signal. The number of coefficients required for either filter is moderate, but the sampling frequency may be well over 400 KHz.

5. Adaptive notch filter

In certain situations, the primary input is a broadband signal corrupted by undesired narrowband (sinusoidal) interference. The conventional method of eliminating such sinusoidal interference is using a notch filter that is tuned to the frequency of the interference. To design the filter, we need the precise frequency of the interference. The adaptive notch filter has the capability to track the frequency of the interference, and thus is especially useful when the interfering sinusoid drifts in frequency. A single-frequency adaptive notch filter with two adaptive weights is illustrated in figure 5, where the input signal is a cosine signal as

For a sinusoidal signal, two filter coefficients are needed. The reference input is used to estimate the composite sinusoidal interfering signal contained in the primary input d(n). The center frequency of the notch filter is equal to the frequency of the primary sinusoidal noise. Therefore, the noise at that frequency is attenuated. This adaptive notch filter provides a simple method for eliminating sinusoidal interference.

6. Noise canceller

The noise cancellers are used to eliminate intense background noise. This configuration is applied in mobile phones and radio communications, because in some situations these devices are used in high-noise environments. Figure 6 shows an adaptive noise cancellation system.

The canceller employs a directional microphone to measure and estimate the instantaneous amplitude of ambient noise r'(n), and another microphone is used to take the speech signal which is contaminated with noise d(n) + r(n). The ambient noise is processed by the adaptive filter to make it equal to the noise contaminating the speech signal, and then is subtracted to cancel out the noise in the desired signal. In order to be effectively the ambient noise must be highly correlated with the noise components in the speech signal, if there is no access to the instantaneous value of the contaminating signal, the noise cannot be cancelled out, but it can be reduced using the statistics of the signal and the noise process.

The frequency analysis of the signals used in the noise canceller system can be seen on the spectrograms of the figure 8. The figure shows that the output signal has some additional frequency components with respect to the input signal.

The output of the noise canceller is the error signal, the figure 9 shows the error signal obtained when it is used an LMS algorithm. With the spectrogram of the signal it is shown that all the undesired frequency components were eliminated.

The adaptive noise canceller system is used in many applications of active noise control (ANC), in aircrafts is used to cancel low-frequency noise inside vehicle cabins for passenger comfort. Most major aircraft manufacturers are developing such systems, mainly for noisy propeller-driven airplanes. In the automobile industry there are active noise cancellation systems designed to reduce road noise using microphones and speakers placed under the vehicle’s seats.

Another application is active mufflers for engine exhaust pipes, which have been in use for a while on commercial compressors, generators, and such. With the price for ANC solutions dropping, even automotive manufacturers are now considering active mufflers as a replacement of the traditional baffled muffler for future production cars. The resultant reduction in engine back pressure is expected to result in a five to six percent decrease in fuel consumption for in-city driving.

Another application that has achieved widespread commercial success are active headphones to cancel low-frequency noise. The active headphones are equipped with microphones on outside of the ear cups that measure the noise arriving at the headphones. This noise is then being cancelled by sending the corresponding ”anti-noise” to the headphones’ speakers. For feed forward ANC, the unit also includes a microphone inside each ear cup to monitor the error -

the part of the signal that has not been canceled by the speakers in order to optimize the ANC algorithm. Very popular with pilots, active headphones are considered essential in noisy helicopters and propeller-powered airplanes.