

SKP Engineering College

Tiruvannamalai – 606611

A Course Material

on

Digital Communication



By

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Quality Certificate

This is to Certify that the Electronic Study Material

Subject Code: EC6501

Subject Name: Digital Communication

Year/Sem:III/V

Being prepared by me and it meets the knowledge requirement of the University curriculum.

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EC6501 DIGITAL COMMUNICATION L T P C 3 0 0 3

OBJECTIVES:

- To know the principles of sampling & quantization
- To study the various waveform coding schemes
- To learn the various baseband transmission schemes
- To understand the various Band pass signaling schemes
- To know the fundamentals of channel coding

UNIT I SAMPLING & QUANTIZATION 9

Low pass sampling – Aliasing- Signal Reconstruction-Quantization - Uniform & non-uniform quantization - quantization noise - Logarithmic Companding of speech signal- PCM
– TDM

UNIT II WAVEFORM CODING 9

Prediction filtering and DPCM - Delta Modulation - ADPCM & ADM principles- Linear Predictive Coding.

UNIT III BASEBAND TRANSMISSION 9

Properties of Line codes- Power Spectral Density of Unipolar / Polar RZ & NRZ – Bipolar NRZ - Manchester- ISI – Nyquist criterion for distortionless transmission – Pulse shaping – Correlative coding - Mary schemes – Eye pattern – Equalization

UNIT IV DIGITAL MODULATION SCHEME 9

Geometric Representation of signals - Generation, detection, PSD & BER of Coherent BPSK, BFSK & QPSK - QAM - Carrier Synchronization - structure of Non-coherent Receivers - Principle of DPSK.

UNIT V ERROR CONTROL CODING 9

Channel coding theorem - Linear Block codes - Hamming codes - Cyclic codes - Convolutional codes - Vitterbi Decoder

TOTAL: 45 PERIODS

TEXT BOOK:

1. S. Haykin, "Digital Communications", John Wiley, 2005

REFERENCES:

1. B. Sklar, "Digital Communication Fundamentals and Applications", 2nd Edition, Pearson Education, 2009
2. B.P.Lathi, "Modern Digital and Analog Communication Systems" 3rd Edition, Oxford University Press 2007.
3. H P Hsu, Schaum Outline Series - "Analog and Digital Communications", TMH 2006
4. J.G Proakis, "Digital Communication", 4th Edition, Tata Mc Graw Hill Company, 2001.

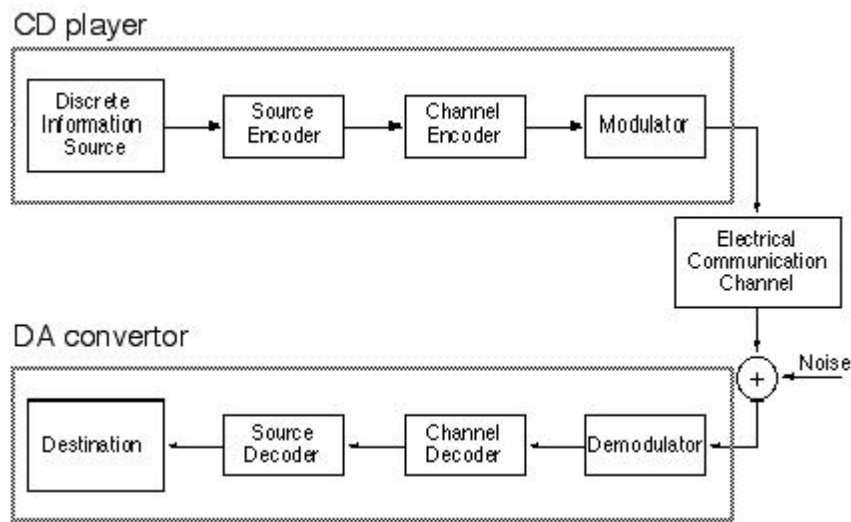
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Unit-I

Sampling & QuantizationPart A

1. Draw the typical digital communication system [AUC NOV/DEC2011]
[AUC NOV/DEC2012]



2. How can BER of an system be improved.

- Increasing the transmitted signal power
- Employing modulation and demodulation technique
- Employing suitable coding and decoding methods
- Reducing noise interference with help of improved filtering

3. Give an example for time limited and time unlimited signals [AUC APR/MAY 2011]

Time limited- rectangular pulse, triangular pulse

Time unlimited signals - sinusoidal signal, exponential signal and step signal.

4. Give the advantages and disadvantages of digital communication. [AUC APR/MAY 2011]

Advantage

Speech, video and other data can be transmitted simultaneously. Wide dynamic range is possible since data is digital.

Disadvantage

Digital communication requires synchronization.

Data rates are very high.

5. Which parameter is called figure of merit of a digital communication system and why?

The ratio E_b/N_0 or bit energy to noise power spectral density is called figure of merit of a digital communication system.

6. What is meant by distortion less transmission? [AUC NOV/DEC 2010]

For distortion less transmission, the transfer function of the system is given as, $H(\omega) = Ke^{-j\omega t_0}$

K- Constant magnitude response

The transfer function imposes two requirements on the system

1. The system response must have constant magnitude response
2. The system phase shift response must be linear with frequency

7. Why prefiltering is done before sampling [AUC NOV/DEC 2010]

The signal must be limited to some highest frequency W Hz before sampling.

Then the signal is sampled at the frequency of $f_s = 2W$ or higher. Hence the signal should be prefiltered at higher than W Hz. If the signal is not prefiltered, then frequency components higher than W Hz will generate aliasing in the sampled signal spectrum.

8. Define quantization noise power [AUC NOV/DEC 2010]

Quantisation noise power is the noise power due to quantisation noise. Let the quantisation

noise has the pdf of $f_{\epsilon}(\epsilon)$. Then Quantisation noise power is given as, $E[\epsilon]^2 = \int_{-\infty}^{\infty} \epsilon^2 f_{\epsilon}(\epsilon) d\epsilon$.

9. State sampling theorem. [AUC APR/MAY 2011] [AUC APR/MAY 2012]

A band limited signal of finite energy, which has no frequency components higher than W Hz, may be completely recovered from the knowledge of its samples taken at the rate of $2W$ samples per second.

10. What is quantization error [AUC APR/MAY 2011]

Because of quantization inherent error is introduced in the signal. The error is called Quantization error $\epsilon = x_q(nT_s) - x(nT_s)$

$x_q(nT_s)$ - quantised value of the signal

$x(nT_s)$ - value of the sample before

quantization

11. Compare uniform and non uniform quantization [AUC NOV/DEC 2011]

S.No	Uniform Quantization	Non-uniform Quantization
1	The quantization step size	The quantization step size varies with

	remains same throughout the dynamic range of the signal	the amplitude of the input signal.
2	SNR ratio varies with the input signal amplitude	SNR ratio can be maintained constant

12. What is meant by quantization? [AUC APR/MAY 2012]

The conversion of analog sample of the signal into digital form is called quantizing process

13. Differentiate the principle of temporal waveform coding and model based coding

TEMPORAL WAVEFORM CODING

The signal which varying with time can be digitized by periodic time sampling and amplitude quantization. This process is called temporal waveform coding .DM,ADM,DPCM are example of temporal waveform coding

MODEL BASED CODING

The signal is characterised in various parameter. This parameter represent the model of the signal.LPC is an example model based coding.

14. What is meant by aliasing effect?

Aliasing effect takes place when sampling frequency is less than Nyquist rate.Under such condition, the spectrum of the sampled signal overlaps with itself.Hence higher frequencies take the form of lower frequencies. This interference of the frequency components is called aliasing effects.

15. What is the effects of aliasing? How it is reduced?

- (i) Since high and low frequencies interfere with each other,distortion is generated.
- (ii) The data is lost and it can not be recovered.

Aliasing can be avoided by two methods:

- (i) Sampling rate $f_s \geq 2W$ samples/sec. Where $W \rightarrow$ Max.frequency present in the signal

(ii) Strictly bandlimit the signal to „W“Hz.

16. What is the function of Low pass filter on sampling?

A low pass filter basically a reconstruction of filter. This filter should pass all the frequencies between $(-W, W)$, Since original signal was having maximum frequency of „W“Hz. Therefore cut-off frequency of this low pass reconstruction filter will be „W“Hz.

17. Define Non-uniform quantization. (AU-Apr 2015)

In non-uniform quantization, the step size is not fixed. It varies according to certain level of input signal amplitude. Step size is small at low input signal levels and the step size is higher at high input levels. Hence signal to noise power ratio remains almost same throughout the input signal.

18. What is meant by compander?

The non-uniform quantization (variable stepsize „ ρ “) becomes very difficult to implement.

Therefore the signal is amplified at low signal levels and attenuated high signal levels. After this process, uniform quantization is used. This is equivalent to more stepsize at low signal levels and small step size at high signal levels. At the receiver a reverse process is done. That is the signal is attenuated at low signal levels and amplified at high signal levels to get original signal. Thus the compression of signal at transmitter and expansion at receiver is called combinedly as Companding.

Part - B

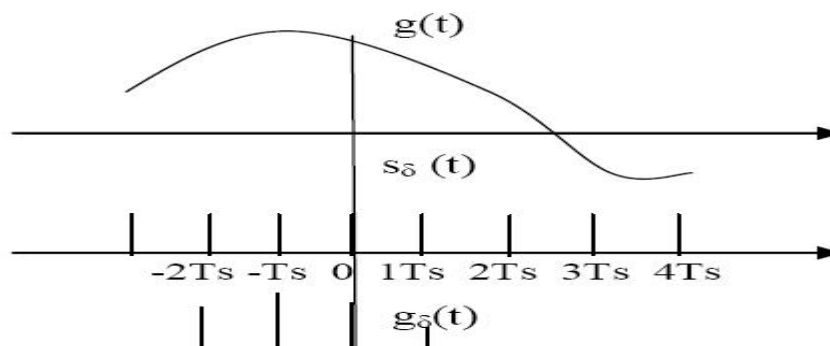
1. State and prove Sampling Theorem.

SAMPLING:

A message signal may originate from a digital or analog source. If the message signal is analog in nature, then it has to be converted into digital form before it can transmit by digital means. The process by which the continuous-time signal is converted into a discrete-time signal is called Sampling. Sampling operation is performed in accordance with the sampling theorem.

SAMPLING THEOREM FOR LOW-PASS SIGNALS:-

Statement: - "If a band-limited signal $g(t)$ contains no frequency components for $|f| > W$, then it is completely described by instantaneous values $g(kT_s)$ uniformly spaced in time with period $T_s \leq 1/2W$. If the sampling rate, f_s is equal to the Nyquist rate or greater ($f_s \geq 2W$), the signal $g(t)$ can be exactly reconstructed.



Proof:- Part - I If a signal $x(t)$ does not contain any frequency component beyond W Hz, then the signal is completely described by its instantaneous uniform samples with sampling interval (or period) of $T_s < 1/(2W)$ sec.

Part – II The signal $x(t)$ can be accurately reconstructed (recovered) from the set of uniform instantaneous samples by passing the samples sequentially through an ideal (brick-wall) lowpass filter with bandwidth B , where $W \leq B < f_s - W$ and $f_s = 1/T_s$.

If $x(t)$ represents a continuous-time signal, the equivalent set of instantaneous uniform samples $\{x(nT_s)\}$ may be represented as,

$$\{x(nT_s)\} \equiv x_s(t) = \sum x(t) \cdot \delta(t - nT_s) \text{-----} \quad \begin{array}{l} 1. \\ 1 \end{array}$$

where $x(nT_s) = x(t)|_{t=nT_s}$, $\delta(t)$ is a unit pulse singularity function and „n“ is an integer. The continuous-time signal $x(t)$ is multiplied by an (ideal) impulse train to obtain $\{x(nT_s)\}$ and can be rewritten as,

$$x_s(t) = x(t) \cdot \sum \delta(t - nT_s) \text{-----} \quad \begin{array}{l} 1. \\ 2 \end{array}$$

Now, let $X(f)$ denote the Fourier Transform $F(T)$ of $x(t)$, i.e.

$$X(f) = \int_{-\infty}^{+\infty} x(t) \cdot \exp(-j2\pi ft) dt$$

Now, from the theory of Fourier Transform, we know that the F.T of $\sum \delta(t - nT_s)$, the impulse train in time domain, is an impulse train in frequency domain:

$$F\{\sum \delta(t - nT_s)\} = (1/T_s) \cdot \sum \delta(f - n/T_s) = f_s \cdot \sum \delta(f - n f_s) \text{-- -----} \quad 1.3$$

If $X_s(f)$ denotes the Fourier transform of the energy signal $x_s(t)$, we can write using Eq. (1.2.4) and the convolution property:

$$\begin{aligned}
 X_s(f) &= X(f) * F\{\sum \delta(t - nT_s)\} \\
 &= X(f) * [f_s \cdot \sum \delta(f - nf_s)] \\
 &= f_s \cdot X(f) * \sum \delta(f - nf_s) \\
 &= f_s \cdot \int_{-\infty}^{+\infty} X(\lambda) \cdot \sum \delta(f - nf_s - \lambda) d\lambda = f_s \cdot \sum \int X(\lambda) \cdot \delta(f - nf_s - \lambda) d\lambda = f_s \cdot \sum X(f - nf_s)
 \end{aligned}$$

-----1.4

Aliasing and signal reconstruction:

Nyquist's theorems as stated above and also helps to appreciate their practical implications.

Let us note that while writing Eq.(1.4), we assumed that $x(t)$ is an energy signal so that its Fourier transform exists. With this setting, if we assume that $x(t)$ has no appreciable frequency component greater than W Hz and if $f_s > 2W$, then Eq.(1.4) implies that $X_s(f)$, the Fourier Transform of the sampled signal $X_s(t)$ consists of infinite number of replicas of $X(f)$, centered at discrete frequencies $n \cdot f_s$, $-\infty < n < \infty$ and scaled by a constant $f_s = 1/T_s$.

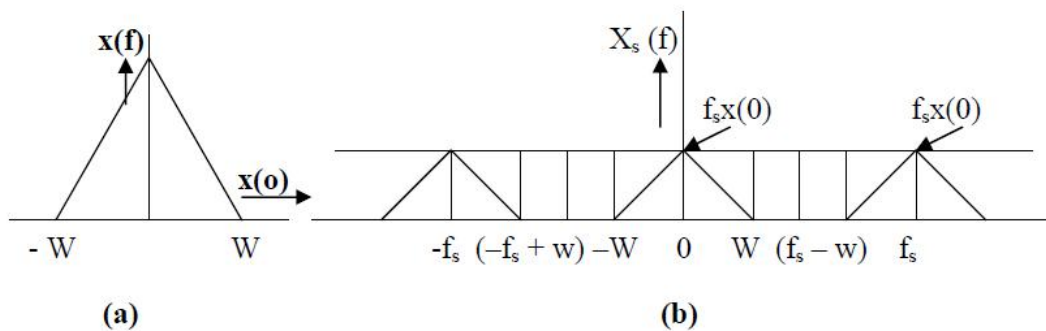


Fig. 1.2.1 Spectra of (a) an analog signal $x(t)$ and (b) its sampled version

Fig. 1.2.1 indicates that the bandwidth of this instantaneously sampled wave $x_s(t)$ is infinite while the spectrum of $x(t)$ appears in a periodic manner, centered at discrete frequency values $n \cdot f_s$. Part – I of the sampling theorem is about the condition $f_s > 2 \cdot W$ i.e. $(f_s - W) > W$ and $(-f_s + W) < -W$. As seen from Fig. 1.2.1, when this condition is satisfied, the spectra of $x_s(t)$, centered at $f = 0$ and $f = \pm f_s$ do not overlap and hence, the spectrum of $x(t)$ is present in $x_s(t)$ without any distortion. This implies that $x_s(t)$, the appropriately sampled version of $x(t)$, contains all information about $x(t)$ and thus represents $x(t)$.

The second part suggests a method of recovering $x(t)$ from its sampled version $x_s(t)$

by using an ideal lowpass filter. As indicated by dotted lines in Fig. 1.2.1, an ideal lowpass filter (with brick-wall type response) with a bandwidth $W \leq B < (f_s - W)$, when fed with $x_s(t)$, will allow the portion of $X_s(f)$, centered at $f = 0$ and will reject all its replicas at $f = n f_s$, for $n \neq 0$. This implies that the shape of the continuous time signal $x_s(t)$, will be retained at the output of the ideal filter.

2. Explain Quantization in Detail?

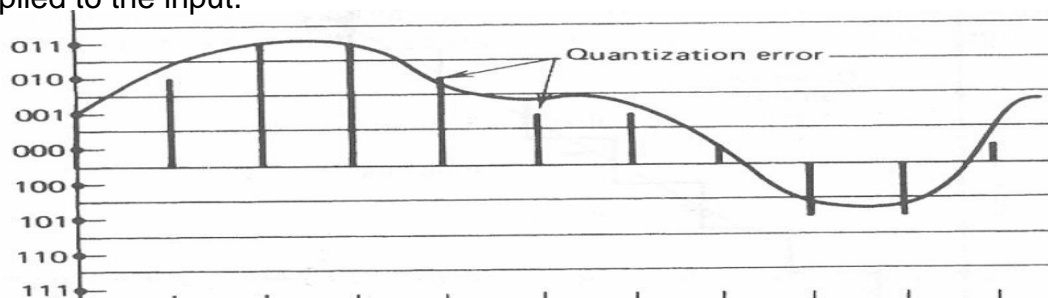
Quantization:

The process of transforming Sampled amplitude values of a message signal into a discrete amplitude value is referred to as Quantization.

The quantization Process has a two-fold effect:

1. the peak-to-peak range of the input sample values is subdivided into a finite set of decision levels or decision thresholds that are aligned with the risers of the staircase, and
2. the output is assigned a discrete value selected from a finite set of representation levels that are aligned with the treads of the staircase.

A quantizer is memory less in that the quantizer output is determined only by the value of a corresponding input sample, independently of earlier analog samples applied to the input.



Types of Quantizers:

1. Uniform Quantizer
2. Non- Uniform Quantizer

Uniform Quantizer: In Uniform type, the quantization levels are uniformly spaced, whereas in non-uniform type the spacing between the levels will be unequal and mostly the relation is logarithmic.

Types of Uniform Quantizers: (based on I/P - O/P Characteristics)

1. Mid-Rise type Quantizer
2. Mid-Tread type Quantizer

In the stair case like graph, the origin lies the middle of the tread portion in Mid – Tread type where as the origin lies in the middle of the rise portion in the Mid-Rise type.

Mid – tread type: Quantization levels – odd number.

Mid – Rise type: Quantization levels – even number.

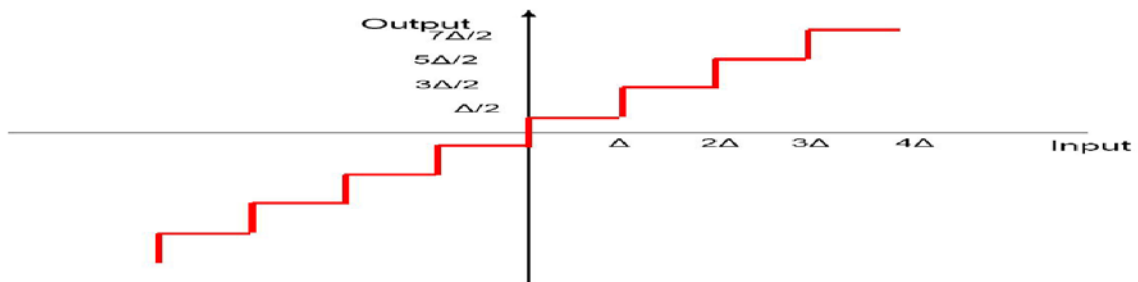


Fig:2.12 Input-Output Characteristics of a Mid-Rise type Quantizer

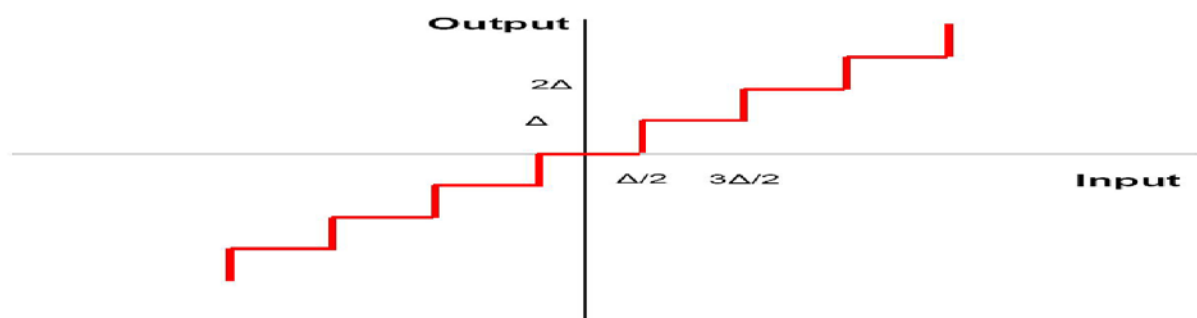


Fig:2.13 Input-Output Characteristics of a Mid-Tread type Quantizer

Quantization Noise and Signal-to-Noise:

“The Quantization process introduces an error defined as the difference between the input signal, $x(t)$ and the output signal, $y(t)$. This error is called the Quantization Noise.”

$$q(t) = x(t) - y(t)$$

Quantization noise is produced in the transmitter end of a PCM system by rounding off sample values of an analog base-band signal to the nearest permissible representation levels of the quantizer. As such quantization noise differs from channel noise in that it is signal dependent.

Let “ Δ ” be the step size of a quantizer and L be the total number of quantization levels. Quantization levels are $0, \pm \Delta, \pm 2\Delta, \pm 3\Delta, \dots$. The Quantization error, Q is a random variable and will have its sample values bounded by $[-(\Delta/2) < q < (\Delta/2)]$. If Δ is small, the quantization error can be assumed to a uniformly distributed random variable.

Consider a memory less quantizer that is both uniform and symmetric.

L = Number of quantization levels

X = Quantizer input

Y = Quantizer output

The output y is given by

$$Y=Q(x)$$

which is a staircase function that befits the type of mid tread or mid riser quantizer of interest.

Companding of Speech signal:

Compander = Compressor + Expander

In Non – Uniform Quantizer the step size varies. The use of a non – uniform quantizer is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer. The resultant signal is then transmitted.

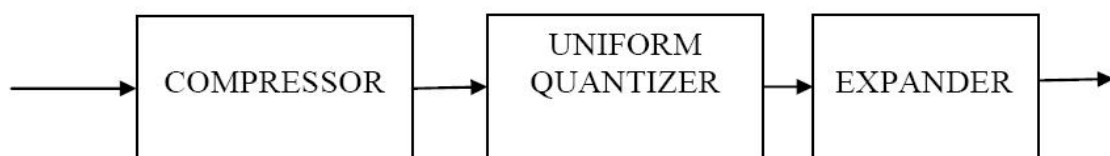


Fig: 2.14 MODEL OF NON UNIFORM QUANTIZER

At the receiver, a device with a characteristic complementary to the compressor

called Expander is used to restore the signal samples to their correct relative level. The Compressor and expander take together constitute a Componder.

Advantages of Non – Uniform Quantization:

1. Higher average signal to quantization noise power ratio than the uniform quantizer when the signal pdf is non uniform which is the case in many practical situation.

RMS value of the quantizer noise power of a non – uniform quantizer is substantially proportional to the sampled value and hence the effect of the quantizer noise is reduced.

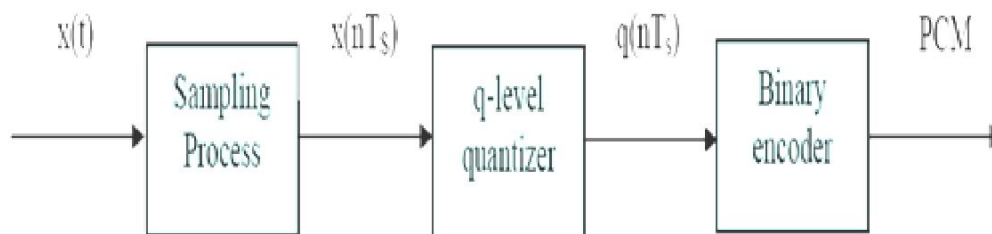
3. Explain Pulse code modulation in detail?

Pulse code Modulation:

The pulse code modulator technique samples the input signal $x(t)$ at a sampling frequency. This sampled variable amplitude pulse is then digitalized by the analog to digital converter. Figure. shows the PCM generator.

the signal is first passed through sampler which is sampled at a rate of (f_s) where:

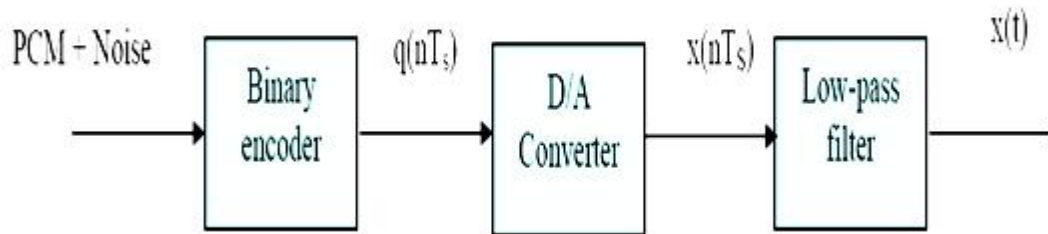
The output of the sampler $x(nT_s)$ which is discrete in time is fed to a „q“ level quantizer. The quantizer compares the input $x(nT_s)$ with it's fixed levels. It assigns any one of the digital level to $x(nT_s)$ that results in minimum distortion or error. The error is called quantization error, thus the output of the quantizer is a digital level called $q(nT_s)$. The quantized



$$f_s \geq 2f_m$$

signal level $q(nT_s)$ is binary encode. The encoder converts the input signal to v digits binary word. The receiver starts by reshaping the received pulses, removes the noise and then converts the binary bits to analog.

The received samples are then filtered by a low pass filter; the cut off frequency is at f_c . $f_c = f_m$.



It is impossible to reconstruct the original signal $x(t)$ because of the permanent quantization error introduced during quantization at the transmitter. The quantization error can be reduced by the increasing quantization levels. This corresponds to the increase of bits persample (more information). But increasing bits (v) increases the signaling rate and requires a large transmission bandwidth. The choice of the parameter for the number of quantization levels must be acceptable with the quantization noise (quantization error).

Signaling Rate in PCM

Let the quantizer use ' v ' number of binary digits to represent each level. Then the number of levels that can be represented by v digits will be : $q=2^v$

The number of bits per second is given by : (Number of bits per second) = (Number of bits per samples) \times (number of samples per second) = v (bits per sample) \times f_s (samples per second).

The number of bits per second is also called signaling rate of PCM and is Signaling rate = $v f_s$

Quantization Noise in PCM System

Errors are introduced in the signal because of the quantization process. This error is called "quantization error".

$$\epsilon = x_q(nT_s) - x(nT_s)$$

Let an input signal $x(nT_s)$ have an amplitude in the range of x_{max} to $-x_{max}$. The total amplitude range is : Total amplitude = $x_{max} - (-x_{max}) = 2x_{max}$

If the amplitude range is divided into ' q ' levels of quantizer, then the step size ' Δ '. $\Delta = q/2 \times x_{max}$. If the minimum and maximum values are equal to 1, $x_{max} = 1$, $-x_{max} = -1$, $\Delta = q/2$.

Signal to Quantization Noise ratio in PCM

The signal to quantization noise ratio is given as:

$$\frac{S}{N_q} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

$$= \frac{\text{Normalized signal power}}{\frac{\Delta^2}{12}}$$

The number of quantization value is equal to: $q=2^v$

$$\frac{S}{N_q} = \frac{\text{Normalized signal power}}{\left[\frac{2 X_{\max}}{2^v} \right]^2 * \frac{1}{12}}$$

Let the normalized signal power is equal to P then the signal to quantization noise will be given by:

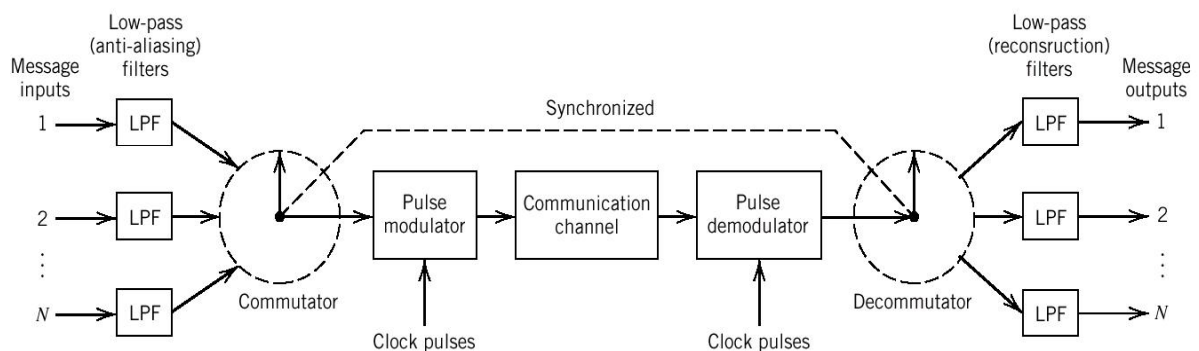
$$\frac{S}{N_q} = \frac{3 P * 2^{2v}}{X_{\max}^2}$$

Advantages of PCM

1. Effect of noise is reduced.
2. PCM permits the use of pulse regeneration.
3. Multiplexing of various PCM signals is possible.

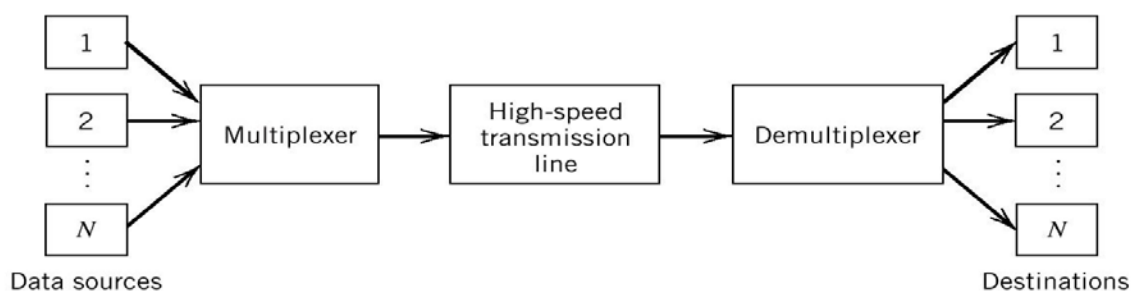
4. Explain Time Division Multiplexing?

Time-Division Multiplexing (TDM):



5. Explain Digital Multiplexers?

Digital Multiplexers:



Unit-II**Waveform Coding****Part A****1. What is meant by temporal waveform coding [AUC NOV/DEC 2011]**

The signal which varies with time can be digitized by periodic time sampling and amplitude quantization. This process is called temporal waveform coding. DM, ADM, DPCM are examples of temporal waveform coding.

2. Differentiate the principle of temporal waveform coding and model based coding.**TEMPORAL WAVEFORM CODING [AUC NOV/DEC 2012]**

The signal which varies with time can be digitized by periodic time sampling and amplitude quantization. This process is called temporal waveform coding. DM, ADM, DPCM are examples of temporal waveform coding.

MODEL BASED CODING

The signal is characterized in various parameters. This parameter represents the model of the signal. LPC is an example of model based coding.

3. What is meant by DPSK?

In DPSK, the input sequence is modified. Let input sequence be $d(t)$ and output sequence be $b(t)$. Sequence $b(t)$ changes level at the beginning of each interval in which $d(t)=1$ and it does not change level when $d(t)=0$.

When $b(t)$ changes level, phase of the carrier is changed. And as stated above, $b(t)$ changes to its level only when $d(t)=1$. This means phase of the carrier is changed only if $d(t)=1$. Hence the technique is called Differential PSK.

4. Mention the merits of DPCM.

1. Bandwidth requirement of DPCM is less compared to PCM.
2. Quantization error is reduced because of prediction filter.
3. Numbers of bits used to represent one sample value are also reduced compared to PCM.

5. What is the main difference in DPCM and DM?

DM encodes the input sample by one bit. It sends the information about $+\delta$ or $-\delta$, i.e. step rise or fall. DPCM can have more than one bit of encoding the sample. It

sends the information about difference between actual sample value and the predicted sample value.

6. What is the advantage of delta modulation over pulse modulation schemes?

Delta modulation encodes one bit per samples. Hence signalling rate is reduced in DM.

7. What is meant by adaptive delta modulation?

In adaptive delta modulation, the step size is adjusted as per the slope of the input signal. Step size is made high if slope of the input signal is high. This avoids slope overload distortion.

8. What are the two limitations of delta modulation?

- 1 Slope of overload distortion.
2. Granular noise.

9. How does Granular noise occurs?

It occurs due to large step size and very small amplitude variation in the input signal.

10. What are the advantages of the Delta modulation?

1. Delta modulation transmits only one bit for one sample. Thus the signaling rate and transmission channel bandwidth is quite small for delta modulation.
2. The transmitter and receiver implementation is very much simple for delta modulation. There is no analog to digital converter involved in delta modulation.

11. What do you understand from adaptive coding?

In adaptive coding, the quantization step size and prediction filter coefficients are changed as per properties of input signal. This reduces the quantization error and number of bits to represent the sample value. Adaptive coding is used for speech coding at low bits rates.

12. Mention the use of adaptive quantize in adaptive digital waveform coding schemes.

Adaptive quantizer changes its step size according variance of the input signal. Hence quantization error is significantly reduced due to the adaptive quantization. ADPCM uses adaptive quantization. The bit rate of such schemes is reduced due to adaptive quantization.

Part - B

1. Explain in detail about Linear Predictive Coding.

Linear Predictive Coding:

Linear predictive coding (LPC) is a tool used mostly in audio signal processing and speech processing for representing the spectral envelopment of a digital signal of speech in compressed form, using the information of a linear prediction model.

Linear prediction is a mathematical operation where future values of a discrete time signal are estimated as a linear function of previous samples.

In digital signal processing, linear prediction is often called linear predictive coding (LPC) and can thus be viewed as a subset of filter theory.

Filter design is the process of designing a signal processing filter that satisfies a set of requirements, some of which are contradictory. The purpose is to find a realization of the filter that meets each of the requirements to a sufficient degree to make it useful.

The filter design process can be described as an optimization problem where each requirement contributes to an error function which should be minimized. Certain parts of the design process can be automated, but normally an experienced electrical engineer is needed to get a good result.

In system analysis linear prediction can be viewed as a part of mathematical modeling or optimization.

Optimization is the selection of a best element (with regard to some criteria) from some set of available alternatives.

In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function. The generalization of optimization theory and techniques to other formulations comprises a large area of applied mathematics. More generally, optimization includes finding "best available" values of some objective function given a defined domain or a set of constraints, including a variety of different types of objective functions and different types of domains.

LPC starts with the assumption that a speech signal is produced by a buzzer at the end of a tube (voiced sounds), with occasional added hissing and popping sounds. Although apparently crude, this model is actually a close approximation of the reality of speech production. The glottis (the space between the vocal folds) produces the buzz, which is characterized by its intensity (loudness) and frequency (pitch). The vocal tract (the throat and mouth) forms the tube, which is characterized by its resonances, which give rise to formants, or enhanced frequency bands in the sound produced. Hisses and pops are generated by the action of the tongue, lips and throat during

sibilants and plosives.

LPC analyzes the speech signal by estimating the formants, removing their effects from the speech signal, and estimating the intensity and frequency of the remaining buzz. The process of removing the formants is called inverse filtering, and the remaining signal after the subtraction of the filtered modeled signal is called the residue.

The numbers which describe the intensity and frequency of the buzz, the formants, and the residue signal, can be stored or transmitted somewhere else. LPC synthesizes the speech signal by reversing the process: use the buzz parameters and the residue to create a source signal, use the formants to create a filter (which represents the tube), and run the source through the filter, resulting in speech.

Because speech signals vary with time, this process is done on short chunks of the speech signal, which are called frames; generally 30 to 50 frames per second give intelligible speech with good compression. It is one of the most powerful speech analysis techniques, and one of the most useful methods for encoding good quality speech at a low bit rate and provides extremely accurate estimates of speech parameters.

2. Explain in detail about Differential pulse code Modulation.

Differential Pulse Code Modulation (DPCM)

For the signals which do not change rapidly from one sample to next sample, the PCM scheme is not preferred. When such highly correlated samples are encoded the resulting encoded signal contains redundant information. By removing this redundancy before encoding an efficient coded signal can be obtained. One of such scheme is the DPCM technique. By knowing the past behavior of a signal up to a certain point in time, it is possible to make some inference about the future values. The transmitter and receiver of the DPCM scheme is shown in the below fig. Respectively. Transmitter: Let $x(t)$ be the signal to be sampled and $x(nT_s)$ be its samples. In this scheme the input to the quantizer is a signal.

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad (3.31)$$

where $\hat{x}(nT_s)$ is the prediction for unquantized sample $x(nT_s)$. This predicted value is produced by using a predictor whose input, consists of a quantized versions of the input signal $x(nT_s)$. The signal $e(nT_s)$ is called the prediction error.

By encoding the quantizer output, in this method, we obtain a modified version of the PCM called differential pulse code modulation (DPCM).

Quantizer output,

$$\begin{aligned} v(nT_s) &= Q[e(nT_s)] \\ &= e(nT_s) + q(nT_s) \quad (3.32) \end{aligned}$$

Predictor input is the sum of quantizer output and predictor

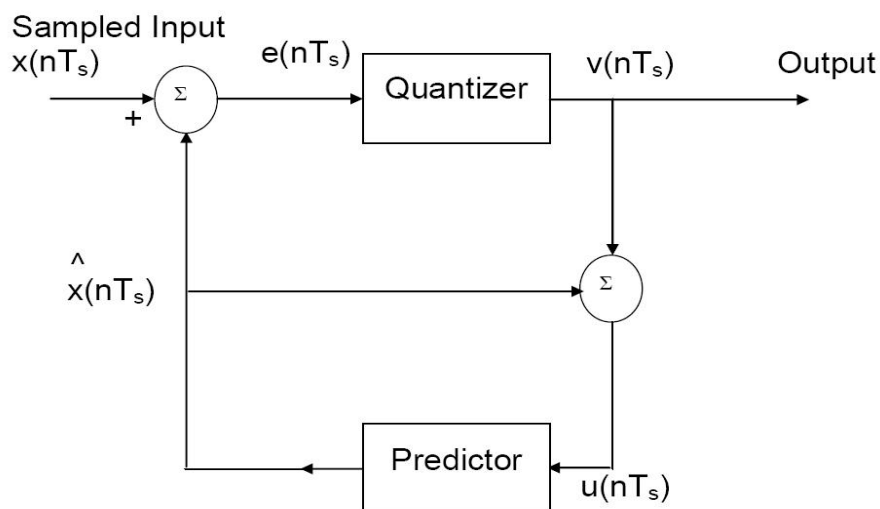
$$\text{output, } u(nT_s) = \hat{x}(nT_s) + v(nT_s) \text{ ---- (3.33)}$$

Using 3.32 in 3.33,

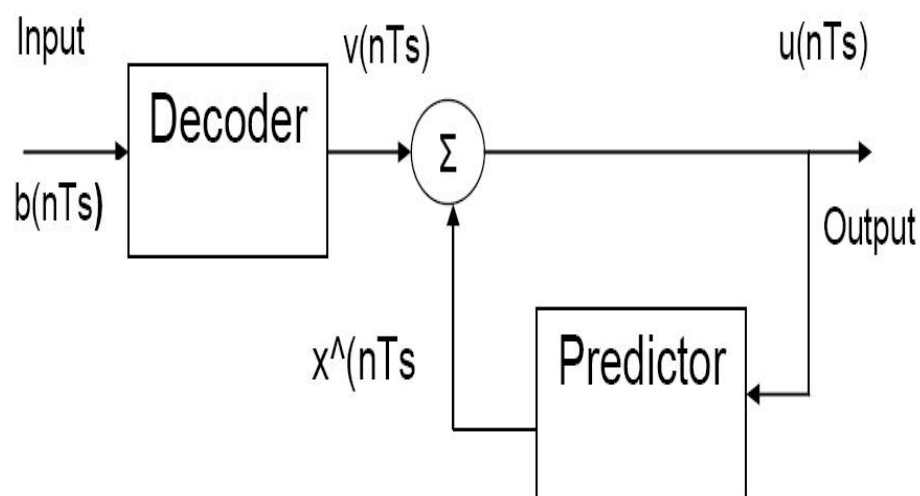
$$u(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \text{ ----}$$

$$(3.34) \quad u(nT_s) = \hat{x}(nT_s) + q(nT_s) \text{ ----(3.35)}$$

The receiver consists of a decoder to reconstruct the quantized error signal. The quantized version of the original input is reconstructed from the decoder output using the same predictor as used in the transmitter. In the absence of noise the encoded signal at the receiver input is identical to the encoded signal at the transmitter output. Correspondingly the receive output is equal to $u(nT_s)$, which differs from the input $x(nT_s)$ only by the quantizing error $q(nT_s)$.



Block diagram of DPCM Transmitter



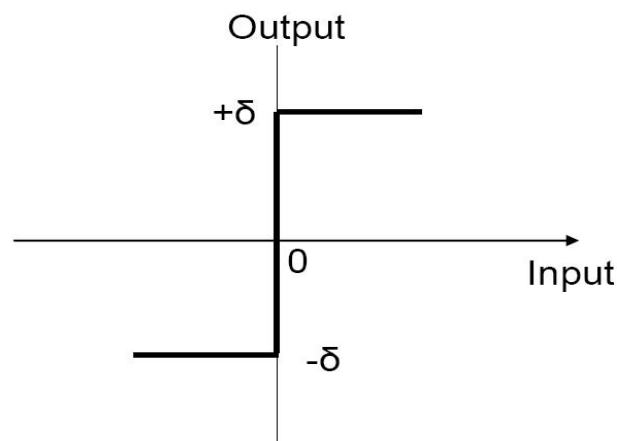
Block diagram of DPCM Receiver.

3. Explain in detail about Delta Modulation.

Delta Modulation (DM)

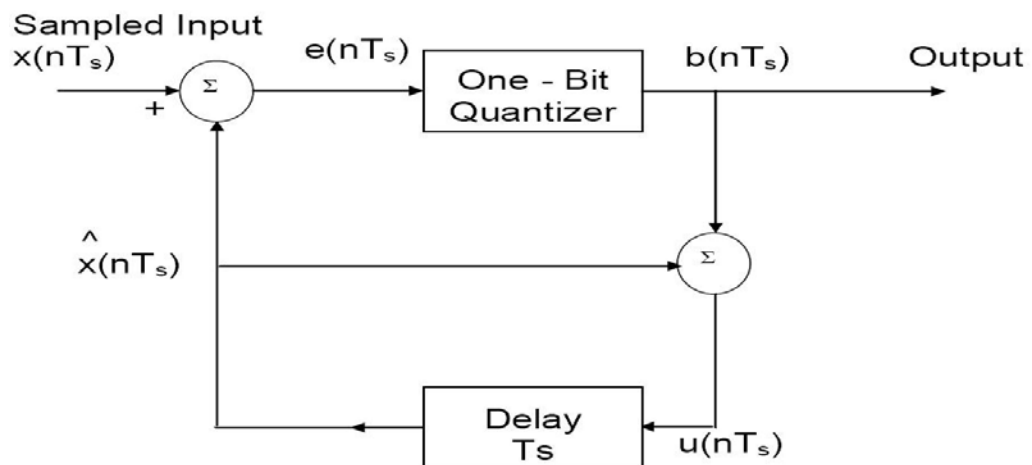
Delta Modulation is a special case of DPCM. In DPCM scheme if the base band signal is sampled at a rate much higher than the Nyquist rate purposely to increase the correlation between adjacent samples of the signal, so as to permit the use of a simple quantizing strategy for constructing the encoded signal, Delta modulation (DM) is precisely such as scheme. Delta Modulation is the one-bit (or two-level) versions of DPCM.

DM provides a staircase approximation to the over sampled version of an input base band signal. The difference between the input and the approximation is quantized into only two levels, namely, $\pm\delta$ corresponding to positive and negative differences, respectively, Thus, if the approximation falls below the signal at any sampling epoch, it is increased by δ . Provided that the signal does not change too rapidly from sample to sample, we find that the stair case approximation remains within $\pm\delta$ of the input signal. The symbol δ denotes the absolute value of the two representation levels of the one-bit quantizer used in the DM.



Input-Output characteristics of the delta modulator.

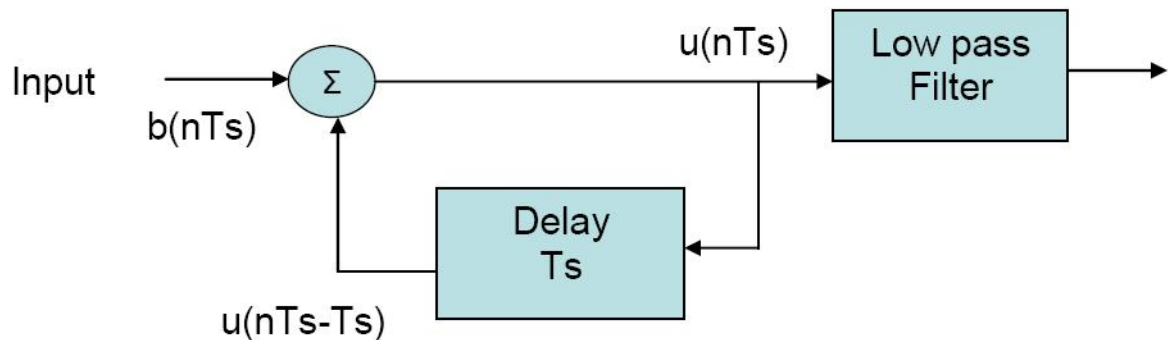
Let the input signal be $x(t)$ and the staircase approximation to it is $u(t)$.



Block diagram for Transmitter of a DM system

In the receiver the stair case approximation $u(t)$ is reconstructed by passing the incoming sequence of positive and negative pulses through an accumulator in a manner similar to that used in the transmitter. The out-of-band quantization noise in the high frequency staircase waveform $u(t)$ is rejected by passing it through a low-pass filter with a band-width equal to the original signal bandwidth. Delta modulation offers two unique features:

1. No need for Word Framing because of one-bit code word.
2. Simple design for both Transmitter and Receiver



Block diagram for Receiver of a DM system

Disadvantage of DM: Delta modulation systems are subject to two types of quantization error:

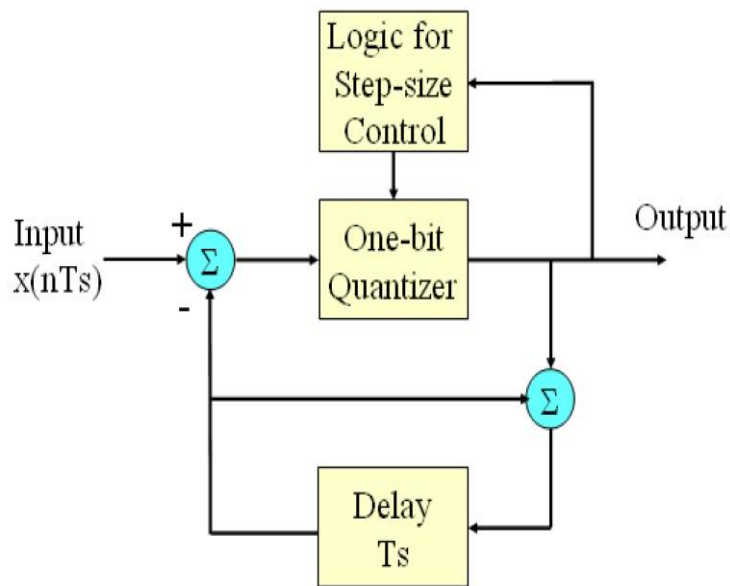
- (1) slope –overload distortion, and (2) granular noise.

4. Explain in detail about Adaptive Delta Modulation.

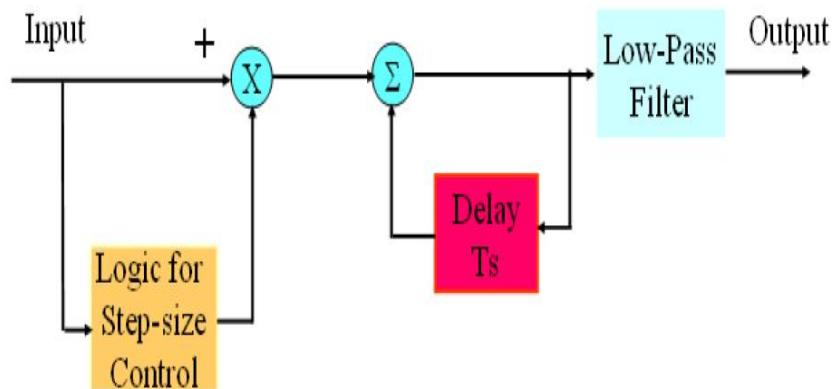
Adaptive Delta Modulation:

The performance of a delta modulator can be improved significantly by making the step size of the modulator assume a time-varying form. In particular, during a steep segment of the input signal the step size is increased. Conversely, when the input signal is varying slowly, the step size is reduced. In this way, the size is adapted to the level of the input signal. The resulting method is called adaptive delta modulation (ADM). There are several types of ADM, depending on the type of scheme used for adjusting the step size. In this ADM, a discrete set of values is provided for the step size.

A D M - Transmitter



A D M - Receiver



Block Diagram of ADM Receiver

Unit-III

Baseband Transmission

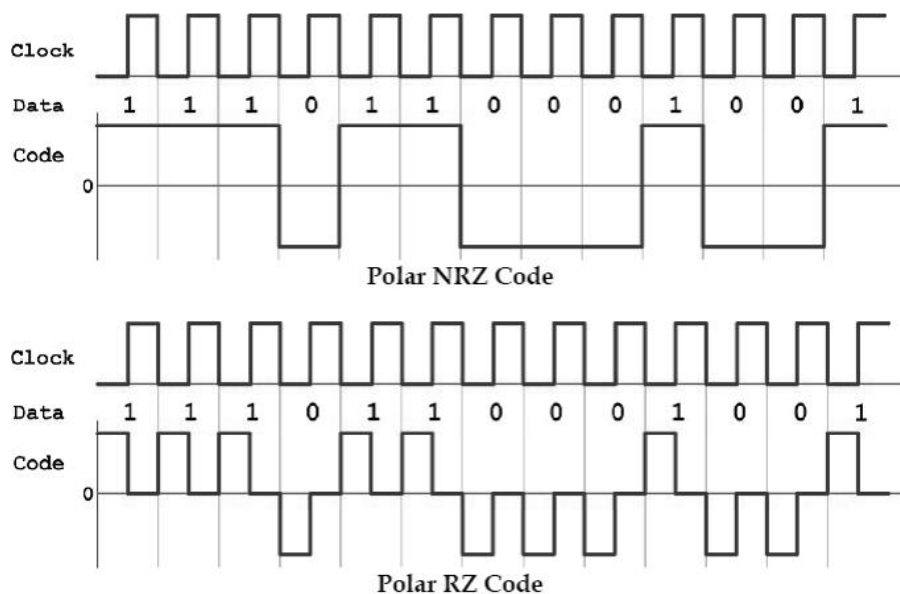
Part - A

1. What is meant by transparency with respect to line codes [AUC APR/MAY 2011]

The line code is said to be transparent if the synchronization between the transmitter and receiver is maintained for any type of input data sequence.

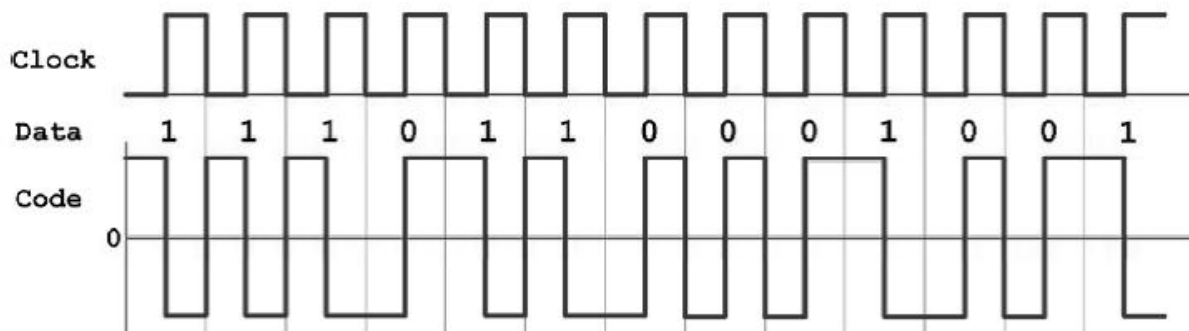
2. Draw the NRZ and RZ code for the digital data 10110001 [AUC APR/MAY 2010] [OR]

Draw the RZ bipolar line code format for the information {10110}[AUC NOV/DEC2011]



3. What is Manchester code? Draw the Manchester format for the data stream 10110? [AUC APR/MAY 2012]

In Manchester code each bit of data is signified by at least one transition. Manchester encoding is therefore considered to be self-clocking, which means that accurate clock recovery from a data stream is possible. In addition, the DC component of the encoded signal is zero. Although transitions allow the signal to be self-clocking, it carries significant overhead as there is a need for essentially twice the bandwidth of a simple NRZ or NRZI encoding



4. State any four desirable properties of line code [AUC NOV/DEC 2012]

The PAM signal should have adequate timing content,

The PAM signal should be immune to channel noise and interference. The PAM signal should allow error detection and error correction.

The PAM signal should be transparent to digital data being transmitted.

5. What is intersymbol interference in baseband binary PAM systems?

In baseband binary PAM, symbols are transmitted one after another. These symbols are separated by sufficient time durations. The transmitter, channel and receiver acts as a filter to this baseband data. Because of the filtering characteristics, transmitted PAM pulses are spread in time.

6. What is correlative coding?

Correlative level coding is used to transmit a baseband signal with the signalling rate of $2B_0$ over the channel of bandwidth B_0 . This is made physically possible by allowing ISI in the transmitted signal in a controlled manner. This ISI is known to the receiver. The correlative coding is implemented by duobinary signalling and modified duobinary signalling.

7. Define Duobinary baseband PAM system.

Duobinary encoding reduces the maximum frequency of the baseband signal. The word „duo“ means to double the transmission capacity of the binary system. Let the PAM signal a_k represent the k th bit. Then the encoder encodes the new waveform as $C_k = a_k + a_{k-1}$.

Thus two successive bits are added to get the encoded value of the k th bit. Hence C_k becomes a correlated signal even though a_k is not correlated. This introduces intersymbol interference in a controlled manner to reduce the bandwidth.

8. What are eye patterns?

Eye pattern is used to study the effect of ISI in baseband transmission.

- 1) Width of eye opening defines the interval over which the received wave can be sampled without error from ISI.
- 2.) The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling time is varied.
- 3) Height of the eye opening at sampling time is called margin over noise.

9. How is eye pattern obtained on the CRO?

Eye pattern can be obtained on CRO by applying the signal to one of the input channels and given an external trigger of $1/Tb$ Hz. This makes one sweep of beam equal to Tb seconds.

10. Why do you need adaptive equalization in a switched telephone network.

In switched telephone network the distortion depends upon

- 1) Transmission characteristics of individual links.

- 2) Number of links in connection.

Hence fixed pair of transmit and receive filters will not serve the equalization problem. The transmission characteristics keep on changing. Therefore adaptive equalization is used.

11. What are the necessity of adaptive equalization?

Most of the channels are made up of individual links in switched telephone network, the distortion induced depends upon

- 1) transmission characteristics of individual links

- 2) number of links in connection

12. Define the principle of adaptive equalization?

The filters adapt themselves to the dispersive effects of the channel that is the coefficients of the filters are changed continuously according to the received data. The filter coefficients are changed in such a way that the distortion in the data is reduced.

13. Define the term ISI?

Ans. The presence of outputs due to other bits interference with the output of required bit. This effect is called inter symbol interference (ISI).

14. Write the performance of data transmission system using eye pattern technique?

The width of the eye opening defines the interval over which the received wave can be sampled without error from inter symbol interference.

The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling time is varied.

15. What is the necessity of equalization?

When the signal is passed through the channel distortion is introduced in terms of 1) amplitude 2) delay this distortion creates problem of ISI. The

detection of the signal also become difficult this distraction can be compensated with the help of equalizer.

16. What is raised cosine spectrum?

In the raised cosine spectrum, the frequency response $P(f)$

decreases towards zero gradually That is there is no abrupt transition.

17. What is nyquist Bandwidth?

The B.is called nyquist bandwidth. .The nyquist bandwidth is the minimum transmission bandwidth for zero ISI.

18. Give two applications for Eye pattern. [AUC APR/MAY 2011]. [AUC NOV/DEC 2012]

- To determine an interval over which the received wave van be sampled without error due ot ISI.
- To determine the sensitivity of the system to timing error
- The margin over the noise is determined from eye pattern

19. What are the information that can be obtained from eye pattern regarding the signal quality? [AUC APR/MAY 2012]

- To determine an interval over which the received wave van be sampled without
- error due ot ISI.
- To determine the sensitivity of the system to timing error
- The margin over the noise is determined from eye pattern

20. A 64 kbps binary PCM polar NRZ signal is passed through a communication system with a raised-cosine filter with roll-off factor 0.25. Find the bandwidth of a filtered PCM signal. [AUC NOV/DEC 2012]

$$\begin{aligned}
 F_b &= 64 \text{ kbps} \\
 B_0 &= F_b/2 = 32 \text{ kbps} \\
 \alpha &= 0.25 \\
 B &= B_0(1+\alpha) = 32 \times 10^3 (1+0.25) = 40 \text{ k} \\
 &\text{Hz}
 \end{aligned}$$

PART B

1. Explain the properties of Line Codes.

Properties of Line Codes:

DC Component:

Eliminating the dc energy from the single power spectrum enables the transmitter to be ac coupled. Magnetic recording system or system using transformer coupling are less sensitive to low frequency signal components. Low frequency component may lost, if the presence of dc or near dc spectral component is significant in the code itself.

Self synchronization

Any digital communication system requires bit synchronization. Coherent detector requires carrier synchronization.

For example Manchester code has a transition at the middle of every bit interval irrespective of whether a 1 or 0 is being sent. This guaranteed transmitter provides a clocking signal at the bit level.

Error detection

Some codes such as duo binary provide the means of detecting data error without introducing additional error detection bits into the data sequence.

Band width compression:

Some codes such as multilevel codes increase the efficiency of the bandwidth utilization by allowing a reduction in required bandwidth for a given data rate, thus more information transmitted per unit band width.

2. Explain in detail about Differential Encoding.

DIFFERENTIAL ENCODING

This technique is useful because it allows the polarity of differentially encoded waveform to be inverted without affecting the data detection. In communication system where waveform to be inverted having great advantage.

NOISE IMMUNITY

For same transmitted energy some codes produce lesser bit detection error than others in the presence of noise. For ex. The NRZ waveforms have better noise performance than the RZ type.

SPECTRAL COMPATABILITY WITH CHANNEL:

One aspect of spectrum matching is dc coupling. Also transmission bandwidth of the code must be sufficiently small compared to channel bandwidth so that ISI is not a problem.

TRANSPARENCY

A line code should be so designed that the receiver does not go out of synchronization for any line sequence of data symbol. A code is not transparent if for some sequence of symbol, the clock is lost.

Power spectral density of unipolar NRZ line code:

Line coding:

Line coding refers to the process of representing the bit stream (1s and 0s) in the form of voltage or current variations optimally tuned for the specific properties of the physical channel being used. The selection of a proper line code can help in so many ways: One possibility is to aid in clock recovery at the receiver. A clock signal is recovered by observing transitions in the received bit sequence, and if enough transitions exist, a good recovery of the clock is guaranteed, and the signal is said to be **self-clocking**.

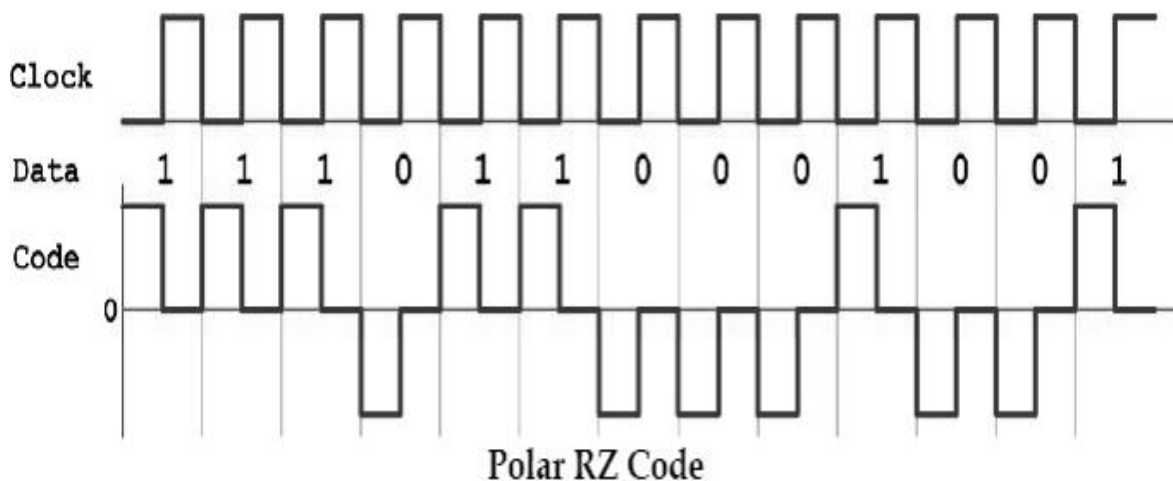
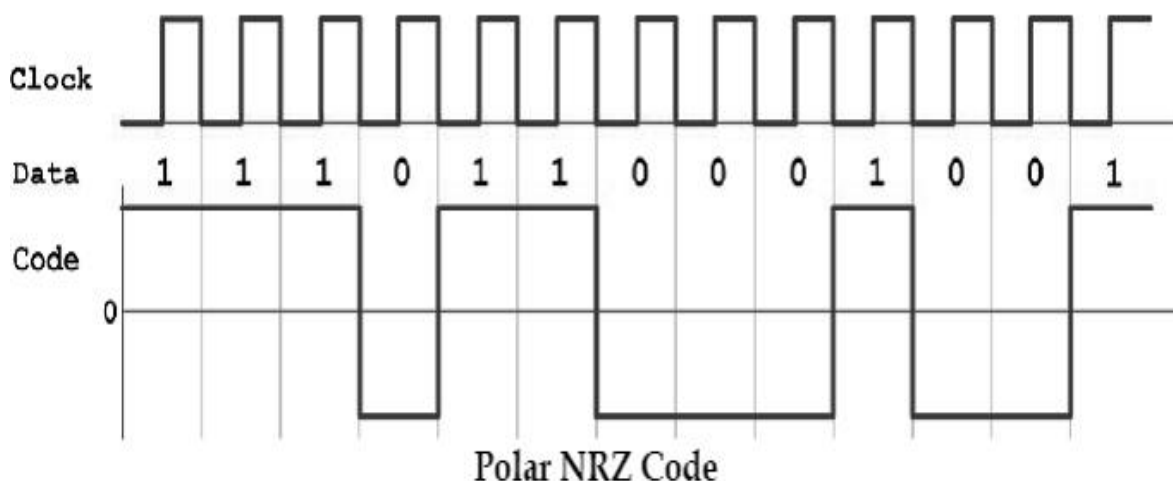
Another advantage is to get rid of DC shifts. The DC component in a line code is called the *bias* or the *DC coefficient*. Unfortunately, most long-distance communication channels cannot transport a DC component. This is why most line codes try to eliminate the DC component before being transmitted on the channel. Such codes are called *DC balanced*, *zero-DC*, *zero-bias*, or *DC equalized*. Some common types of line encoding in common-use nowadays are unipolar, polar, bipolar, Manchester, MLT-3 and Duobinary encoding. These codes are explained here: **1. Unipolar** (Unipolar **NRZ** and Unipolar **RZ**):

Unipolar is the simplest line coding scheme possible. It has the advantage of being compatible with TTL logic. Unipolar coding uses a positive rectangular pulse $p(t)$ to represent binary **1**, and the absence of a pulse (i.e., zero voltage) to represent a binary **0**. Two possibilities for the pulse $p(t)$ exist: Non-Return-to-Zero (NRZ) rectangular

pulse and Return-to-Zero (RZ) rectangular pulse. The difference between Unipolar NRZ and Unipolar RZ codes is that the rectangular pulse in NRZ stays at a positive value (e.g., +5V) for the full duration of the logic **1** bit, while the pulse in RZ drops from +5V to

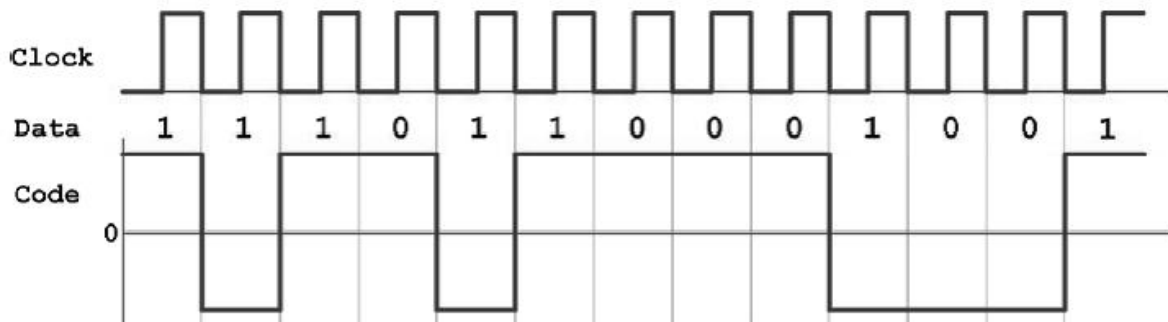
0V in the middle of the bit time. A drawback of unipolar (RZ and NRZ) is that its average value is not zero, which means it creates a significant DC-component at the receiver

(see the impulse at zero frequency in the corresponding power spectral density (PSD) of this line code



that polar signals have more power than unipolar signals, and hence have better SNR at the receiver. Actually, polar NRZ signals have more power compared to polar RZ signals. The drawback of polar NRZ, however, is that it lacks clock information especially when a long sequence of **0**'s or **1**'s is transmitted.

Non-Return-to-Zero, Inverted (NRZI): NRZI is a variant of Polar NRZ. In NRZI there are two possible pulses, $p(t)$ and $-p(t)$. A transition from one pulse to the other happens if the bit being transmitted is logic **1**, and no transition happens if the bit being transmitted is a logic **0**.



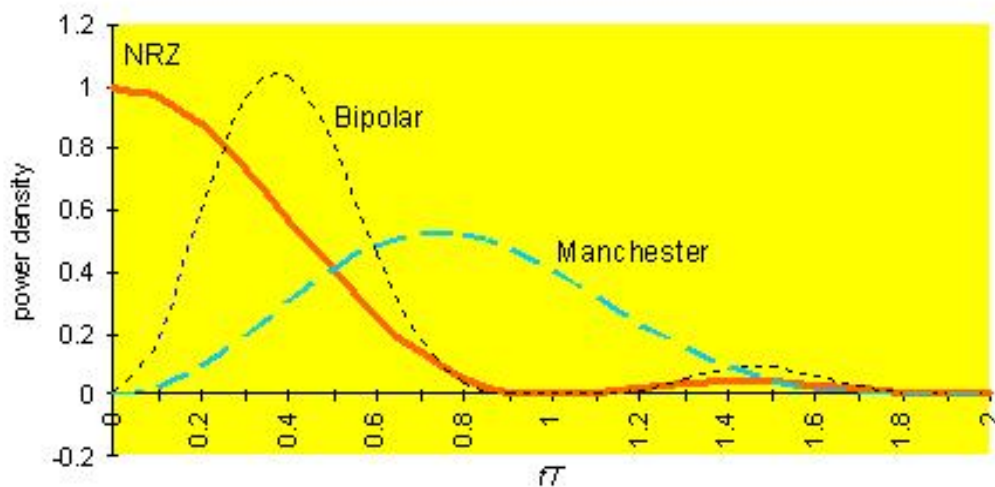
This is the code used on compact discs (CD), USB ports, and on fiber-based Fast Ethernet at 100-Mbit/s.

Manchester encoding:

In Manchester code each bit of data is signified by at least one transition. Manchester encoding is therefore considered to be self-clocking, which means that accurate clock recovery from a data stream is possible. In addition, the DC component of the encoded signal is zero. Although transitions allow the signal to be self-clocking, it carries significant overhead as there is a need for essentially twice the bandwidth of a simple NRZ or NRZI encoding.

The power spectral density of polar signalling:

POWER SPECTRA OF LINE CODE:



Unipolar most of signal power is centered on origin and there is waste of power due to DC component that is present.

Polar format most of signal power is centered on origin and they are simple to implement.

- Bipolar format does not have DC component and does not demand more bandwidth, but power requirement is double than other formats.
- Manchester format does not have DC component but provides proper clocking.

Training mode

A known sequence $d(nT)$ is transmitted and synchronized version of it is generated in the receiver applied to adaptive equalizer. This training sequence has maximal length PN Sequence, because it has large average power and large SNR, resulting response sequence (Impulse) is observed by measuring the filter outputs at the sampling instants. The difference between resulting response $y(nT)$ and desired response $d(nT)$ is error signal which is used to estimate the direction in which the coefficients of filter are to be optimized using algorithms.

3. Describe Nyquist Criterion for Zero-ISI.

Nyquist Criterion for Zero-ISI

Nyquist proposed a condition for pulses $p(t)$ to have zero-ISI when transmitted through a channel with sufficient bandwidth to allow the spectrum of all the transmitted signal to pass. Nyquist proposed that a zero-ISI pulse $p(t)$ must satisfy the condition

$$p(t) = \begin{cases} 1 & t = 0 \\ 0 & t = \pm T_b, \pm 2T_b, \pm 3T_b, \dots \end{cases}$$

A pulse that satisfies the above condition at multiples of the bit period T_b will result in zero-ISI if the whole spectrum of that signal is received. The reason for which these zero-ISI pulses (also called Nyquist-criterion pulses) cause no ISI is that each of these pulses at the sampling periods is either equal to 1 at the center of pulse and zero the points other pulses are centered.

In fact, there are many pulses that satisfy these conditions. For example, any square pulse that occurs in the time period $-T_b$ to T_b or any part of it (it must be zero at $-T_b$ and T_b) will satisfy the above condition.

Also, any triangular waveform („Δ“ function) with a width that is less than $2T_b$ will also satisfy the condition. A sinc function that has zeros at $t = T_b, 2T_b, 3T_b, \dots$ will also satisfy this condition. The problem with the sinc function is that it extends over a very long period of time resulting in a lot of processing to generate it. The square pulse required a lot of bandwidth to be transmitted. The triangular pulse is restricted in time but has relatively large bandwidth.

There is a set of pulses known as raised-cosine pulses that satisfy the Nyquist criterion and require slightly larger bandwidth than what a sinc pulse (which requires the minimum bandwidth ever) requires.

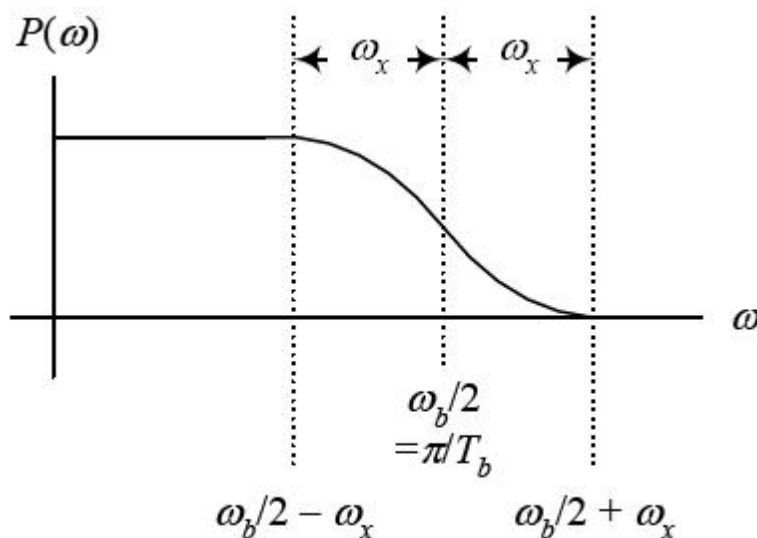
The spectrum of these pulses is given by

$$P(\omega) = \begin{cases} \frac{1}{2} \left[1 - \sin \left(\frac{\pi}{2\omega_x} \left(\omega - \frac{\omega_b}{2} \right) \right) \right] & \left| \omega - \frac{\omega_b}{2} \right| < \omega_x \\ 0 & \left| \omega \right| > \frac{\omega_b}{2} + \omega_x \\ 1 & \left| \omega \right| < \frac{\omega_b}{2} + \omega_x \end{cases}$$

Where ω_b is the frequency of bits in rad/s ($\omega_b = 2/Tb$), and x is called the excess bandwidth and it defines how much bandwidth would be required above the minimum bandwidth that is required when using a sinc pulse. The excess bandwidth ω_x for this type of pulses is restricted between

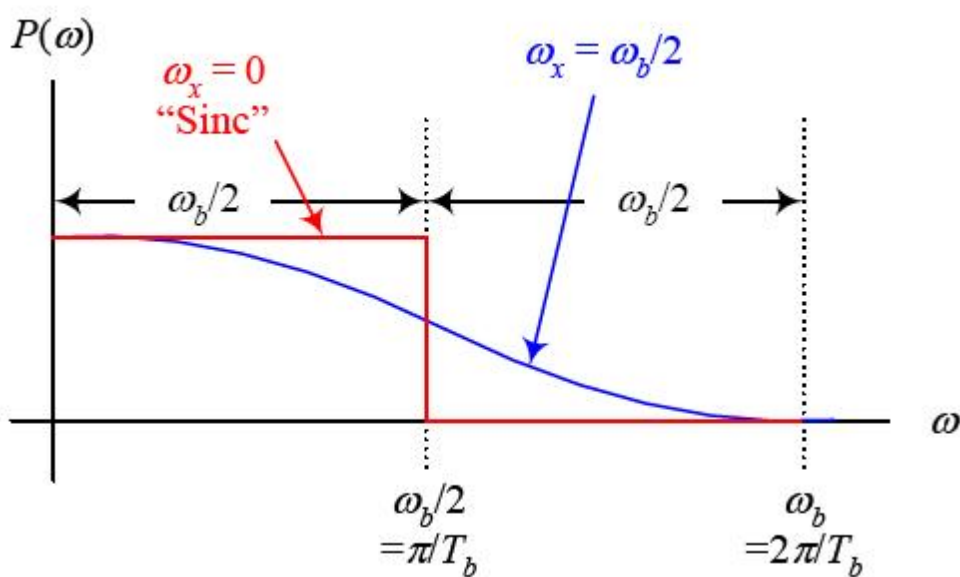
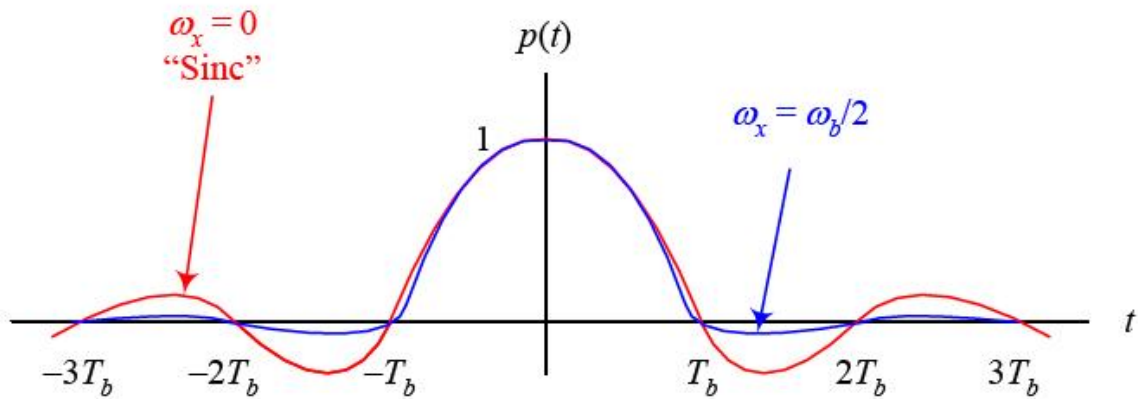
$$0 \leq \omega_x \leq \frac{\omega_b}{2}.$$

Sketching the spectrum of these pulses we get



We can easily verify that when $\omega_x = 0$, the above spectrum becomes a rect function, and therefore the pulse $p(t)$ becomes the usual sinc function. For $\omega_x = b/2$, the spectrum is similar to a sinc function but decays (drops to zero) much faster than

the sinc (it extends over 2 or 3 bit periods on each side). The expense for having a pulse that is short in time is that it requires a larger bandwidth than the sinc function (twice as much for $\omega_x = \omega_b/2$). Sketch of the pulses and their spectrum for the two extreme cases of $\omega_x = \omega_b/2$ and $\omega_x = 0$ are shown below.



We can define a factor r called the roll-off factor to be

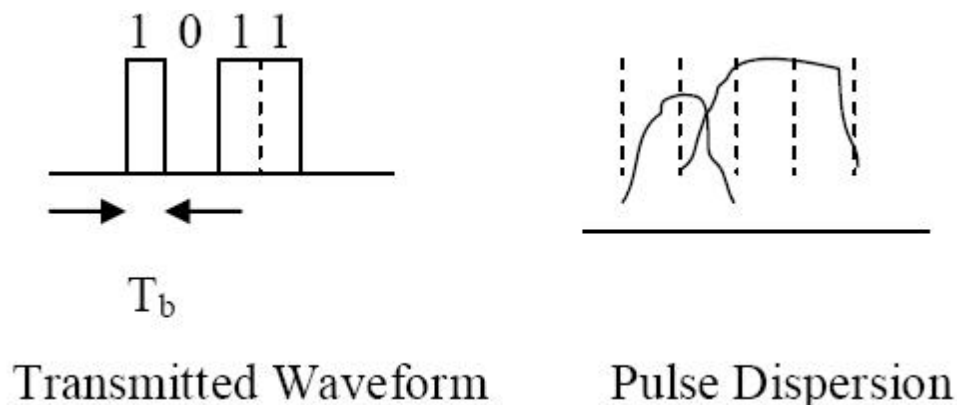
$$r = \frac{\text{Excess Bandwidth}}{\text{Minimum Bandwidth}} = \frac{\omega_x}{\omega_b / 2} = \frac{2\omega_x}{\omega_b}.$$

The roll-off factor r specifies the ratio of extra bandwidth required for these pulses compared to the minimum bandwidth required by the sinc function.

Inter symbol Interference

Generally, digital data is represented by electrical pulse, communication channel is always band limited. Such a channel disperses or spreads a pulse carrying digitized samples passing through it. When the channel bandwidth is greater than bandwidth of pulse, spreading of pulse is very less. But when channel bandwidth is close to signal bandwidth, i.e. if we transmit digital data which demands more bandwidth which exceeds channel bandwidth, spreading will occur and cause signal pulses to overlap. This overlapping is called **Inter Symbol Interference**. In short it is called ISI. Similar to interference caused by other sources, ISI causes degradations of signal if left uncontrolled. This problem of ISI exists strongly in Telephone channels like coaxial cables and optical fibers.

The main objective is to study the effect of ISI, when digital data is transmitted through band limited channel and solution to overcome the degradation of waveform by properly shaping pulse.



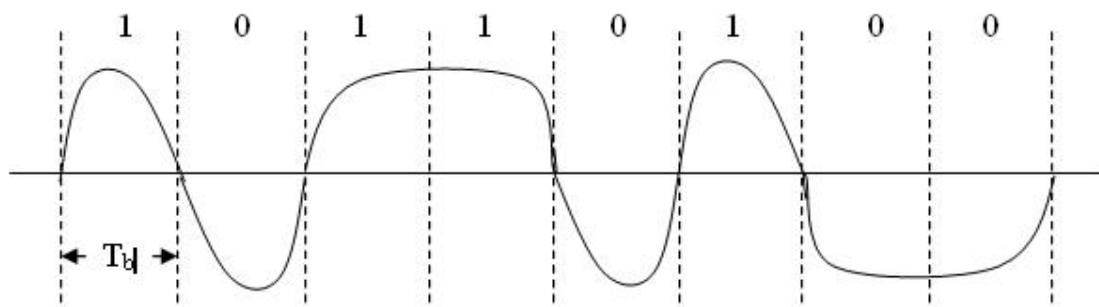
The effect of sequence of pulses transmitted through channel is shown in fig. The Spreading of pulse is greater than symbol duration, as a result adjacent pulses interfere. i.e. pulses get completely smeared, tail of smeared pulse enter into adjacent symbol intervals making it difficult to decide actual transmitted pulse. First let us have look at different formats of transmitting digital data. In base band transmission best way is to map digits or symbols into pulse waveform. This waveform is generally termed as **Line codes**.

4. Explain about Eye Pattern.

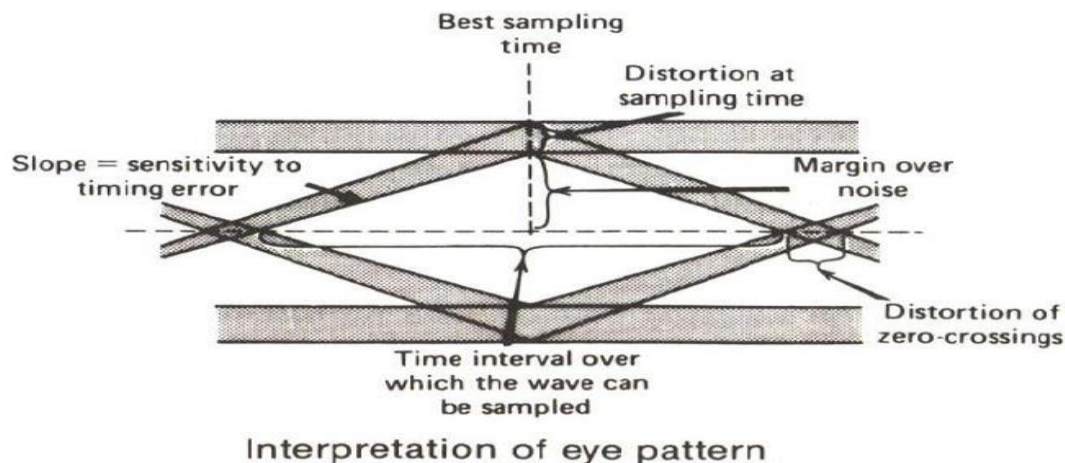
EYE PATTERN

The quality of digital transmission systems are evaluated using the bit error rate. Degradation of quality occurs in each process modulation, transmission, and detection. The eye pattern is experimental method that contains all the information concerning the degradation of quality. Therefore, careful analysis of the eye pattern is important in analyzing the degradation mechanism.

- Eye patterns can be observed using an oscilloscope. The received wave is applied to the vertical deflection plates of an oscilloscope and the sawtooth wave at a rate equal to transmitted symbol rate is applied to the horizontal deflection plates, resulting display is eye pattern as it resembles human eye.
- The interior region of eye pattern is called eye opening



We get superposition of successive symbol intervals to produce eye pattern as shown below.



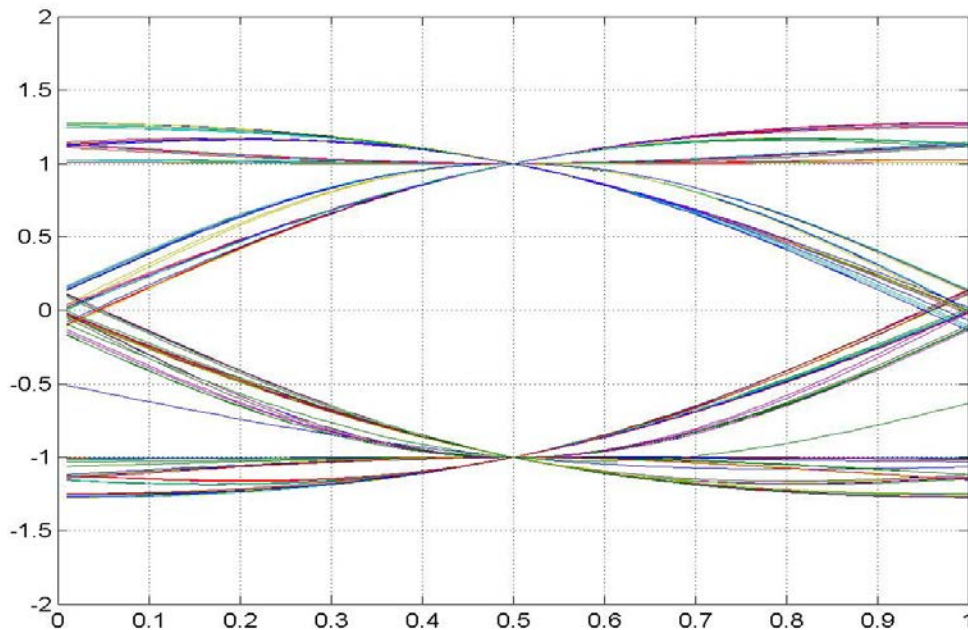
- The width of the eye opening defines the time interval over which the received wave can be sampled without error from ISI
- The optimum sampling time corresponds to the maximum eye opening
- The height of the eye opening at a specified sampling time is a measure of the margin over channel noise.

The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling time is varied. Any non linear transmission distortion would

reveal itself in an asymmetric or squinted eye. When the effect of ISI is excessive, traces from the upper portion of the eye pattern cross traces from lower portion with the result that the eye is completely closed.

Example of eye pattern:

Binary-PAM Perfect channel (no noise and no ISI)



Example of eye pattern: Binary-PAM with noise no ISI

5. Explain about Equalization Filter.

EQUALISING FILTER

Adaptive equalization

- An equalizer is a filter that compensates for the dispersion effects of a channel. Adaptive equalizer can adjust its coefficients continuously during the transmission of data.

Pre channel equalization

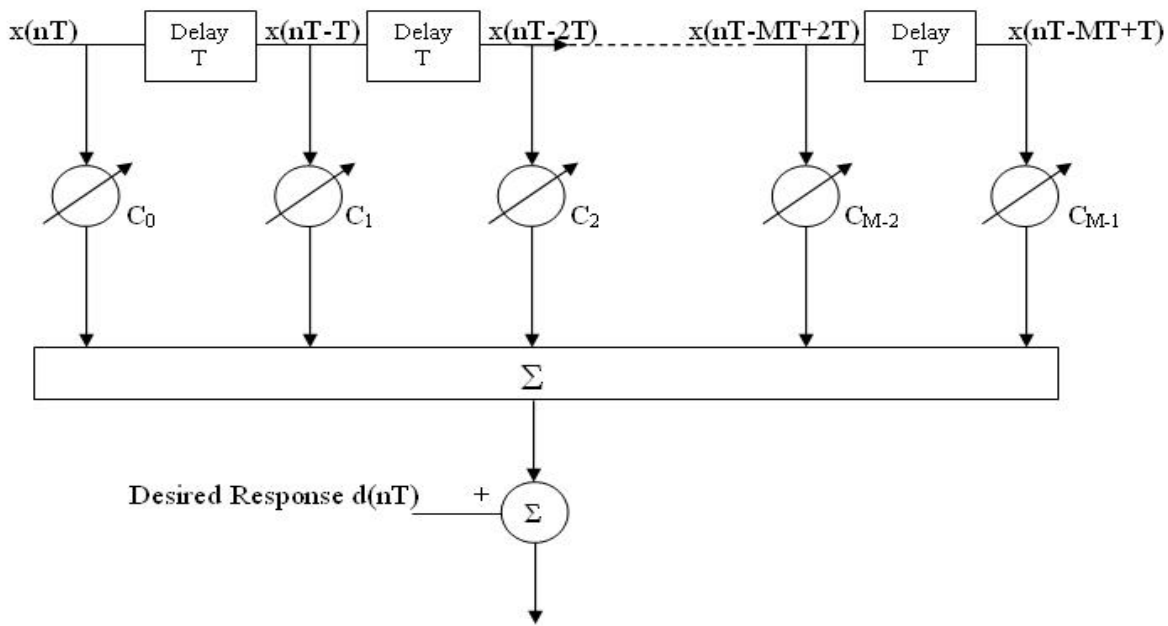
requires feed back channel causes burden on transmission.

Post channel equalization

Achieved prior to data transmission by training the filter with the guidance of a training sequence transmitted through the channel so as to adjust the filter parameters to optimum values.

Adaptive equalization

It consists of tapped delay line filter with set of delay elements, set of adjustable multipliers connected to the delay line taps and a summer for adding multiplier outputs



The output of the Adaptive equalizer is given by

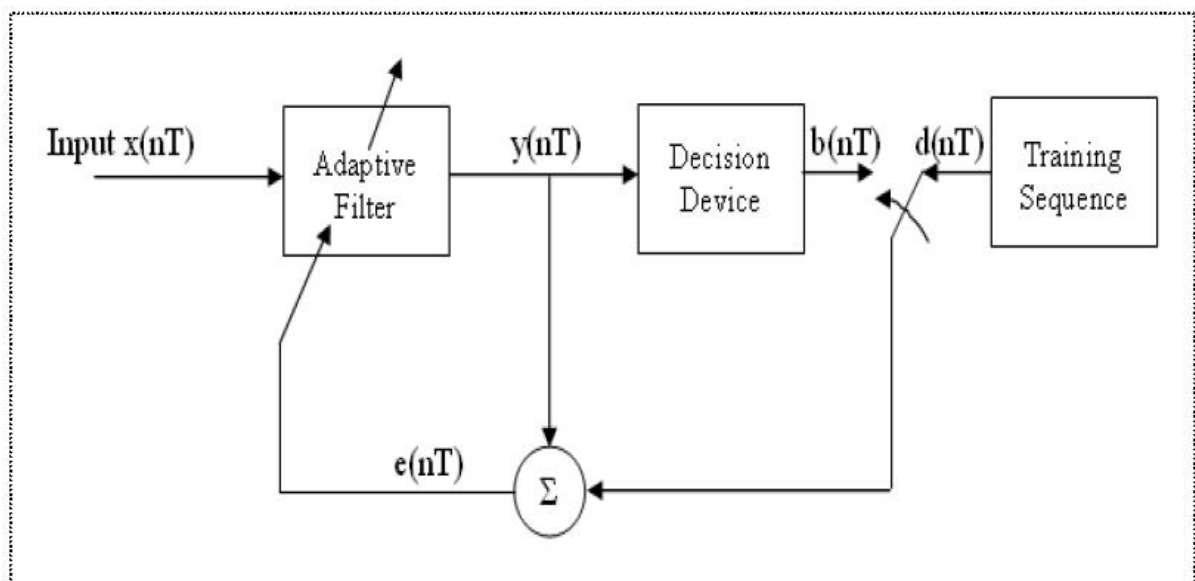
$$Y(nT) = \sum C_i x(nT - iT)$$

C_i is weight of the i th tap Total number of taps are M . Tap spacing is equal to symbol duration T of transmitted signal In a conventional FIR filter the tap weights are constant and particular designed response is obtained. In the adaptive equaliser the C_i 's are variable and are adjusted by an algorithm.

Two modes of operation

1. Training mode
2. Decision directed mode

Mechanism of adaptation



Training mode

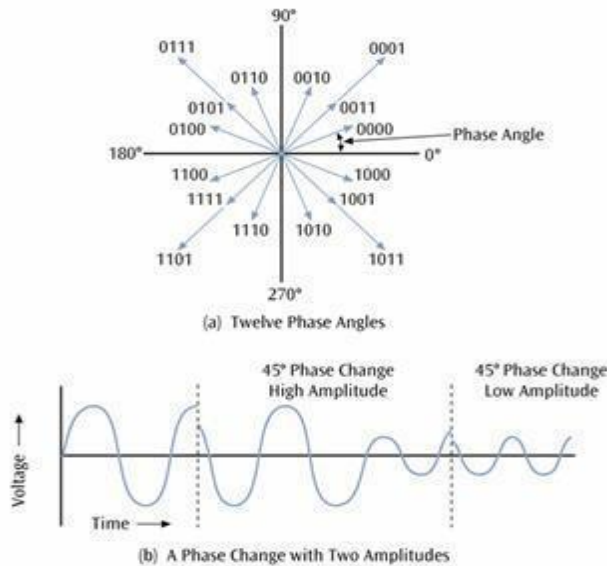
A known sequence $d(nT)$ is transmitted and synchronized version of it is generated in the receiver applied to adaptive equalizer. This training sequence has maximal length PN Sequence, because it has large average power and large SNR, resulting response sequence (Impulse) is observed by measuring the filter outputs at the sampling instants. The difference between resulting response $y(nT)$ and desired response $d(nT)$ is error signal which is used to estimate the direction in which the coefficients of filter are to be optimized using algorithms.

Unit-IV

Digital Modulation Scheme

Part - A

1. Define QAM and draw its constellation diagram. ? [AUC NOV/DEC 2010]



2. A binary frequency shift keying system employs two signaling frequencies $1f$ and $2f$. The lower frequency $1f$ is 1200 Hz and signaling rate is 500 Baud. Calculate $2f$. ?

[AUC NOV/DEC 2010]

For binary FSK
 $\text{baud} = f_b$ $f_b = 500 \text{ Hz}$

Consider the FN modulation index (h) of 1 in FSK $f_m - f_s / f_b = h = 1$

$$f_m - f_s = f_b$$

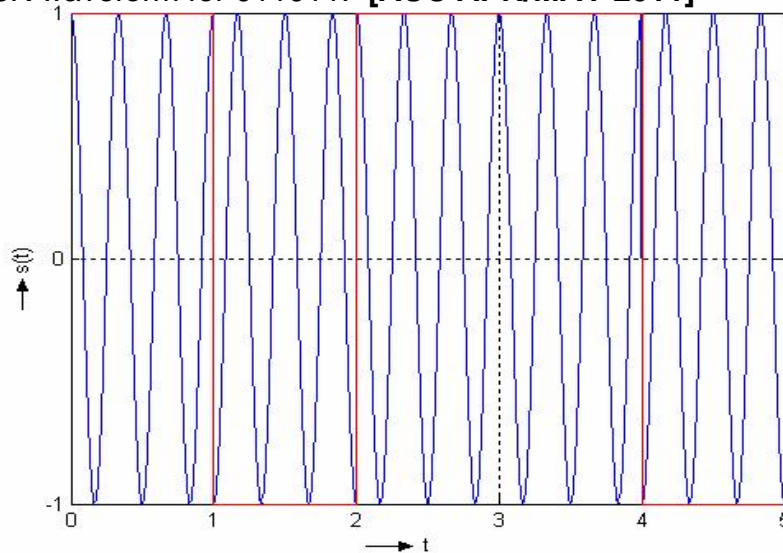
$$f_s = f_1 = 1200 \text{ Hz}$$

$$f_m -$$

$$1200 \text{ Hz} = 500 \text{ Hz}$$

$$f_m = 1700 \text{ Hz}, f_2 = f_m = 1700 \text{ Hz}.$$

3. Draw the PSK waveform for 011011. [AUC APR/MAY 2011]



4. What is meant by coherent detection system? **[AUC APR/MAY 2011]**

In coherent ASK, correlation receiver is used to detect the signal. Locally generated carrier is correlated with incoming Ask signal. The locally generated carrier is in exact phase either transmitted carrier. Coherent Ask is also called synchronous ASK.

5. Why is PSK always preferable over ASK in coherent detection? **[AUC NOV/DEC 2011]**

ASK is on-off signaling where as the modulated carrier is continuously transmitted in PSK. Hence peak power requirement is more in ASK, where it is reduces in PSK.

6. Differentiate between coherent and non-coherent detection **[AUC NOV/DEC 2011]**
[AUC APR/MAY 2012]

In coherent detection the local carrier generated at the receiver is phase locked with the carrier at the transmitter. Hence it is also called synchronous detection. In non coherent detection the local carrier generated at the receiver not be phase locked with the carrier at the transmitter. It is simple, but it has higher probability of error.

7. What are the drawbacks of binary PSK system? **[AUC APR/MAY 2012]** It is difficult to detect $+b(t)$ and $-b(t)$ because of squaring in the receiver Problem, of ISI and inter channel interference are present.

8. A BPSK system makes errors at the average rate of 1000 errors per delay. Data rate is 1 kbps .

The single-sided noise power spectral density is 10-20 W/Hz. Assuming the system to be wide sense stationary, what is the average bit error probability?

[AUC NOV/DEC 2012]

$$24 \times 60 \times 60 = 86400s$$

$$ec \ 86.4 \times 10^6$$

Bit error probability

$$Pe = 100 / 86.4 \times 10^6 = 1.1157 \times 10^{-6}$$

9. What is meant by DPSK?

In DPSK, the input sequence is modified. Let input sequence be $d(t)$ and output Sequence be $b(t)$. Sequence $b(t)$ changes level at the beginning of each interval in which $d(t)=1$ and it does not changes level when $d(t)=0$.

When $b(t)$ changes level, phase of the carrier is changed. And as stated above, $b(t)$ changes t =its level only when $d(t) =1$. This means phase of the carrier is changed only if $d(t)=1$. Hence the technique is called Differential PSK.

10. Explain coherent detection?

In coherent detection, the local carrier generated at the receiver is phase locked with the carrier at the transmitter. The detection is done by correlating received noisy signal and locally generated carrier. The coherent detection is a synchronous detection.

11. Bring out the difference between coherent & non coherent binary modulation scheme. **a. Coherent detection:**

In this method the local carrier generated at the receiver is phase locked with the carrier At the transmitter. Hence it is called synchronous detection

b. Non coherent detection:

In this method, the receiver carrier need not be phase locked with transmitter carrier. Hence it is called envelope detection.

12. Write the expression for bit error rate for coherent binary FSK. Bit error rate for coherent binary FSK is given as,

$$P_e = 1/2 \operatorname{erfc} \sqrt{0.6E/N_0}$$

13. What is Signal constellation diagram?

Suppose that in each time slot of duration T seconds, one $s_1(t), \dots, s_M(t)$ is transmitted with equal probability, $1/M$ For geometric representation, the signal $s_i(t)$, $i = 1, 2, \dots, M$, is applied to a bank of correlators. The correlator outputs define the signal vector s_i . The set of message points corresponding to the set of transmitted signals $\{s_i(t)\}_{i=1..M}$ is called a signal constellation.

14. What is meant by memory less modulations? **[AUC NOV/DEC 2012]**

When the digital symbol modulates amplitude, phase or frequency of the carrier without any reference to previous symbol, it is called memory less modulations. Eg.:ASK,PSK,FSK,QPSK etc.

15. Define QPSK.

- In QPSK two successive bits in the data sequence are grouped together. This combination of two bits forms four distinct symbols. When the symbol is changed to next symbol the phase of the carrier is changed by 45° (or $\pi/4$).
- Because of combination of two bits there will be four symbols. Hence the phase shift will be $\pi/4, 3\pi/4, 5\pi/4$ or $7\pi/4$.
QPSK reduces amplitude variations and required transmission bandwidth.

PART B**1. Explain the Geometric representation of Signals.**

Derive Geometrical representation of signal.

combinations of two orthonormal basis functions $\phi_1(t)$ and $\phi_2(t)$.

- $\phi_1(t)$ and $\phi_2(t)$ are orthonormal if:

$$\int_0^{T_b} \phi_1(t)\phi_2(t)dt = 0 \text{ (orthogonality),}$$

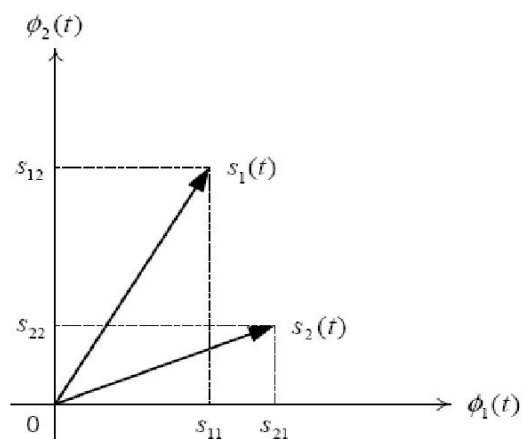
$$\int_0^{T_b} \phi_1^2(t)dt = \int_0^{T_b} \phi_2^2(t)dt = 1 \text{ (normalized to have unit energy).}$$

- The representations are

$$s_1(t) = s_{11}\phi_1(t) + s_{12}\phi_2(t),$$

$$s_2(t) = s_{21}\phi_1(t) + s_{22}\phi_2(t).$$

$$\text{where } s_{ij} = \int_0^{T_b} s_i(t)\phi_j(t)dt, \quad i, j \in \{1, 2\},$$



$$s_1(t) = s_{11}\phi_1(t) + s_{12}\phi_2(t),$$

$$s_2(t) = s_{21}\phi_1(t) + s_{22}\phi_2(t),$$

$$s_{ij} = \int_0^{T_b} s_i(t)\phi_j(t)dt, \quad i, j \in \{1, 2\},$$

- $\int_0^{T_b} s_i(t)\phi_j(t)dt$ is the projection of $s_i(t)$ onto $\phi_j(t)$.

Basis Vectors

The set of basis vectors $\{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_n\}$ of a space are chosen such that: Should be complete or span the vector space: any vector \mathbf{a} can be expressed as a linear combination of these vectors.

Each basis vector should be orthogonal to all others

Each basis vector should be normalized:

A set of basis vectors satisfying these properties is also said to be a complete

orthonormal basis

- In an **n-dim** space, we can have at most n basis vectors

Signal Space

Basic Idea: If a signal can be represented by n-tuple, then it can be treated in much the same way as a n-dim vector.

Let $\phi_1(t), \phi_2(t), \dots, \phi_n(t)$ be n signals

Consider a signal $x(t)$ and suppose that If every signal can be written as above \Rightarrow ~ ~ **basis functions** and we have a **n-dim signal space**

Orthonormal Basis

Signal set $\{\phi_k(t)\}_n$ is an **orthogonal** set if

If $c_j \neq 1 \forall j \Rightarrow \{\phi_k(t)\}$ is an **orthonormal** set.

$$\int_{-\infty}^{\infty} \phi_j(t) \phi_k(t) dt = \begin{cases} 0 & j \neq k \\ 1 & j = k \end{cases}$$

Consider a set of M signals (M -ary symbol) $\{s_i(t), i=1, 2, \dots, M\}$ with finite energy. That is

$$\int_{-\infty}^{\infty} s_i^2(t) dt < \infty$$

Then, we can express each of these waveforms as weighted linear combination of orthonormal signals

$$s_i(t) = \sum_{j=1}^N s_{ij} \phi_j(t) \quad \text{for } i = 1, \dots, M$$

$$\{\phi_j(t)\}_1^N$$

where $N \leq M$ is the dimension of the signal space and are called the orthonormal basis functions

Let, for a convenient set of $\{\phi_j(t)\}, j = 1, 2, \dots, N$ and $0 \leq t < T$,

$$s_i(t) = \sum_{j=1}^N s_{ij} \phi_j(t), \quad i= 1, 2, \dots, M \text{ and } 0 \leq t < T, \text{ such that,}$$

$$s_{ij} = \int_0^T s_i(t) \phi_j(t) dt$$

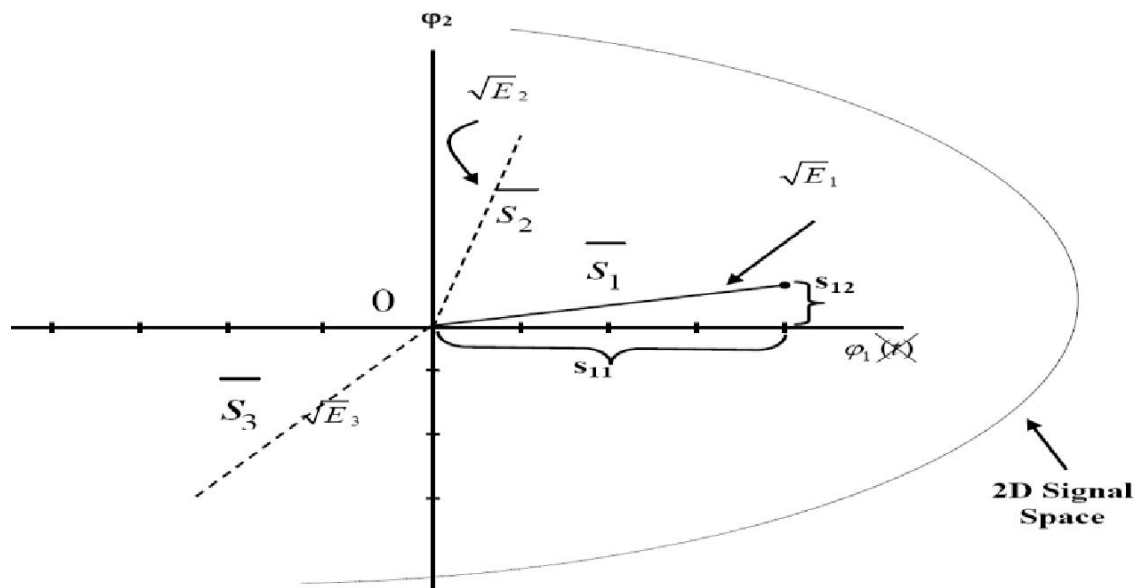
Now, we can represent a signal $s_i(t)$ as a column vector whose elements are the scalar coefficients

$$s_{ij}, \quad j = 1, 2, \dots, N \quad :$$

$$\vec{s}_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix}_{1 \times N} ; i = 1, 2, \dots, M$$

These M energy signals or vectors can be viewed as a set of M points in an N – dimensional

Euclidean space, known as the „Signal Space’. Signal Constellation is the collection of M signals points (or messages) on the signal space.



Now, the length or *norm* of a vector is denoted as $\|\vec{s}_i\|$. The squared norm is the inner product of the vector:

$$\|\vec{s}_i\|^2 = (\vec{s}_i, \vec{s}_i) = \sum_{j=1}^N s_{ij}^2$$

The cosine of the angle between two vectors is defined as:

$$\cos(\text{angle between } \vec{s}_i \text{ \& } \vec{s}_j) = \frac{(\vec{s}_i, \vec{s}_j)}{\|\vec{s}_i\| \|\vec{s}_j\|}$$

$\therefore \vec{s}_i$ & \vec{s}_j are orthogonal to each other if $(\vec{s}_i, \vec{s}_j) = 0$.

If E_i is the energy of the i -th signal vector,

$$\begin{aligned}
 E_i &= \int_0^T s_i^2(t) dt = \int_0^T \left[\sum_{j=1}^N s_{ij} \phi_j(t) \right] \left[\sum_{k=1}^N s_{ik} \phi_k(t) \right] dt \\
 &= \sum_{j=1}^N \sum_{k=1}^N s_{ij} s_{ik} \int_0^T \phi_j(t) \phi_k(t) dt \quad \text{as } \{\phi_j(t)\} \text{ forms an ortho-normal set} \\
 &= \sum_{j=1}^N s_{ij}^2 = \|\vec{s}_i\|^2
 \end{aligned}$$

For a pair of signals $s_i(t)$ and $s_k(t)$, $\|\vec{s}_i - \vec{s}_k\|^2 = \sum_{j=1}^N (s_{ij} - s_{kj})^2 = \int_0^T [s_i(t) - s_k(t)]^2 dt$

It may now be guessed intuitively that we should choose $s_i(t)$ and $s_k(t)$ such that the Euclidean distance between them, i.e. $\|\vec{s}_i - \vec{s}_k\|$ is as much as possible to ensure that their detection is more robust even in presence of noise. For example, if $s_1(t)$ and $s_2(t)$ have same energy E , (i.e. they are equidistance from the origin), then an obvious choice for maximum distance of separation is, $s_1(t) = -s_2(t)$.

2. Explain the Generation and Coherent Detection of Bpsk Signals

(i) Generation

To generate the BPSK signal, we build on the fact that the BPSK signal is a special case of DSB-SC modulation. Specifically, we use a product modulator consisting of two components.

(i) Non-return-to-zero level encoder, whereby the input binary data sequence is encoded in polar form with symbols 1 and 0 represented by the constant-amplitude.

(ii) Product modulator, which multiplies the level encoded binary wave by the sinusoidal carrier of amplitude to produce the BPSK signal. The timing pulses used to generate the level encoded binary wave and the sinusoidal carrier wave are usually, but not necessarily, extracted from a common master clock.

(ii) Detection

To detect the original binary sequence of 1s and 0s, the BPSK signal at the channel output is applied to a receiver that consists of four sections

(a) Product modulator, which is also supplied with a locally generated reference signal that is a replica of the carrier wave

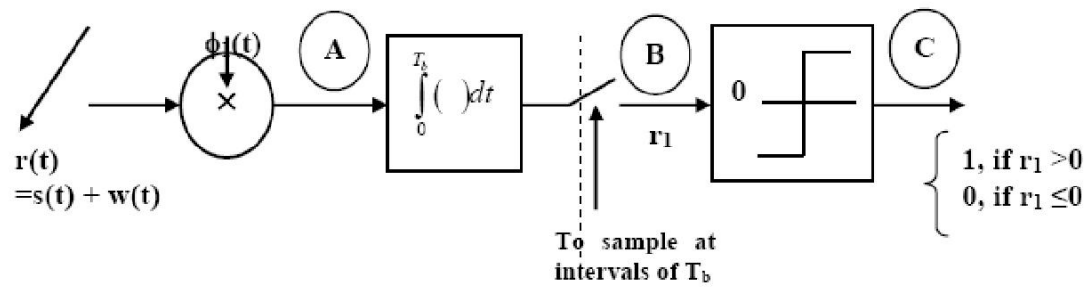
(b) Low-pass filter, designed to remove the double-frequency components of the product modulator output (i.e., the components centered on ω_c) and pass the zero-frequency components.

(c) Sampler, which uniformly samples the output of the low-pass filter at where; the local clock governing the operation of the sampler is *synchronized* with the clock responsible for bit-timing in the transmitter.

(d) Decision-making device, which compares the sampled value of the low-pass filters output to an externally supplied *threshold*, every seconds. If the threshold is exceeded, the device decides in favor of symbol 1; otherwise, it decides in favor of symbol 0. levels.

Decision
Boundary





Power Spectrum for BPSK Modulated Signal

Continuing with our simplifying assumption of zero initial phase of the carrier and with no pulse shaping filtering, we can express a BPSK modulated signal as:

$$s(t) = \sqrt{\frac{E_b \cdot 2}{T_b}} \cdot d(t) \cos \omega_c t, \text{ where } d(t) = \pm 1 \quad 5.24.6$$

The baseband equivalent of $s(t)$ is,

$$\tilde{u}(t) = u_I(t) = \sqrt{\frac{2E_b}{T_b}} \cdot d(t) = \pm g(t), \quad 5.24.7$$

$$\text{where } g(t) = \sqrt{\frac{2E_b}{T_b}} \text{ and } u_Q(t) = 0.$$

Now, $u_I(t)$ is a random sequence of $+\sqrt{\frac{2E_b}{T_b}}$ and $-\sqrt{\frac{2E_b}{T_b}}$ which are equi-probable. So, the power spectrum of the base band signal is:

$$\rightarrow U_B(f) = \frac{2E_b \cdot \sin^2(\pi T_b f)}{(\pi T_b f)^2} = 2 \cdot E_b \cdot \text{sinc}^2(T_b f) \quad 5.24.8$$

Now, the power spectrum $S(f)$ of the modulated signal can be expressed in terms of $U_B(f)$ as:

$$S(f) = \frac{1}{4} [U_B(f - f_c) + U_B(f + f_c)] \quad 5.24.9$$

'Fig.5.24.4 shows the normalized base band power spectrum of BPSK modulated signal. The spectrum remains the same for arbitrary non-zero initial phase of carrier oscillator.'

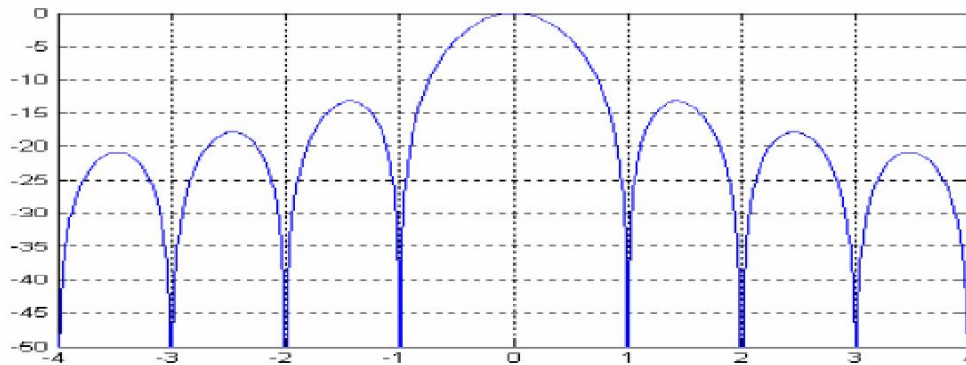


Fig.5.24.4: Normalized base band power spectrum of BPSK modulated signal

Frequency Shift Keying Modulation

Frequency Shift Keying (FSK) modulation is a popular form of digital modulation used in low-cost applications for transmitting data at moderate or low rate over wired as well as wireless channels. In general, an M-ary FSK modulation scheme is a power efficient modulation scheme and several forms of M-ary FSK modulation are becoming popular for spread spectrum communications and other wireless applications. In this lesson, our discussion will be limited to binary frequency shift keying (BFSK).

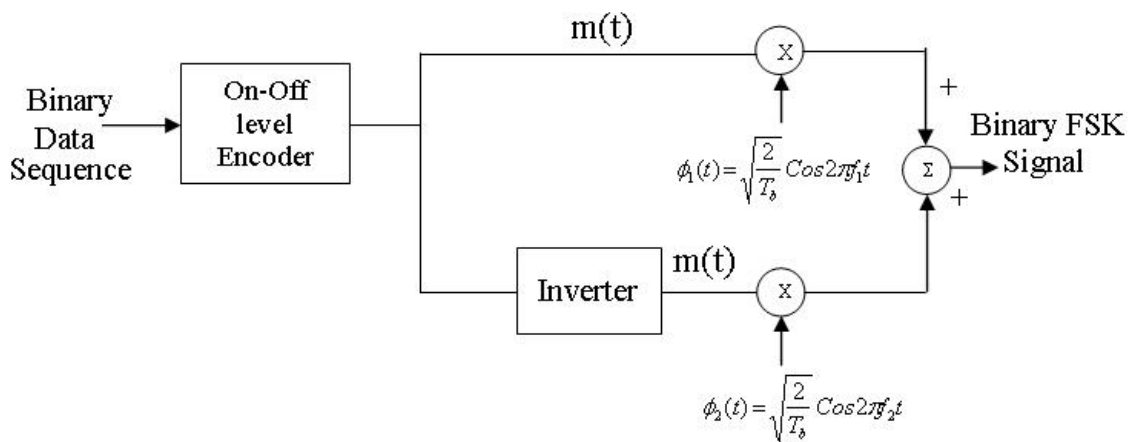
Two carrier frequencies are used for binary frequency shift keying modulation. One frequency is called the 'mark' frequency (f_2) and the other as the space frequency (f_1). By convention, the 'mark' frequency indicates the higher of the two carriers used. If T_b indicates the duration of one information bit, the two time-limited signals can be expressed as :

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_i t, & 0 \leq t \leq T_b, i=1,2 \\ 0, & \text{elsewhere.} \end{cases} \quad 5.23.2$$

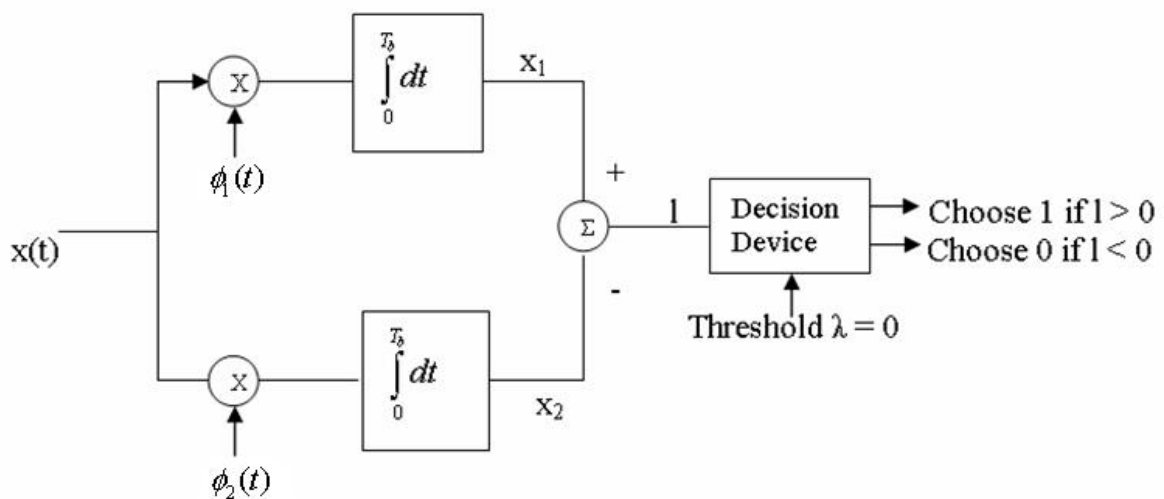
The binary scheme uses two carriers and for special relationship between the two frequencies one can also define two orthonormal basis functions as shown below.

$$\phi_j(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_j t \quad ; \quad 0 \leq t \leq T_b \quad \text{and } j=1,2 \quad 5.23.3$$

Generation and Detection:-



FSK Transmitter



FSK receiver

A binary FSK Transmitter is as shown, the incoming binary data sequence is applied to on-

“0”. When we have symbol 1 the upper channel is switched on with oscillator frequency f_1 , for symbol “0”, because of inverter the lower channel is switched on with oscillator frequency f_2 . These two frequencies are combined using an adder circuit and then transmitted. The transmitted signal is nothing but required BFSK signal. The detector consists of two correlators. The incoming noisy BFSK signal $x(t)$ is common to both correlator. The

Coherent reference signal $\phi_1(t)$ & $\phi_2(t)$ are supplied to upper and lower correlators respectively.

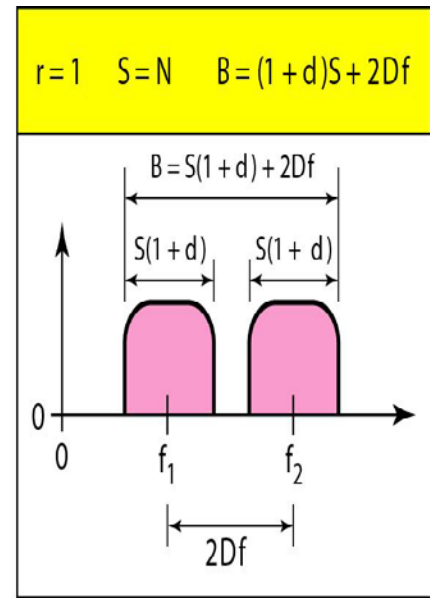
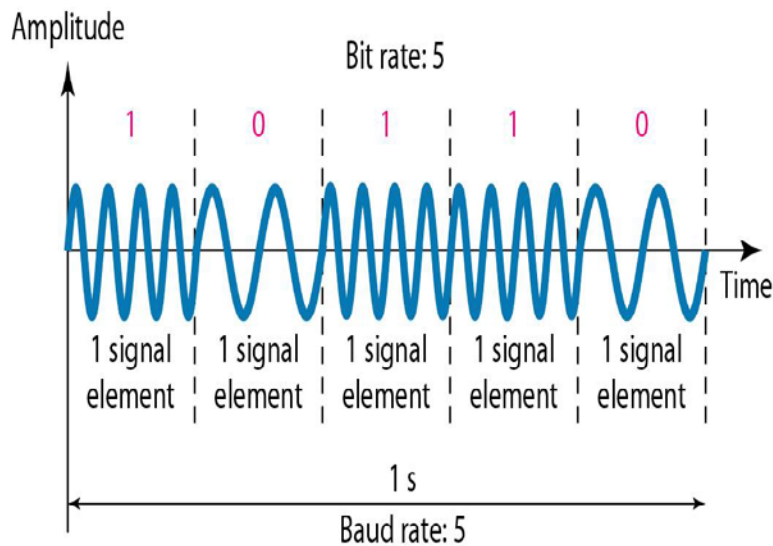
The correlator outputs are then subtracted one from the other and resulting a random vector

“l” ($l = x_1 - x_2$). The output “l” is compared with threshold of zero volts.

If $I > 0$, the receiver decides in favour of symbol 1. $I < 0$, the receiver decides in favour of symbol 0.

FSK Bandwidth:

- Limiting factor: Physical capabilities of the carrier
- Not susceptible to noise as much as ASK



- Applications
 - On voice-grade lines, used up to 1200bps
 - Used for high-frequency (3 to 30 MHz) radio transmission
 - used at higher frequencies on LANs that use coaxial cable.

Therefore Binary FSK system has 2 dimensional signal space with two messages $S_1(t)$ and $S_2(t)$, $[N=2, m=2]$ they are represented,

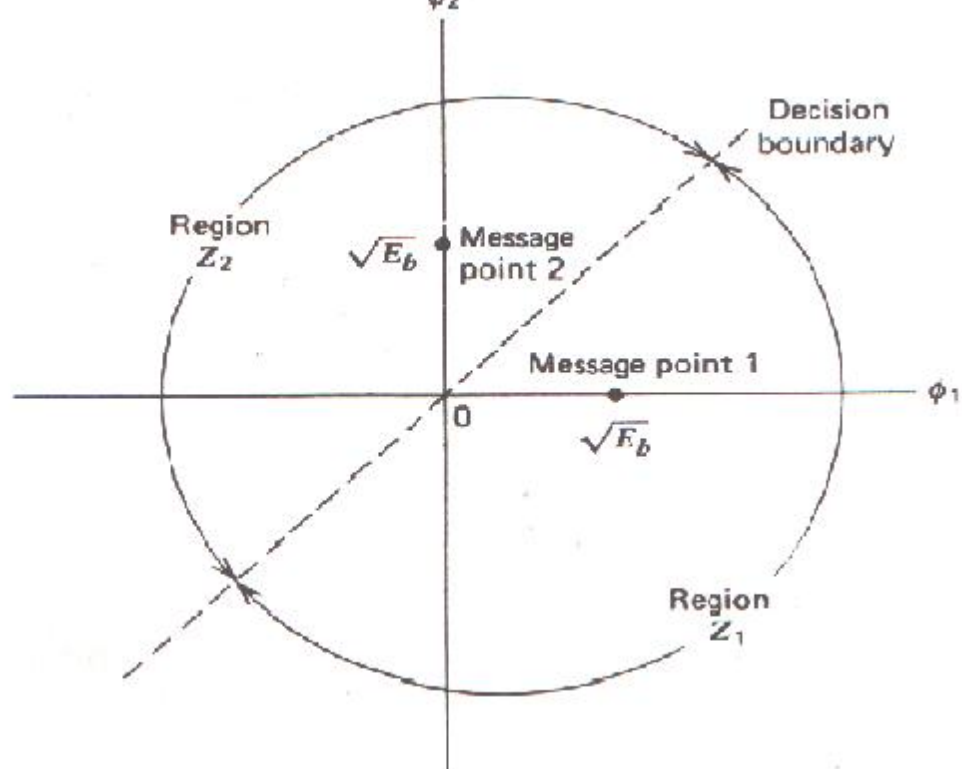


Fig. Signal Space diagram of Coherent binary FSK system.

3. Explain About Quadrature Phase – Shift Keying (Qpsk)

In a sense, QPSK is an expanded version from binary PSK where in a symbol consists of two bits and two orthonormal basis functions are used. A group of two bits is often called a

“dibit”. So, four dibits are possible. Each symbol carries same energy. Let, E : Energy per Symbol and T : Symbol duration = $2 \cdot T_b$, where T_b : duration of 1 bit.

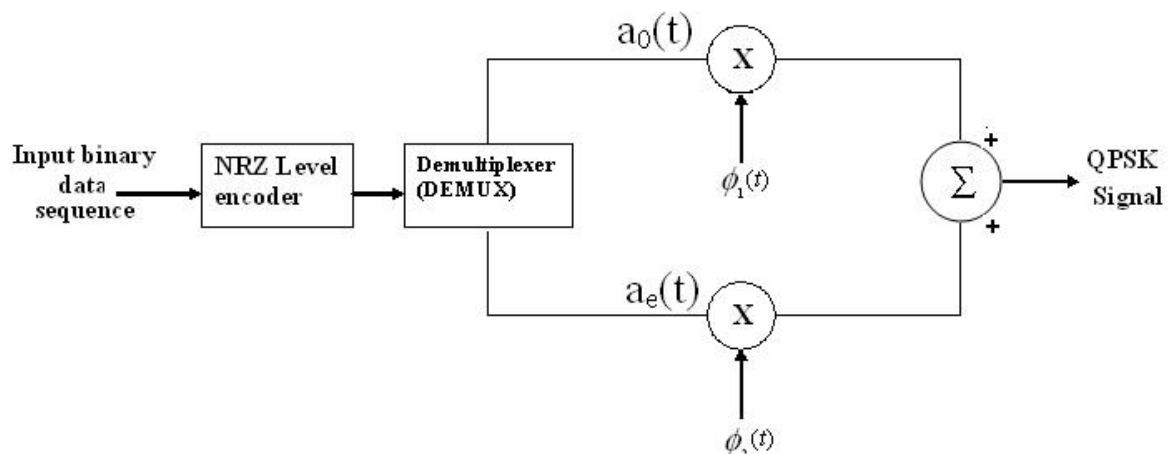


Fig. (a) QPSK Transmitter

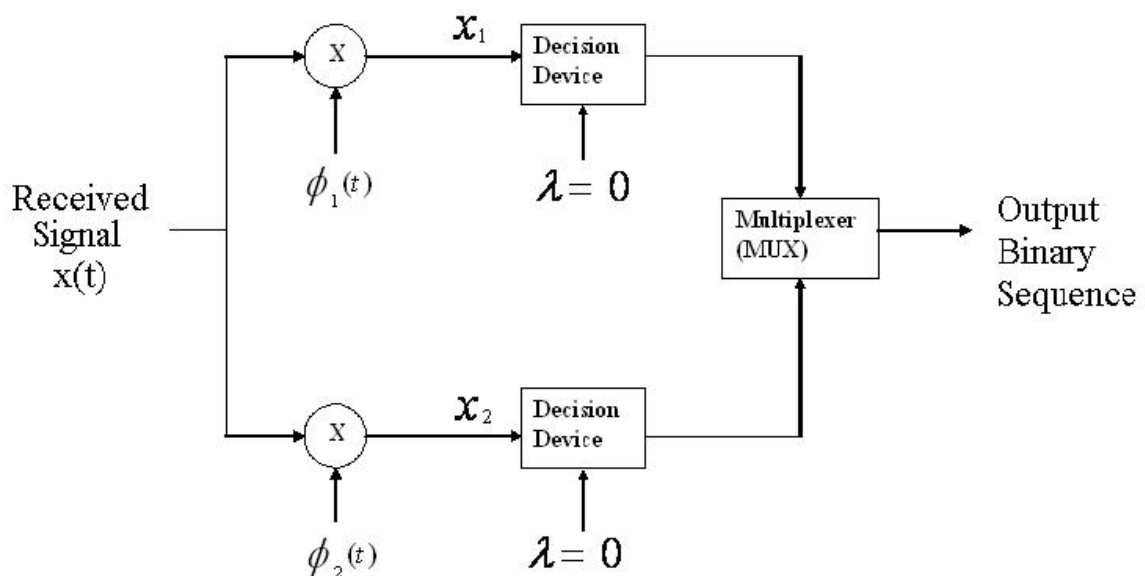


Fig. (b) QPSK Receiver

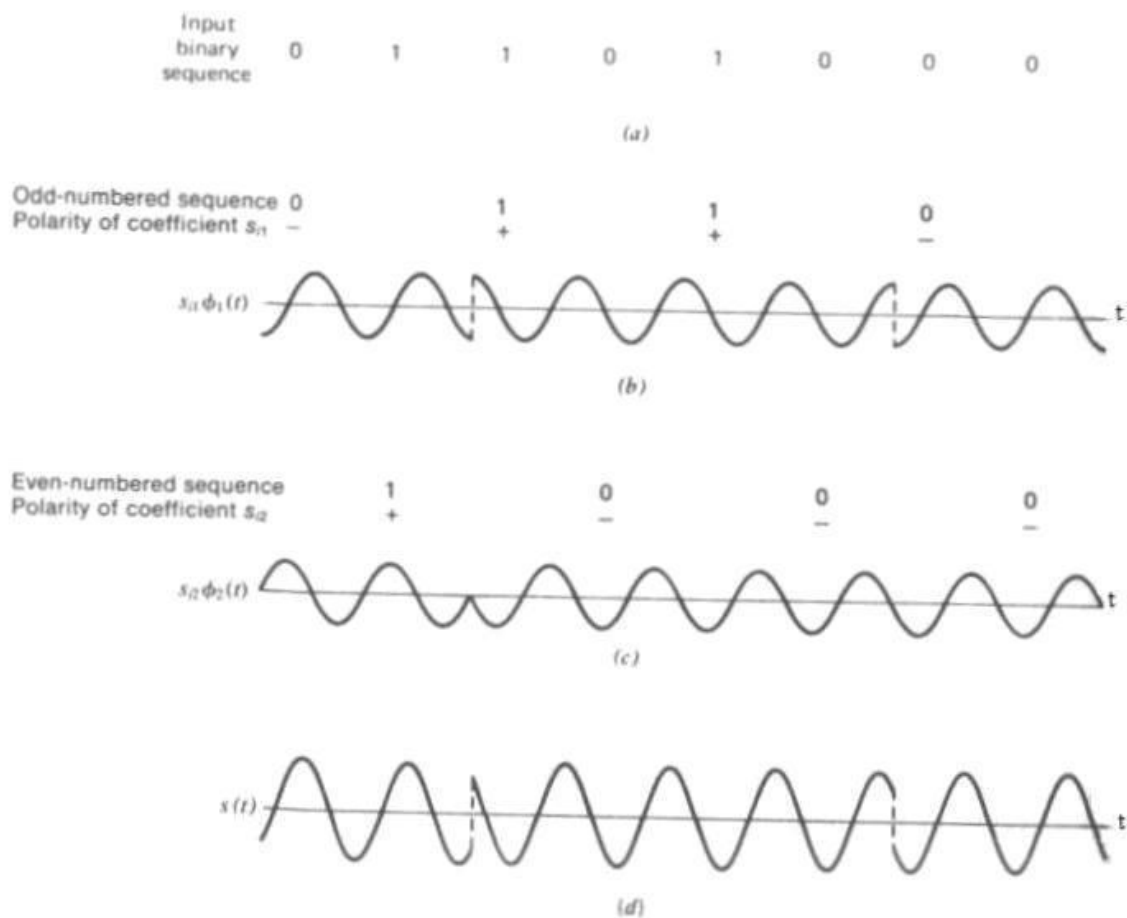


Fig. QPSK Waveform

In QPSK system the information carried by the transmitted signal is contained in the phase.

QPSK Receiver:-

The QPSK receiver consists of a pair of correlators with a common input and supplied with a

locally generated pair of coherent reference signals $\phi_1(t)$ & $\phi_2(t)$ as shown in fig(b). The correlator outputs x_1 and x_2 produced in response to the received signal $x(t)$ are each compared with a threshold value of zero.

The in-phase channel output:

If $x_1 > 0$ a decision is made in favour of symbol 1 $x_1 < 0$ a decision is made in favour of symbol 0.

Similarly quadrature channel output:

If $x_2 > 0$ a decision is made in favour of symbol 1 and $x_2 < 0$ a decision is made in favour of symbol 0. Finally these two binary sequences at the in phase and

quadrature channel outputs are combined in a multiplexer (Parallel to Serial) to reproduce the original binary sequence

Input	Dibit		Phase of QPSK	Coordinates of signal points		
	(b ₀)	(b _e)		s _{i1}	s _{i2}	i
\bar{s}_1	1	0	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$	1
\bar{s}_2	0	0	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$	2
\bar{s}_3	0	1	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$	3
\bar{s}_4	1	1	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$	4

Probability of error:-

A QPSK system is in fact equivalent to two coherent binary PSK systems working in parallel and using carriers that are in-phase and quadrature. The in-phase channel output x_1 and the Q-channel output x_2 may be viewed as the individual outputs of the two coherent binary PSK systems. Thus the two binary PSK systems may be characterized as follows.

- The signal energy per bit $\bar{\sqrt{}}$
- The noise spectral density is $N_0/2$

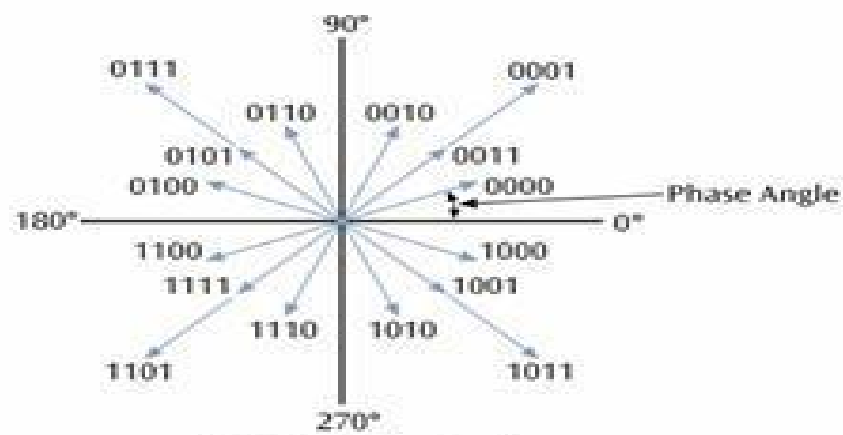
The bit errors in the I-channel and Q-channel of the QPSK system are statistically independent. The I-channel makes a decision on one of the two bits constituting a symbol (di bit) of the QPSK signal and the Q-channel takes care of the other bit.

4. Explain about QAM (Quadrature Amplitude Modulation).

- QAM is a combination of ASK and PSK

Two different signals sent simultaneously on the same carrier frequency ie, $M=4, 16, 32, 64, 128, 256$.

As an example of QAM, 12 different phases are combined with two different amplitudes. Since only 4 phase angles have 2 different amplitudes, there are a total of 16 combinations. With 16 signal combinations, each baud equals 4 bits of information ($2^4 = 16$). Combine ASK and PSK such that each signal corresponds to multiple bits. More phases than amplitudes. Minimum bandwidth requirement of QAM is same as ASK or PSK.



(a) Twelve Phase Angles



(b) A Phase Change with Two Amplitudes

Unit-V**Error Control Coding****Part-A**

1. Mention the properties of cyclic codes [AUC NOV/DEC 2011]

Linearity property

The sum of any two code words is also a valid code word

Cyclic property

Every cyclic shift of a valid code vector produces another valid code vector.

2. Define hamming distance. [AUC APR/MAY 2011]

The hamming distance between two code vectors is equal to the number of elements in which they differ. For example, let the two code words be,

$X = (101)$ and $Y = (110)$ These two code words differ in second and third bits. Therefore the hamming distance between X and Y is two.

3. What is meant by transparency with respect to line codes [AUC APR/MAY 2011] The line code is said to be transparent if the synchronization between the transmitter and receiver is maintained for any type of input data sequence.

4. Define hamming distance and calculate its value for two code words 11100 and 11011 [AUC APR/MAY 2010]

The hamming distance between two code vectors is equal to the number of elements in which they differ. For example, let the two code words be,

$X = (11100)$ and $Y = (11011)$

$D = 2$ These two code words differ in second and third bits. Therefore the hamming distance between X and Y is two.

5. What is convolution code? How is it different from block codes? [AUC APR/MAY 2012]

Fixed number of input bits is stored in the shift register & they are combined with the help of mod 2 adders. This operation is equivalent to binary convolution coding.

6. State any four desirable properties of line code [AUC NOV/DEC 2012]

- The PAM signal should have adequate timing content,
- The PAM signal should be immune to channel noise and interference
- The PAM signal should allow error detection and error correction
- The PAM signal should be transparent to digital data being transmitted

7. Find the hamming distance 101010 and 010101. If the minimum hamming distance of a (n,k) linear block code is 3, what is its minimum hamming weight? [AUC NOV/DEC 2012]

$$d(x_1, x_2) = x_1 \oplus x_2 = 111111$$

$$d(x_1, x_2) = 6$$

$d_{min}=3$ then $W_{min}=d_{min}=3$

8. What is meant by syndrome of linear block code?

The non zero output of the product YH^T is called syndrome & it is used to detect errors in y . Syndrome is denoted by S & given as,

$$S=YH^T$$

9. What is convolutional code? Explain the fundamental difference between block codes and convolutional codes.

Block codes take k number of bits simultaneously form n -bit code vector. This code vector is also called block. Convolutional code takes one message bits at a time and generates two or more encoded bits. Thus convolutional codes generate a string of encoded bits for input message string.

10. What is hamming distance?

The hamming distance between two code vectors is equal to the number of elements in which they differ. For example, let the two code words be,

$$X = (101) \text{ and } Y = (110)$$

These two code words differ in second and third bits. Therefore the hamming distance between X and Y is two.

11. Define code efficiency.

The code efficiency is the ratio of message bits in a block to the transmitted bits for that block by the encoder i.e.,

$$\text{Code efficiency} = \frac{k}{n}$$

$k = \text{message bits}$

$n = \text{transmitted bits.}$

12. What are the error detection and correction capabilities of hamming codes ?

The minimum distance (d_{min}) of hamming codes is 3 . Hence it can be used to detect double errors or correct single errors. Hamming codes are basically linear block codes with $d_{min} = 3$.

13. What is meant by linear code?

A code is linear if modulo-2 sum of any two code vectors produces another code vector. This means any code vector can be expressed as linear combination of other code vectors.

14. What is meant by cyclic codes?

Cyclic codes are the subclasses of linear block codes. They have the property that a cyclic shift of one codeword produces another code word.

15. How syndrome is calculated in Hamming codes and cyclic codes?

In hamming codes the syndrome is calculated as, $S=YH^T$

Here Y is the received and H is the transpose of parity check matrix

16. What is difference between block codes and convolutional codes?

Block codes take "k" number of bits simultaneously form "n"-bit code vector. This code vector is also called block. Convolutional code takes one message bits at a time and generates two or more encoded bits. Thus convolutional codes generate a string of encoded bits for input message string.

PART B

1. Explain about Linear Block Codes.

Block codes operate on a block of bits. Block codes are referred to as (n, k) codes. A block of k information bits are coded to become a block of n bits. But before we go any further with the details, let's look at an important concept in coding called Hamming distance. Let's say that we want to code the 10 integers, 0 to 9 by a digital sequence. Sixteen unique sequences can be obtained from four bit words. We assign the first ten of these, one to each integer. Each integer is now identified by its own unique sequence of bits.

Hamming Weight: The Hamming weight of this code scheme is the largest number of 1's in a valid codeword. This number is 3 among the 10 codewords we have chosen. (the ones in the white space).

Concept of Hamming Distance: In continuous variables, we measure distance by Euclidean concepts such as lengths, angles and vectors. In the binary world, distances are measured between two binary words by something called the Hamming distance. The Hamming distance is the number of disagreements between two binary sequences of the same size. The Hamming distance between sequences 001 and 101 is = 1. The Hamming distance between sequences 0011001 and 1010100 is = 4. Hamming **distance** and **weight** are very important and useful concepts in coding. The knowledge of Hamming distance is used to determine the capability of a code to detect and correct errors. Although the Hamming **weight** of our chosen code set is 3, the minimum Hamming **distance** is 1. We can generalize this to say that the maximum number of error bits that can be detected is

$$t = d_{\min} - 1$$

Where d_{\min} is Hamming distance of the codewords. For a code with $d_{\min} = 3$, we can both detect 1 and 2 bit errors. So we want to have a code set with as large a Hamming distance as possible since this directly effects our ability to detect errors. The number of errors that we can correct is given by

$$t(\text{int}) = \frac{d_{\min} - 1}{2}$$

Creating block codes: The block codes are specified by (n,k). The code takes k information bits and computes (n-k) parity bits from the code generator matrix. Most block codes are systematic in that the information bits remain unchanged with parity bits attached either to the front or to the back of the information sequence.

- * Hamming code, a simple linear block code
- * Hamming codes are most widely used linear block codes.

- * A Hamming code is generally specified as $(2n-1, 2n-n-1)$.
- * The size of the block is equal to $2n-1$.
- * The number of information bits in the block is equal to $2n-n-1$ and the number of overhead bits is equal to n . All Hamming codes are able to detect three errors and correct one.

Reed-Solomon Codes: Reed Solomon (R-S) codes form an important sub-class of the family of Bose- Chaudhuri-Hocquenghem (BCH) codes and are very powerful linear non-binary block codes capable of correcting multiple random as well as burst errors. They have an important feature that the generator polynomial and the code symbols are derived from the same finite field. This enables to reduce the complexity and also the number of computations involved in their implementation. A large number of R-S codes are available with different code rates.

An R-S code is described by a generator polynomial $g(x)$ and other usual important code parameters such as the number of message symbols per block (k), number of code symbols per block (n), maximum number of erroneous symbols (t) that can surely be corrected per block of received symbols and the designed minimum symbol Hamming distance (d). A parity-check polynomial $h(X)$ of order k also plays a role in designing the code. The symbol x , used in polynomials is an indeterminate which usually implies unit amount of delay.

For positive integers m and t , a primitive (n, k, t) R-S code is defined as below:
 Number of encoded symbols per block: $n = 2^m - 1$
 Number of message symbols per block: k
 Code rate: $R = k/n$
 Number of parity symbols per block: $n - k = 2t$
 Minimum symbol Hamming distance per block: $d = 2t + 1$. It can be noted that the block length n of an (n, k, t) R-S code is bounded by the corresponding finite field $GF(2^m)$. Moreover, as $n - k = 2t$, an (n, k, t) R-S code has optimum error correcting capability.

2. Explain about Convolution codes.

Convolutional codes are widely used as channel codes in practical communication systems for error correction. *The encoded bits depend on the current k input bits and a few past input bits. * The main decoding strategy for convolutional codes is based on the widely used Viterbi algorithm. *Convolutional codes are commonly described using two parameters: the code rate and the constraint length. The code rate, k/n , is expressed as a ratio of the number of bits into the convolutional encoder (k) to the number of channel symbols output by the convolutional encoder (n) in a given encoder cycle. *The constraint length parameter, K , denotes the "length" of the convolutional encoder, i.e. how many k -bit stages are available to feed the combinatorial logic that produces the output symbols. Closely related to K is the parameter m , which can be thought of as the memory length of the encoder. A simple convolutional encoder is shown below(fig.). The information bits are fed in small groups of k -bits at a time to a shift register. The output encoded bits are obtained by modulo-2 addition (EXCLUSIVE-OR operation) of the input information bits and the contents of the shift registers which are a few previous information bits.

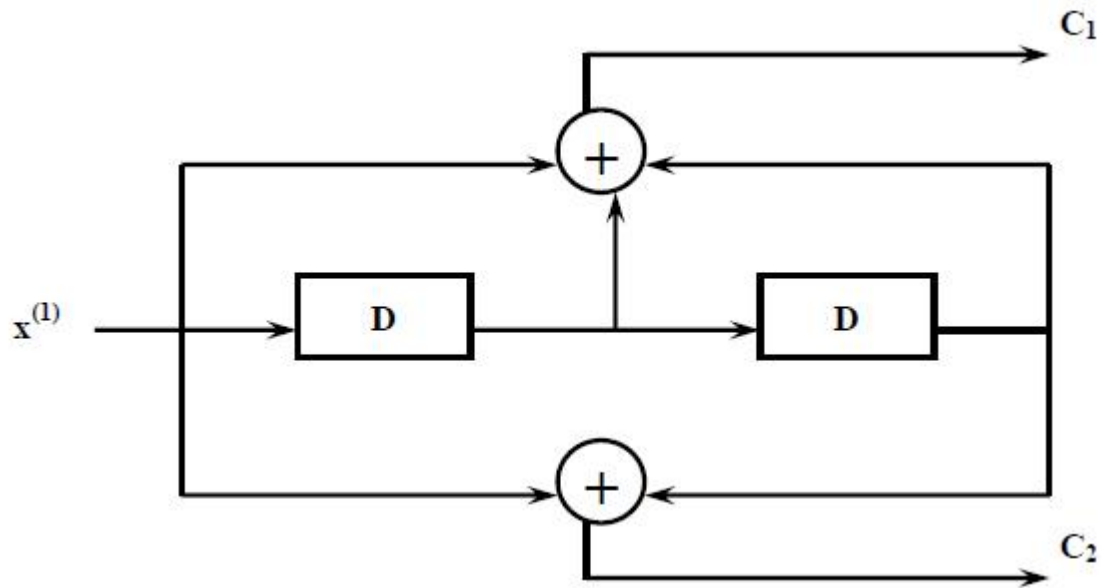


Fig 3.1 . A convolutional encoder with $k=1$, $n=2$ and $r=1/2$

The operation of a convolutional encoder can be explained in several but equivalent ways such as, by

- a) state diagram representation.
- b) tree diagram representation.
- c) trellis diagram representation.

a) State Diagram Representation: A convolutional encoder may be defined as a finite state machine. Contents of the rightmost $(K-1)$ shift register stages define the states of the encoder. So, the encoder in **Fig. 3.1** has four states. The transition of an encoder from one state to another, as caused by input bits, is depicted in the state diagram. **Fig. 3.2** shows the state diagram of the encoder in **Fig. 3.1**. A new input bit causes a transition from one state to another.

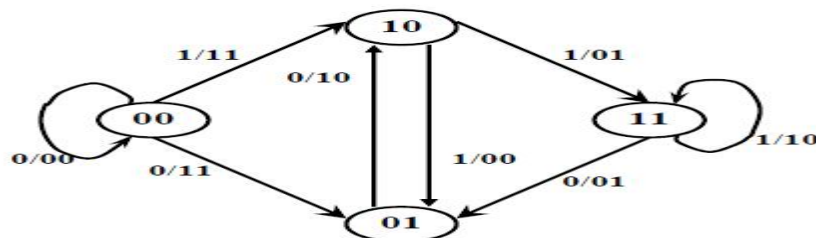
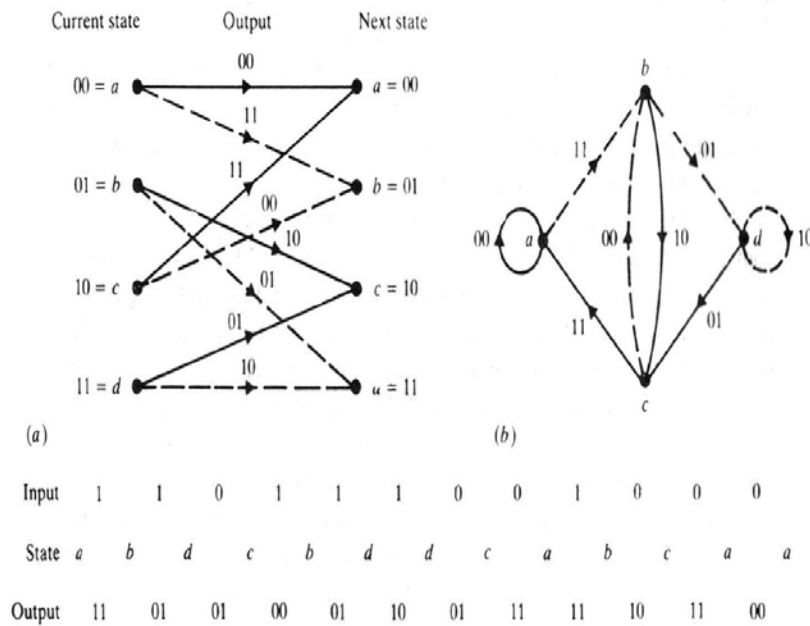


Fig 3.2 State diagram representation for the encoder

b) Tree Diagram Representation: The tree diagram representation shows all possible information and encoded sequences for the convolutional encoder. **Fig. 3.3** shows the tree diagram for the encoder in **Fig. 3.1**. The encoded bits are labeled on the branches of the tree. Given an input sequence, the encoded sequence can be directly read from the tree.

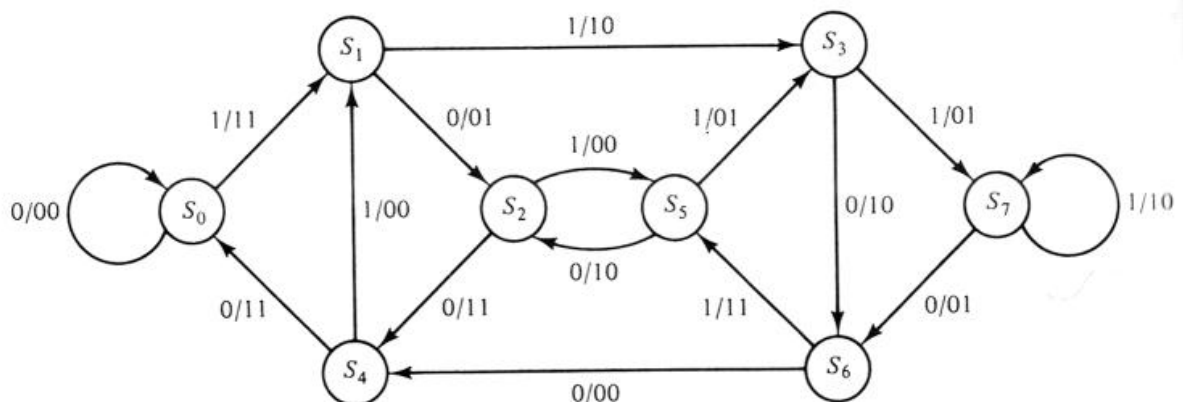
Representing convolutional codes compactly: code trellis and state diagram:

STATE DIAGRAM:



Inspecting state diagram: Structural properties of convolutional codes:

- Each new block of k input bits causes a transition into new state.
- Hence there are $2k$ branches leaving each state.
- Assuming encoder zero initial state, encoded word for any input of k bits can thus be obtained. For instance, below for $\mathbf{u}=(1\ 1\ 1\ 0\ 1)$, encoded word $\mathbf{v}=(1\ 1, 1\ 0, 0\ 1, 0\ 1, 1\ 1, 1\ 0, 1\ 1, 1\ 1)$ is produced:



- Encoder state diagram for $(n,k,L)=(2,1,2)$ code

- Note that the number of states is $2L+1 = 8$.

Distance for some convolutional codes:

n	k	R_c	L	d_f	$R_c d_f/2$
4	1	1/4	3	13	1.63
3	1	1/3	3	10	1.68
2	1	1/2	3	6	1.50
			6	10	2.50
			9	12	3.00
3	2	2/3	3	7	2.33
4	3	3/4	3	8	3.00

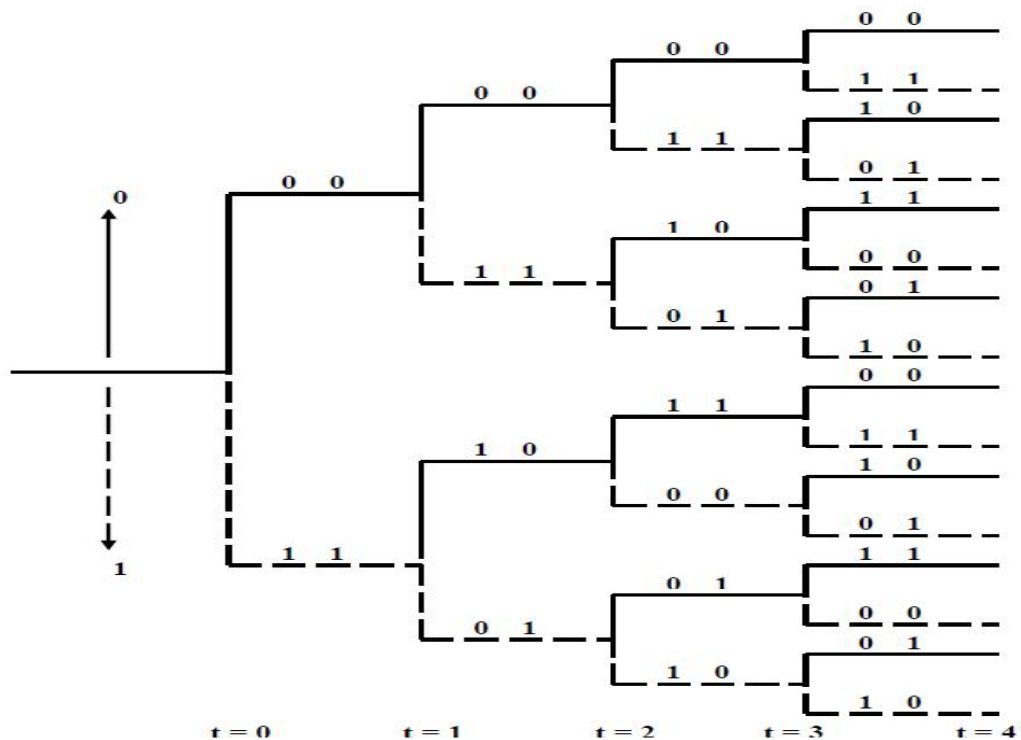
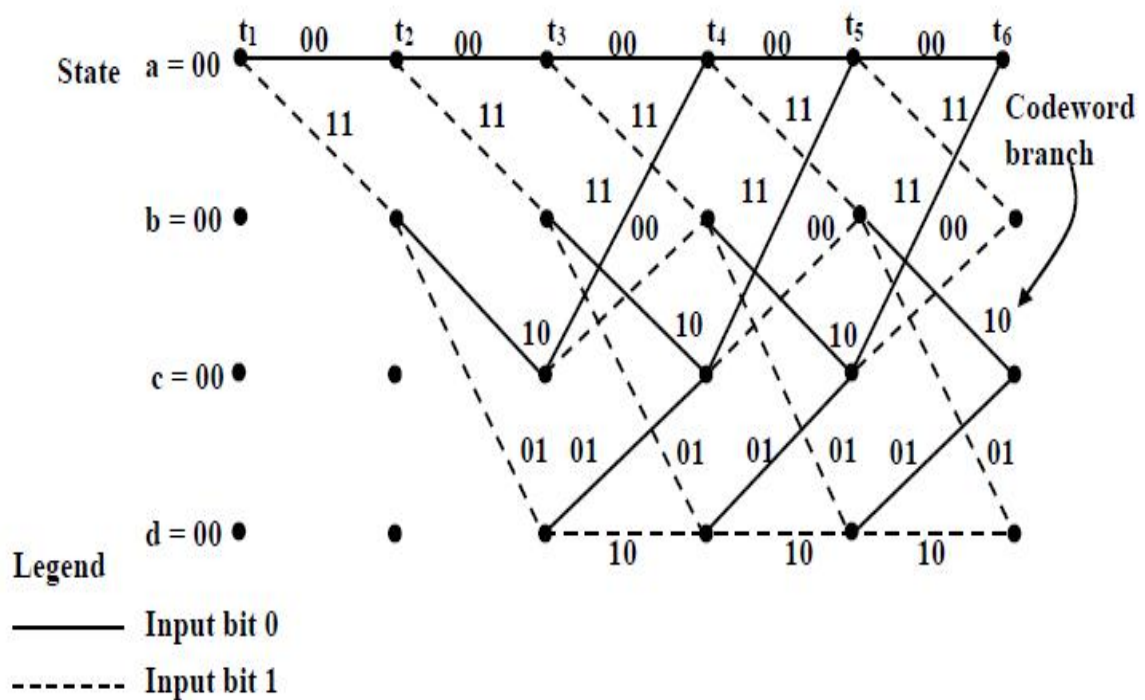


Fig.3.3 A tree diagram for the encoder in Fig. 3.1

c) Trellis Diagram Representation: The trellis diagram of a convolutional code is obtained from its state diagram. All state transitions at each time step are explicitly shown in the diagram to retain the time dimension, as is present in the corresponding tree diagram. Usually, supporting descriptions on state transitions, corresponding input and output bits etc. are labeled in the trellis diagram. It is interesting to note that the trellis diagram, which describes the operation of the encoder, is very convenient for describing the behavior of the corresponding decoder, especially when the famous „Viterbi Algorithm (VA)“ is followed. Fig. 3.4 shows the trellis diagram for the encoder in Figure 3.1.

**Fig.3.4.** Trellis diagram for the encoder in Fig. 3.1

Hamming Code Example:

$$G := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

- H (7,4)
- Generator matrix G: first 4-by-4 identical matrix

- Message information vector p
- Transmission vector x
- Received vector r and error vector e
- Parity check matrix H

$$\mathbf{r} = \mathbf{x} + \mathbf{e}_i$$

$$\mathbf{Gp} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \mathbf{x}$$

$$\mathbf{H} := \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Error Correction:

- If there is no error, syndrome vector z=zeros

$$\mathbf{Hr} = \mathbf{H}(\mathbf{x} + \mathbf{e}_i) = \mathbf{Hx} + \mathbf{He}_i = \mathbf{0} + \mathbf{He}_i = \mathbf{He}_i$$

- If there is one error at location 2
- New syndrome vector z is

$$\mathbf{Hr} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \mathbf{z}$$

$$\mathbf{r} = \mathbf{x} + \mathbf{e}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\mathbf{H}\mathbf{r} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \mathbf{z}$$

1. i) Consider a single error correction (7,4) linear code and the corresponding decoding table.
2. Find the (7,4) linear systematic block code word corresponding to 1101. Assume a suitable generator matrix.

Solution:

Let

$$\mathbf{G} = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array} \right]$$

$$n=7 \quad k=4 \quad q \\ = n-k=3$$

code vector $\mathbf{G}=[\mathbf{I}_k \ \mathbf{P}]$

$$\mathbf{P}_{4 \times 3} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Check matrix $\mathbf{C}=\mathbf{M}\mathbf{P}$

$$\mathbf{C}_1 = m_1+m_2+m_3$$

$$\mathbf{C}_2 = m_2+m_3+m_4$$

$$\mathbf{C}_3 = m_1+m_2+m_4$$

$$\mathbf{C}=[010]$$

Complete code word can be calculated $X=\{M:C\}=\{1\ 1\ 0\ 0\ 0\ 1\ 0\}$

The parity matrix $H=[pT : I] = [I : pT] =$

$$: \begin{bmatrix} 1 & 1 & 1 & 0 & : & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & : & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & : & 0 & 0 & 1 \end{bmatrix}$$

Minimum weight $W(X)=3$

3. Determine the generator polynomial $g(X)$ FOR A (7,4) cyclic code and find the code vector for the following data vector 1010, 1111 and 1000

$$n=7$$

$$k=4$$

$$q=n-$$

$$k=3$$

To obtain the generator polynomial (p^7+1)
 $= (p+1)(p^3+p^2+1)(p^3+p+1)$ Let
 $G(p) = (p^3+p+1)$

To obtain the generator matrix in systematic form

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix}$$

To determine the code vector

1. code vector for $M=1010$

$$X=MG$$

$$X = [1\ 0\ 1\ 0] \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix} = [1\ 0\ 1\ 0\ 0\ 1\ 1]$$

2. code vector for $M=1111$

3. code vector for M=1000

$$X = [1000] \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix} = [10000101]$$

4. Assume a (2,1) convolutional coder with constraint length 6. Draw the tree diagram, state diagram and trellis diagram for the assumed coder

Design block code for a message block of size eight that can correct for single errors Briefly discuss on various error control codes and explain in detail with one example for convolution code.

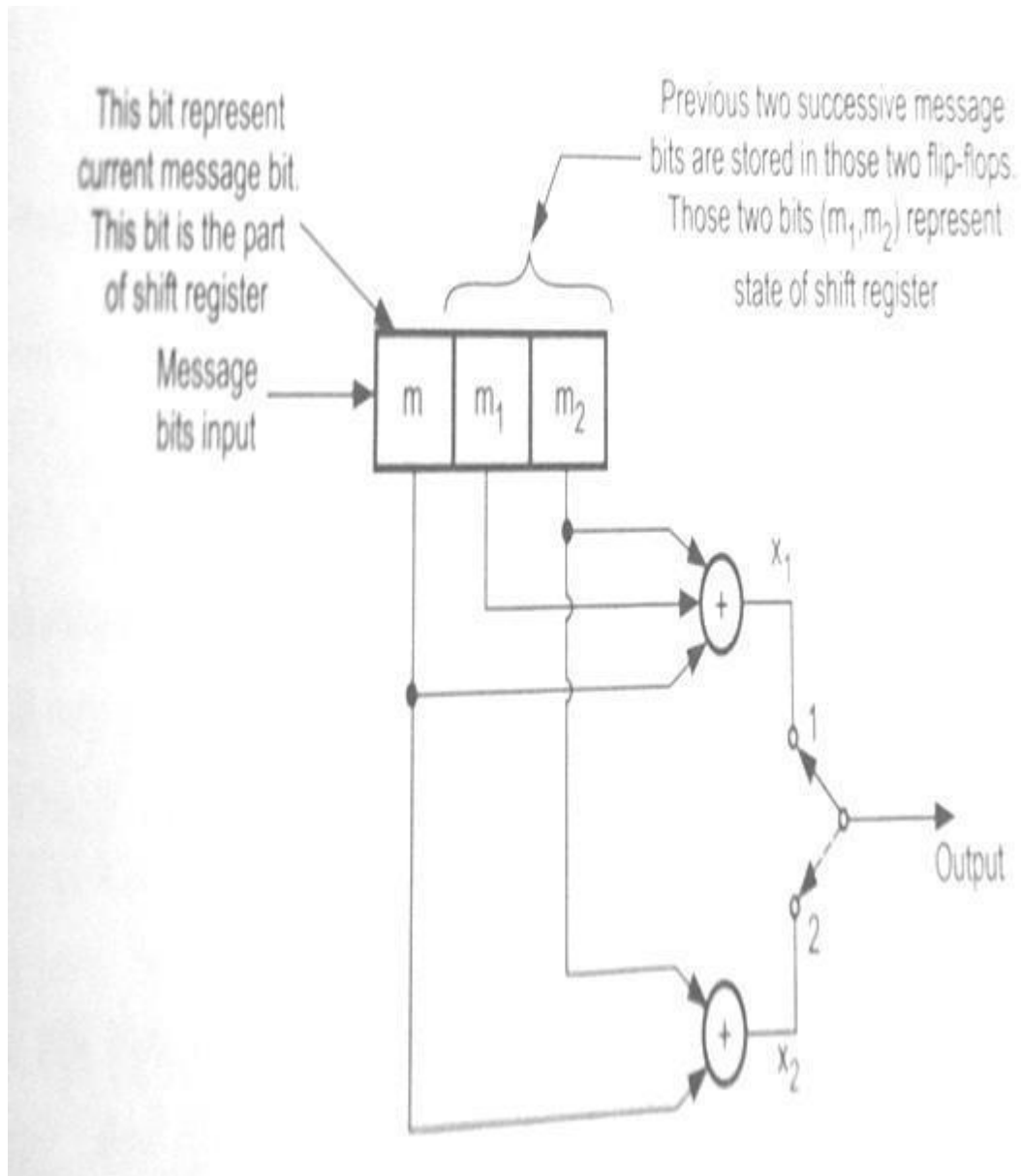
$N=2$, $K=1$ AND $K=6$ (CONSTRAIN LENGTH) $M=K/n=6/2=3$, since constrain length $k=n*M$

3 storage element in shift register $N=2$ two output bits

One set $k=1$ of shift register having 3 storage element the convolutional code structure is easy to draw from its parameters. First draw m boxes representing the m memory register. Then draw n modulo-2 adders to represent the n output bits. Now connect the memory registers to the adders using the generator polynomial

$$g^{(i)}(D) = g_0^{(i)} + g_1^{(i)}D + g_2^{(i)}D^2 + \dots + g_M^{(i)}D^M$$

$$P = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$



Convolutional codes

k = number of bits shifted into the encoder at one time

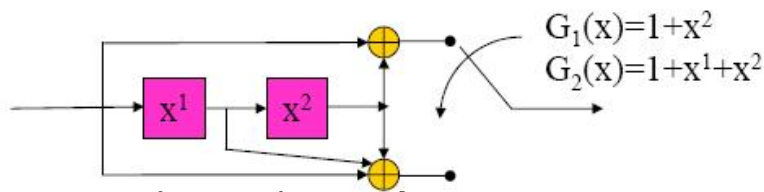
- $k=1$ is usually used!!
- n = number of encoder output bits corresponding to the k information bits
- $r = k/n$ = code rate
- K = constraint length, encoder memory

Each encoded bit is a function of the present input bits and their past ones.

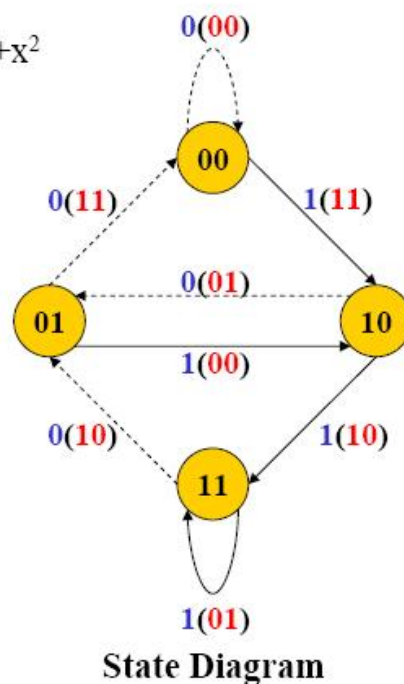
Generator Sequence

Convolutional Codes

An Example – (rate=1/2 with $K=2$)

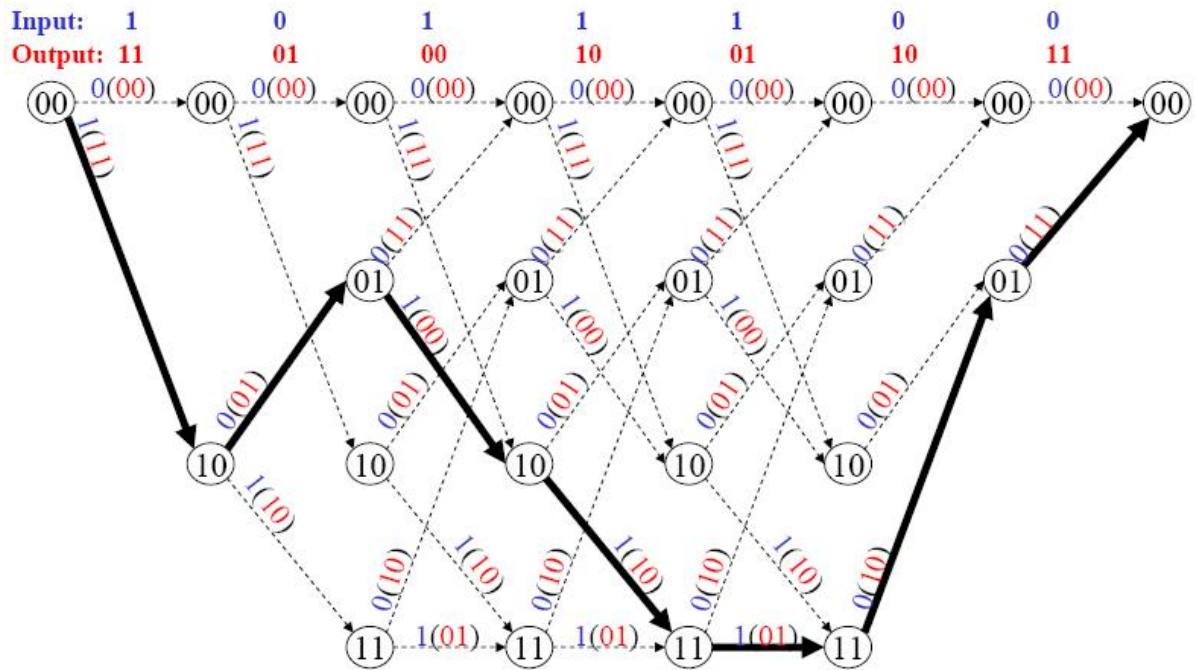


	Present	Next	Output
0	00	00	00
1	00	10	11
0	01	00	11
1	01	10	00
0	10	01	01
1	10	11	10
0	11	01	10
1	11	11	01



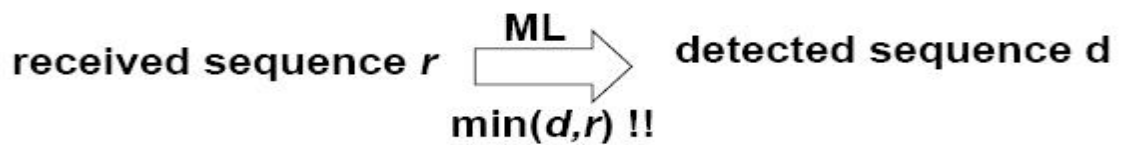
Trellis Diagram Representation

Encoding Process



5. Explain about Viterbi Decoding Algorithm.

Maximum Likelihood (ML) decoding rule



Viterbi Decoding Algorithm

- An efficient search algorithm
- Performing ML decoding rule.
- Reducing the computational complexity.

Basic concept

- Generate the code trellis at the decoder
- The decoder penetrates through the code trellis *level by level* in search for the transmitted code sequence
- At each level of the trellis, the decoder computes and compares the metrics of all the partial paths entering a node
- The decoder *stores* the partial path with the larger metric and *eliminates* all the other partial paths. The stored partial path is called the *survivor*.

Viterbi Decoding Process

