SKP Engineering College

Tiruvannamalai – 606611

A Course Material

on

Signals and Systems

By

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Electronics and Communication Engineering Department

Quality Certificate

This is to Certify that the Electronic Study Material

Subject Code: EC6303

Subject Name: Signals and Systems

Year/Sem: II/III

Being prepared by me and it meets the knowledge requirement of the University curriculum.

Signature of the Author

Name: K.Vijayalakshmi

Designation: Assistant Professor

This is to certify that the course material being prepared by Ms. K.Vijayalakshmi is of the adequate quality. He has referred more than five books and one among them is from abroad author.

Signature of HD

Name:

Seal:

Signature of the Principal

Name: Dr.V.Subramania Bharathi

Seal:

EC6303 SIGNALS AND SYSTEMS L T P C 3 1 0 4

OBJECTIVES:

_ To understand the basic properties of signal & systems and the various methods of classification

_ To learn Laplace Transform &Fourier transform and their properties

_ To know Z transform & DTFT and their properties

_ To characterize LTI systems in the Time domain and various Transform domains

UNIT I CLASSIFICATION OF SIGNALS AND SYSTEMS 9

Continuous time signals (CT signals) - Discrete time signals (DT signals) - Step, Ramp, Pulse, Impulse, Sinusoidal, Exponential, Classification of CT and DT signals - Periodic & Aperiodic signals, Deterministic & Random signals, Energy & Power signals - CT systems and DT systems- Classification of systems – Static & Dynamic, Linear & Nonlinear, Time variant & Time-invariant, Causal & Noncausal, Stable & Unstable.

UNIT II ANALYSIS OF CONTINUOUS TIME SIGNALS 9

Fourier series analysis spectrum of Continuous Time (CT) signals- Fourier and Laplace Transforms in CT Signal Analysis - Properties.

UNIT III LINEAR TIME INVARIANT- CONTINUOUS TIME SYSTEMS 9

Differential Equation-Block diagram representation-impulse response, convolution integrals-Fourier and Laplace transforms in Analysis of CT systems

UNIT IV ANALYSIS OF DISCRETE TIME SIGNALS 9

Baseband Sampling - DTFT –Properties of DTFT - Z Transform – Properties of Z Transform

UNIT V LINEAR TIME INVARIANT-DISCRETE TIME SYSTEMS 9

Difference Equations-Block diagram representation-Impulse response - Convolution sum- Discrete Fourier and Z Transform Analysis of Recursive & Non-Recursive systems

TOTAL (L:45+T:15): 60 PERIODS

OUTCOMES: Upon the completion of the course, students will be able to:

_ Analyze the properties of signals & systems

_ Apply Laplace transform, Fourier transform, Z transform and DTFT in signal analysis

_ Analyze continuous time LTI systems using Fourier and Laplace Transforms

_ Analyze discrete time LTI systems using Z transform and DTFT

TEXT BOOK:

1. Allan V.Oppenheim, S.Wilsky and S.H.Nawab, “Signals and Systems”, Pearson, 2007.

REFERENCES:

1. B. P. Lathi, “Principles of Linear Systems and Signals”, Second Edition, Oxford, 2009.

2. R.E.Zeimer, W.H.Tranter and R.D.Fannin, “Signals & Systems - Continuous and Discrete”, Pearson, 2007.

3. John Alan Stuller, “An Introduction to Signals and Systems”, Thomson, 2007.

4. M.J.Roberts, “Signals & Systems Analysis using Transform Methods & MATLAB”, Tata McGraw Hill, 2007.

CONTENTS

Unit - I

Classification Of Signals and Systems

Part – A

1.What is meant by signal? [CO1-L1]

A signal is formally defined as a function of one or more variable, which conveys the information. It is a physical quantity that varies with time in general or any other independent variable.

2.What are the types of standard signals? [CO1-L1]

1. Unit impulse
2. Unit step
3. Unit ramp
4. Real exponential
5. Sinusoidal signal

3.What are the classification of signals? [CO1-L1-May/June 2012]

1. Continuous – time signal and discrete – time signal
2. Periodic signal and a periodic signal
3. Even signal and odd signal
4. Deterministic signal and random signal
5. Energy signal and power signal.

4. What are the types of basic operations on signals? [CO1-L1]

1. Amplitude scaling of signal
2. Addition of signals

3. Multiplication of signal

4. Differentiation on signal

5. Integration on signal

6. Time scaling of signal

7. Reflection of signal

8. Time shifting of signal.

5.Define unit impulse and unit step signals. [CO1-L1-May/June 2010]

Unit Impulse signal:

Amplitude of unit impulse is 1 as its width approaches zero. Then it has zero value at all other values.

Unit Step Signal:

The unit step signal has amplitude of 1 for positive values of independent variable and amplitude of 0 for negative of independent variable.

6.What is meant by continuous – time signal? Give one example. [CO1-L1]

A signal x (t) is said to be continuous – time signal if it is defined for all time t. For continuous time signals, the amplitude of the signal varies continuously with time.

Ex: the speech signal is a continuous – time signal

7.What is meant by discrete - time signal? Give one example. [CO1-L1]

A signal x (n) is said to be discrete – time signal if it defined for discrete instant of time. For discrete – time signal, the amplitude of the signal varies at every discrete value n, which is generally uniform spaced.

Ex: crime rate for all year.

8.Write expression and graphical representation for the impulse function of CT and DT signal.[Dec2013]

The expression of CT signal is given by

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

δ (t) = 0 for t $\neq$ 0

The expression of DT signal is given by

$$\sum_{-\infty}^{\infty} \delta(n) = 1$$

δ (n) =1 for n =0

9. Write expression and graphical representation for unit step function of CT and DT signals. [CO1-L1-May/June 2012]

CT Signal

$$u(n) = \begin{cases} 1 & \text{when } 0 \le n \le \infty \\ 0 & \text{other wise} \end{cases}$$

Dt signal

$$u(t) = \begin{cases} 1 & \text{when } 0 \le t \le \infty \\ 0 & \text{otherwise} \end{cases}$$

10.Write expression and graphical representation of unit ramp function of CT and DT signals. [CO1-L1-Nov/Dec 2014]

CT Signal

$$r(t) = \begin{cases} t & \text{when } 0 \le t \le \infty \\ 0 & \text{otherwise} \end{cases}$$

DT signal

$$r(n) = \begin{cases} n & \text{when } 0 \le n \le \infty \\ 0 & \text{otherwise} \end{cases}$$

11.State the two properties of unit impulse function. [CO1-L1-Nov/Dec 2014]

i) Shifting property:

$$\int_{-\infty}^{+\infty} x(t)\delta(t)\,dt = x(0)$$

ii) Replication property:

$$\int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,dt = x(t)$$

12. Find the fundamental period of signal [CO1-L1-Nov/Dec 2010]

$$\boldsymbol{x = \sin\left(\frac{7\pi}{3} + t\right)}$$

Solu:

$$x(t) = \sin\left(\frac{7\pi}{3} + t\right)$$

$$time\ period\ T = \frac{2\pi}{\omega}$$

$$\omega = \frac{7\pi}{3}$$

$$= \frac{6}{7}\ \text{sec}$$

13. Check e time whether the discrete signal sin 3n is periodic? [CO1-H2-May/June 2013]

The frequency of the discrete time signal is 3, because it is not a multiple of π. Therefore the signal is Aperiodic.

14. Distinguish between deterministic and random signals. [CO1-L2-May/June 2011]

Random Signal:

It has some degree of uncertainty before it actually occurs. The random signal cannot be defined by mathematical expressions.

Deterministic Signal:

There is no uncertainty occurrence. It is completely represented by mathematical expressions.

15. Determine the period of the signal[CO1-L1-Nov/Dec 2011]

$$x = 2cos(n\pi/4)$$

Solu:

$2\pi f n = n\pi/4$

$f = \frac{\pi}{4} \times \frac{1}{2\pi} = 1/8$

We know that, f=1/T

So T = 8sec

16.Write expression and graphical representation of real exponential function of CT signals. [CO1-L1-Nov/Dec 2009]

A real exponential signal is defined as $x(t) = Ae^{at}$

where both A and a are real.

Depending on the value of 'a' we get different signals.

If 'a' is positive, the signal x (t) is growing exponential.

If 'a' is negative, the signal x (t) is decaying exponential. For 'a' =0, then x (t) is constant.

17.When is a system said to be memory less? Give an example. [CO1-L2-May/June 2010]

If the system output does not depend the previous input, it only depends the present input. Then the system is called memory less or static system.

Eg:

$$y(t) = 2x(t) + x(t)$$
$$y(n) = x(n) + \sqrt{x}(n)$$

18.What is mental by periodic and Aperiodic CT and DT signals? [CO1-L1-May/June 2009]

A continuous – time signal x (t) is said to be periodic if it satisfies the condition.

x (t) = x (t+T) for all t.

Where T = fundamental period

If the condition does not satisfy for at least one value of t, then the CT signal is Aperiodic

A discrete time signal x (n) is said to be Periodic if it satisfies the condition.

x (n) = x (x +N) for all n

Where N = fundamental period.

If the condition does not satisfy for at least one value of n then the discrete – time signal is Aperiodic.

19.Define energy and power signals. [CO1-L1-Nov/Dec 2010]

Energy Signal:

A signal is said to be an energy signal if its normalized energy is non zero and finite.

For an energy signal, P = 0. i.e., $0 < E < \infty$

Power Signal:

A signal is said to be the power signal if it satisfies $0 < P < \infty$

For a power signal, $E = \infty$

20.What is meant by symmetric (or) even signal and anti – symmetric (or) odd for CT signals? [CO1-L1-Nov/Dec 2009]

A continuous – time signal x (t) is said to be symmetric or even if it satisfies the condition x (-t) = x (t) for all 't'

Ex: x (t) = A cos t

A continuous – time signal x (t) is said to be anti – symmetric (or) odd signal if it satisfies the condition x (-t) = -x (t) for all't'

Ex: x (t) = A sin t

21. What is meant by energy and power CT signal? [CO1-L1-Nov/Dec 2011]

A continuous – time signal x (t) is called an energy signal if the energy satisfies 0 < E < ∞ for an energy signal P =0

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt \text{ joules}$$

A continuous – time signal is called an power signal if the power satisfies o < P <∞. For an power signal E = ∞

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x\,(t)|^2 \, dt \text{ watts}$$

22. What is the classification of system? [CO1-L1-Nov/Dec 2009]

The classification of systems is,

(i). Linear and Non-Linear systems

(ii). Time invariant and Time varying systems.

(iii). Causal and Non causal systems.

(iv). Stable and unstable systems.

(v). Static and dynamic systems.

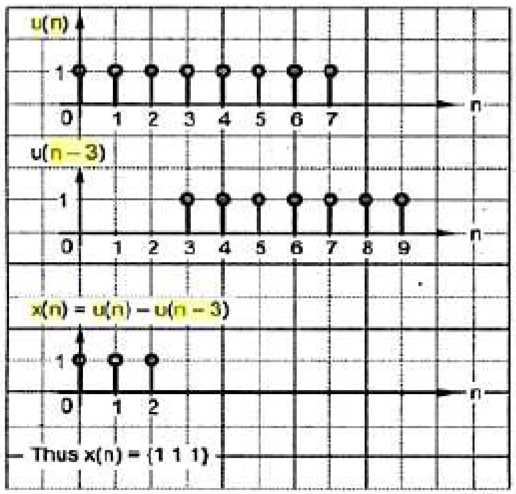
(vi). Invertible and non invertible systems.

23.Verify whether the system described by the equation $y(t) = x(t)^2$ is linear and time invariant. [CO1-L2-May/June 2010]

The system is linear since output is direct function of input.

The system is time variant since time parameter is squared in the given equation.

24. Draw the signal x(n) = u(n) – u(n-3) [CO1-L1-May/June 2011]



25. Check whether the following system is static/dynamic and casual/non casual

y(n) = x(2n) [CO1-H2-Nov/Dec 2009]

If n=1, y(1) = x(2). This means system requires memory. Hence it is dynamic system.

Since y(1) = x(2), the present output depends upon future input. Hence the system is non casual.

PART – B

1.Check for linearity, time invariance, causality and stability for y(n)=x(n^2)[CO1-H2-Nov/Dec 2010]

Solution:

Test for linearity:

For an input x_1(n) and corresponding output y_1(n)

y_1(n)=$x_1(n^2)$

$ll^y y_2$(n)=$x_2(n^2)$

ay_1(n)+by_2(n)=a$x_1(n^2)$+b$x_2(n^2)$

ay_1(n)+by_2(n)= ay_1(n)+by_2(n)

Therefore Linear system

Test for time invariance:

Y(n)=T[x(n)]

=x(n^2)

Output due to delayed input by k unit,

Y(n, k)=T[x(n,-k)]

=x($n^2 - k$) → (1)

Delayed output is,

Y(n-k)=$x(n-k)^2$ (2) →

(1) ≠ (2)

Therefore System is time variant

Test for causality:

Y(n)=x(n^2)

Y(0)=x(0)

Y(-1)=x(1)

Y(1)=x(1)

Y(2)=x(4)

The output depends on future values of input . Hence system is non-causal

Test for stability:

Gn: y(n)=x(n^2)

Let x(n)=δ(n) and

Y(n)=h(n)

h(n)=δ(n^2)

n=0 $\Longrightarrow$ h(0) =δ (0)

=1

n=1 $\Longrightarrow$ h(1)= δ(1)

=0

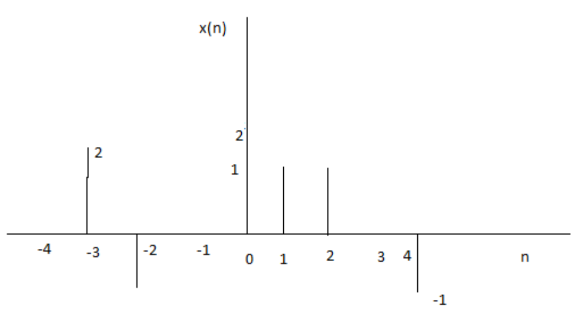
n=-1 $\Longrightarrow$ h(-1)=δ(1)

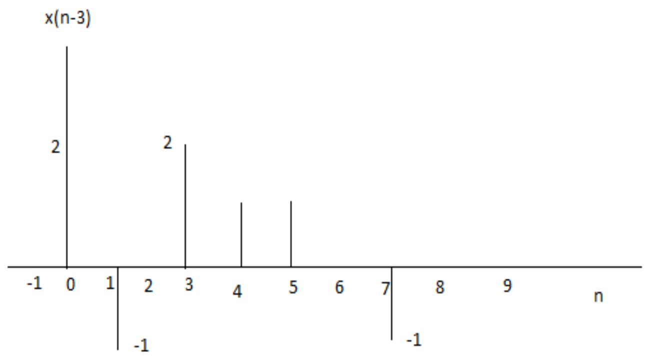
= 0

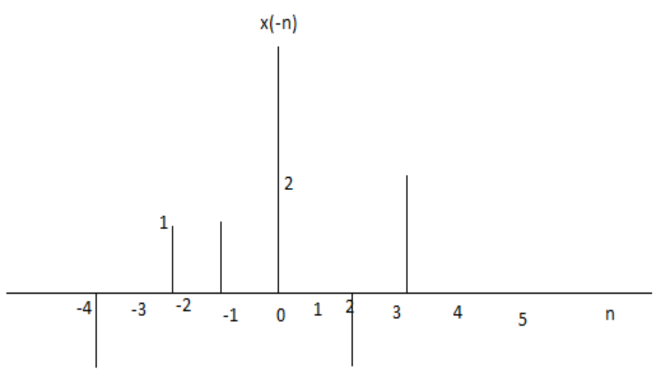
$\sum_{n=-\infty}^{\infty} |h(n)|<\infty$

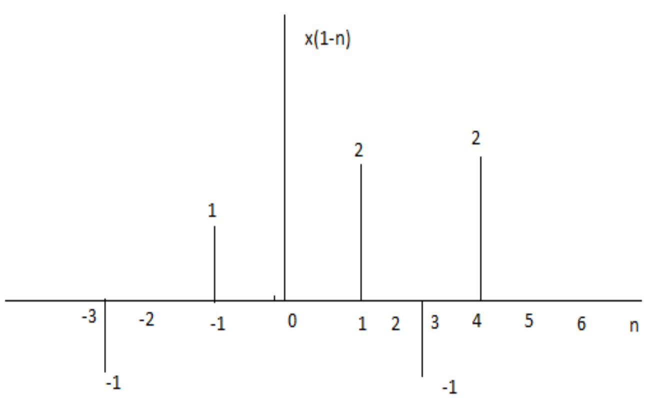
Therefore system is stable

2.x(n)={0,2,-1,0,2,1,1,0-1} what is x(n-3) x(n-1) ? [CO1-L1-Nov/Dec 20111]




x(n-3)
2
2
-1
0
1
2
3
4
5
6
7
8
9
n
-1
-1


x(-n)
2
1
-4
-3
-2
-1
0
1
2
3
4
5
n


x(1-n)
2
2
1
-3
-2
-1
0
1
2
3
4
5
6
n
-1
-1

3. Explain the Classifications of Continuous time signals with Examples. [CO1-L2-Nov/Dec 2014]

A signal is a quantitative description of a physical phenomenon, event or process. More precisely, a signal is a function, usually of one variable in time. However, in general, signals can be functions of more than one variable, e.g., image signals. Signals are functions of one or more variables.

Classifications of Continuous time signals:

Periodic and non-periodic Signals

A periodic function is one which has been repeating an exact pattern for an infinite period of time and will continue to repeat that exact pattern for aninfinite time. That is, a periodic function x(t) is one for which

$x(t) = x(t+nT)$

for any integer value of n, where $T > 0$ is the period of the function and $-\infty < t < \infty$. The signal repeats itself every T sec. Of course, it also repeats every 2 T,3T and nT. Therefore, 2T, 3T and nT are all periods of the function because the function repeats over any of those intervals. The minimum positive interval over which a function repeats itself is called the fundamental period T0.T0 is the smallest value that satisfies the condition $x(t) = x(t+T0)$. The fundamental frequency f 0 of a periodic function is the reciprocal of the fundamental period $f0=1/T0$. It is measured in Hertz and is the number of cycles (periods) per second.The fundamental angular frequency ω0 measured in radians per second is $\omega 0=2\pi T0= 2\pi f0$. A signal that does not satisfy the condition in (2.1) is said to be a periodic or non-periodic.

□□Deterministic and Random Signals

A deterministic signal is one which can be completely represented by Mathematical equation at any time.In a deterministic signal there is no uncertainty with respect to its value at any time.

Eg: $x(t)=\cos\omega t$

$x(n)=2\pi fn$

A random signal is one which cannot be represented by any mathematical equation. Eg: Noise generated in electronic components, transmission channels, cables etc.

Energy and Power Signals:

The signal x(t) is said to be power signal, if and only if the normalized average power p is finite and non-zero. ie. $0<p<\infty$

A signal x(t) is said to be energy signal if and only if the total normalized energy is finite and non-zero. ie. $0<E<\infty$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$P = \lim_{T\_-\infty} 1/T \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

Even and Odd Signal

A continuous – time signal x (t) is said to be symmetric or even if it satisfies the condition x (-t) = x (t) for all 't'

Ex: x (t) = A cos t

A continuous – time signal x (t) is said to be anti – symmetric (or) odd signal if it satisfies the condition x (-t) = -x (t) for all't'

Ex: x (t) = A sin t

Sinusoidal signal:

A continuous time sinusoidal signal is given by, $x(t) = ACos(\Omega 0T + \alpha)$

Where, A – amplitude _- phase angle in radians

Exponential signal:

It is exponentially growing or decaying signal.

Mathematical representation for CT exponential signal is, $x(t) = Ce^{at}$, where C, a Є C

4. Find whether the following signals are periodic or not. [CO1-H1-May/June 2013]

(i) x (t) = 2 cos (10 t +1) – sin (4t – 1)

(ii)x (t) = cos 60 πt + sin 50 πt

(iii) x (t) =2u (t) +2 sin 2 t

(iv) x (t) = 3 cos 4 t + 2 sin 2 πt

(v) x (t) = u (t) – 1/2

(I) Given x_1 (t) = 2 cos (10 t + 1) – sin (4t -1)

Time period of 2 cos (10t +1) is $T_1 = \frac{2\pi}{10} = \frac{\pi}{5}\text{sec}$

Time period of sin (4t -1) is $T_2 = \frac{2\pi}{4} = \frac{\pi}{2}\text{sec}$

Ratio of two periods is $\frac{T_1}{T_2} = \frac{\frac{\pi}{5}}{\frac{\pi}{2}} = \frac{2}{5}$

Is a rational number. There fore the sum of two signals are periodic and the period is given by

$T = 2T_2 = 5T_1 = \pi$ sec

(ii) Given x_2 (t) = cos 60πt + sin 50πt

Period of cos 60πt is $T_1 = \frac{2\pi}{60\pi} = \frac{1}{30}$

Period of sin 50 πt is $T_2 = \frac{2\pi}{50} = \frac{1}{25}$

The ratio $\frac{T_1}{T_2} = \frac{\frac{1}{30}}{\frac{1}{25}} = \frac{5}{6}$

Is a rational number hence x_2 (t) is a periodic signal.

The time period = $6T_1 = 5T_2 = \frac{1}{5}\text{sec}$

(iii) x (t) = 2 u (t) + 2 sin 2t

Period of the signal 2 sin 2t is $T_1 = \frac{2\pi}{2} = \pi$

The signal 2 u (t) is aperiodic

There fore the signal 2 u (t) + 2 sin 2t is an aperiodic

(iv) x (t) = 3 cos 4 t – 2 sin πt

The period of signal 3 cos 4 t is $T_1 = \frac{2\pi}{4} = \frac{\pi}{2}$

The period of signal 2 sin πt is $T_2 = \frac{2\pi}{\pi} = 2$

The ratio of time period $\frac{T_1}{T_2} = \frac{\pi/2}{2} = \frac{\pi}{4}$

Is not a rational number

Therefore the signal is not periodic

(v) x (t) = u (t) – ½

This signal is not periodic

5. Find the even and odd components of the following signals. [CO1-H1-May/June 2012]

(i) x (t) = cost + sin t + cos t sin t

(ii) x (n) = {-2, 1, 2, -1, 3}

(i) Given x (t) = cos t + sin t + cost sin t

x (-t) = cos (-t) +sin (-t) + cos (-t) sin (-t)

x (-t) = cos t – sin t – cost sin t

$x_e(t) = \frac{1}{2}$ [x (t) + x (-t)]

$x_e(t) = \frac{1}{2}$ [(cost + sin t+ cost sin t) + (cost –sin t – cost . sin t)]

$x_e(t) = \frac{1}{2}$ (2 cost) = cost

$x_e(t)$ = cost

$x_0(t) = \frac{1}{2}$ [x (t) – x (-t)]

$x_0(t) = \frac{1}{2}$ [cost + sin t +cos t sin t] + (cost + sin t –cost . sin t)]

$x_0(t) = \frac{1}{2}$[2 sin t + 2 sin t cost]

$x_0(t)$ = sin t + cost sin t

(ii) Given x (n) = {-2, 1, 2, -1, 3}

n = -2, -1 0 1 2

$x_e(n) = \frac{1}{2}$ [x (n) + x (-n)]

At⊥ n = 0

$X_e(0) = \frac{1}{2}$ [x (0) + x (-0)]

$X_e(0) = \frac{1}{2}$ [2+2]

$X_e(0)$ =2

At n =1

$X_e(1) = \frac{1}{2}$[x (1) +x (-1)]

$$= \frac{1}{2}\ [-1+1]$$

$$x_e\ (1) = 0$$

At n =2

$$X_e\ (2) = \frac{1}{2}[x\ (2) + x\ (-2)]$$

$$= \frac{1}{2}\ [3\ -2]$$

$$x_e\ (2) = 0.5$$

$$\Rightarrow x_e\ (n) = \{\ 0.5,\ 0,\ 2,\ 0,\ 0.5\}$$

$$x_0\ (n) = \frac{1}{2}\ [x\ (n) - x\ (-n)]$$

At n =0

$$X_0\ (0) = \frac{1}{2}\ [x\ (0) - x\ (-0)]$$

$$X_0\ (0) = \frac{1}{2}(2 - 2) = 0$$

$$X_0\ (0) = 0$$

At n =1

$$X_0\ (1) = \frac{1}{2}\ [x\ (1) - x\ (-1)]$$

$$X_0\ (1) = \frac{1}{2}\ [-1\ -1]$$

$$X_0\ (1) = -1$$

At n =2

$$X_0\ (2) = \frac{1}{2}\ [x\ (2) - x\ (-2)]$$

$$X_0(2) = \frac{1}{2}[3+2]$$

$$X_0(2) = \frac{5}{2}$$

$\Rightarrow x_0(n) = \{-2.5, 1, 0, -1, 2.5$

6. Find which of the following signals are energy signals, power signals, neither energy nor power signals. [CO1-H1-Nov/Dec 2011]

(i) $x_1(t) = e^{-3t} u(t)$

(ii) $x_2(t) = e^{j(2t + \frac{T}{4})}$

(iii) $x_3(t) = \cos t$

i) Given $x(t) = e^{-3t} u(t)$

The energy of the signal

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x_1(t)|^2 \, dt$$

$$E = \lim_{T\to\infty} \int_{-T}^{T} (e^{-3t})^2 \, dt$$

$$E = \lim_{T\to\infty} \int_{0}^{T} e^{-6t} \, dt$$

$$E = \lim_{T\to\infty} \left[\frac{[e^{-6t}]_0^T}{-6} \right]$$

$$E = \frac{[e^{-6t}]_0^\infty}{-6}$$

$$E = \frac{0-1}{-6}$$

$$E = \frac{1}{6}$$

The power of signal

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_1(t)|^2 \, dt$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} e^{-6t} \, dt$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \frac{[e^{-6t}]_0^T}{-6}$$

$$P = \frac{1}{2(\infty)} \; \frac{[e^{-6t}]_0^T}{-6}$$

$$P = 0$$

The energy of the signal is finite, and the power is 0. There fore the signal x_1 (t) is an energy signal.

(ii) X_2 (t) = $e^{j\,(2t+\pi/u)}$

The energy of the signal

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x_1(t)|^2 \, dt$$

$$E = \lim_{T\to\infty} \int_{-T}^{T} \left| e^{j\,(2t + \frac{\pi}{u})} \right|^2 \, dt$$

$$E = \lim_{T\to\infty} [2T] = \infty$$

The power signal $\left(\therefore \left| e^{j\,(2t+\frac{\pi}{u})} \right| = 1 \right)$

$$p = \lim_{T\to\infty} [\frac{1}{2T}] \int_{-T}^{T} \left| e^{j\,(2t+\frac{\pi}{u})} \right|^2 \, dt$$

$$p = \lim_{T\to\infty} \frac{1}{2T} [2T] = 1$$

The power of the signal is finite and the energy of the signal is infinite. Therefore $x_2(t)$ is a power signal.

(iii) x_3 (t) = cost

$$E = \lim_{T\to\infty} \int_{-T}^{T} \cos^2 t \, dt$$

$$E = \frac{1}{2} \lim_{T\to\infty} \int_{-T}^{T} (1 + \cos 2t) \, dt$$

$$E = \frac{1}{2} \lim_{T\to\infty} \int_{-T}^{T} dt + \lim_{T\to\infty} \int_{-T}^{T} \cos t2 \, dt$$

$$E = \frac{1}{2} [\infty + 0]$$

E = ∞

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \cos^2 t \, dt$$

$$P = \lim_{T\to\infty} \frac{1}{4T} \int_{-T}^{T} (1 + \cos 2 \; t) dt$$

$$P = \lim_{T\to\infty} \left[\frac{1[2T]}{4T} + 0 \right]$$

$$P = \lim_{T\to\infty}\left(\frac{1}{2}\right)$$

P = ½

The signal energy is infinite and power of signal is finite. Therefore $x_3(t)$ is power signal.

7. Check whether the following signals are energy or power signal. [CO1-H2-May/June 2014]

(i) x_1 (n) = $\left(\frac{1}{2}\right)^n u(n)$

(ii) x_2 (n) = $e^{j\left(\frac{\pi}{2}n+\frac{\pi}{8}\right)}$

(iii) x_3 (n) = cos $\left(\frac{\pi}{4}n\right)$

(i) x_1(n) = $\left(\frac{1}{3}\right)^n u(n)$

$$E = \lim_{N\to\infty}\sum_{n=-N}^{N}\left|x(n)\right|^2$$

$$E = \lim_{N\to\infty}\sum_{n=-N}^{N}\left|\left(\frac{1}{3}\right)^n\right|^2 u(n)$$

$$E = \lim_{N\to\infty}\sum_{n=0}^{N}\left(\frac{1}{9}\right)^n$$

$$E = \sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n$$

$$E = \frac{1}{1-1/9}$$

E = 9/8

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{9}\right)^n$$

$$P = \frac{1}{2(\infty)+1} \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n$$

$$P = \frac{9/8}{\infty}$$

P = 0

The energy is finite and power is zero. Therefore x_1 (n) are energy signal.

(ii) $x_2(n) = e^{j\left(\frac{T}{2}n+\frac{T}{8}\right)}$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} \left| e^{j\left(\frac{T}{2}n+\frac{T}{8}\right)} \right|^2$$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} 1$$

$$E = \sum_{n=-\infty}^{\infty} (1)$$

E = ∞

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| e^{j\left(\frac{T}{2}n+\frac{T}{8}\right)} \right|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} (2N+1)$$

P = 1

The energy is in finite and power is finite. Therefore x_2 (n) are power signal.

(iii) $x_3(n) = \text{Cos}(\frac{\pi}{4}n)$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} \cos^2\left(\frac{\pi}{4}n\right)$$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} \frac{1+\cos\left(\frac{\pi}{2}n\right)}{2}$$

$$E = \frac{1}{2}\lim_{N\to\infty} \sum_{n=-N}^{N} \left(1+\cos\frac{\pi}{2}n\right)$$

$$E = \frac{1}{2}\lim_{N\to\infty}(2N+1)$$

$E = \infty$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \cos^2\left(\frac{\pi}{4}n\right)$$

$$P = \frac{1}{2}\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\cos \ \frac{\pi}{2}n+1\right)$$

$$P = \frac{1}{2}\lim_{N\to\infty} \frac{1}{2N+1}[2N+1+0]$$

$P = ½$

The energy is in finite and power is finite. Therefore x_3 (n) are power signal.

8. Explain the classifications of System with an Examples. [CO1-L2-Nov/Dec 2015]

A system is a set of elements or functional block that are connected together and produces an output in response to an input signal.

Eg: An audio amplifier, attenuator, TV set etc.

CT systems are classified according to their characteristics as follows

(i). Linear and Non-Linear systems

(ii). Time invariant and Time varying systems.

(iii). Causal and Non causal systems.

(iv). Stable and unstable systems.

(v). Static and dynamic systems.

Linear and Non-Linear systems

A system is said to be linear if superposition theorem applies to that system. If it does not satisfy the superposition theorem, then it is said to be a nonlinear system.

The continuous system is linear if,

$$F[a1x1(t) + a2x2(t)] = a1y1(t) + a2y2(t)$$

The discrete system is linear if,

$$F[a1x1(n) + a2x2(n)] = a1y1(n) + a2y2(n)$$

□Otherwise the system is non linear.

Causal and non-Causal systems.

A system is said to be a causal if its output at anytime depends upon present and past inputs only.

A system is said to be non-causal system if its output depends upon future inputs also.

Eg. for causal system.

$$Y(t) = x(t) + x(t - 1)$$

$$Y(n) = x(n) + x(n - 3)$$

Eg. For non causal system,

$$Y(t) = x(t+3) + x2(t)$$

$$Y(n) = x(2n)$$

Stable And Unstable Systems.

When the system produces bounded output for bounded input, then the system is called bounded input, bounded output stable.

A system which doesnot satisfy the above condition is called a unstable system.

Time Invariant And Time Varying Systems.

A system is time invariant if the time shift in the input signal results in corresponding time shift in the output. A system which does not satisfy the above condition is time variant system.

Examples of time-invariant systems:

- The RC circuit considered earlier provided the values of R or C are constant.

$y(n) = x(n-1)$

Examples of time-varying systems:

- The RC circuit considered earlier if the values of R or C change over time.

$y(t) = x(2t)$ since

$x(t) = x(2t)$ but $x(t-t_0) = x(2t-t_0)$

Static And Dynamic System.

A system is said to be static or memoryless if its output depends upon the present input only. The system is said to be dynamic with memory if its output depends upon the present and past input values.

Examples of memory less systems:

$y(t) = Rx(t)$ or $y(n) = (2x(n) - x^2(n))^2$

Examples of systems with memory:

$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau$ or $y(n) = X(n-1)$

9. Determine whether the following systems are time invariant or not. [CO1-L1-May/June 2012]

i) Y(t) = tx(t)

ii) Y(n) = x(2n)

Solution:

i) $Y(t) = tx(t)$

$Y(t) = T[x(t)] = tx(t)$

The output due to delayed input is,

$Y(t,T) = T[x(t - T)] = tx(t - t)$

If the output is delayed by T, we get

$Y(t - T) = (t - T)\, x(t - T)$

The system does not satisfy the condition, $y(t,T) = y(t - T)$.

Then the system is time invariant

ii) $Y(n) = x(2n)$

$Y(n) = x(2n)$

$Y(n) = T[x(n)] = x(2n)$

If the input is delayed by K units of time then the output is,

$Y(n,k) = T[x(n-k)] = x(2n-k)$

The output delayed by k units of time is,

$Y(n-k) = x[2(n-k)]$

Therefore, y(n,k) is not equal to y(n-k). Then the system is time variant.

8. Check whether the following system are linear on not [CO1-H1-May/June 2015]

(i) y(n) = A x(n) + B

(ii) y(n) = n x(n)

(iii) $y(n) = 2x(n) + \frac{1}{x(n-1)}$

(iv) $y(t) = e^{x(t)}$

(v) $y(t) = x^2(t)$

(vi) $y(t) = t^2(t)$

(i)

given

$$\begin{aligned} y(n) &= Ax(n) + B \\ y_1(n) &= Ax_1(n) + B \rightarrow (1) \\ y_2(n) &= Ax_2(n) + B \rightarrow (2) \\ y_3(n) &= Ax_3(n) + B \rightarrow (3) \end{aligned}$$

Adding equation (1) & (2)

$$y_1(n) + y_2(n) = A\left[x_1(n) + x_2(n)\right] + 2B \rightarrow (4)$$

equation (4) $\neq$ equation (3)

Then, the system is non-linear

(ii) given

$$\begin{aligned} y(n) &= nx(n) \\ y_1(n) &= nx_1(n) \rightarrow (1) \\ y_2(n) &= nx_2(n) \rightarrow (2) \\ y_3(n) &= nx_3(n) \rightarrow (3) \end{aligned}$$

adding (1) & (2)

$$y_1(n) + y_2(n) = n\left[x_1(n) + x_2(n)\right] \rightarrow (4)$$

equation (3) = equation (4)

Them the system is linear

(iii)

$$y(n) = 2x(n) + \frac{1}{x(n-1)}$$

given

$$y_1(n) = 2x_1(n) + \frac{1}{x_1(n-1)} \rightarrow (1)$$

$$y_2(n) = 2x_2(n) + \frac{1}{x_2(n-1)} \rightarrow (2)$$

$$y_3(n) = 2x_3(n) + \frac{1}{x_3(n-1)} \rightarrow (3)$$

adding (1) & (2)

$$y_1(n) + y_2(n) = 2[x_1(n) + x_2(n)] + \frac{1}{x_1(n-1)} + \frac{1}{x_2(n-1)} \rightarrow (4)$$ Equation (4) $\neq$ equation (3)

Then the system is non-linear

(iv)

y(t) = $e^{x(t)}$

$$y(t) = e^{x(t)}$$
$$y_1(t) = e^{x_1(t)} \rightarrow (1)$$
$$y_2(t) = e^{x_2(t)} \rightarrow (2)$$
$$y_3(t) = e^{x_3(t)} \rightarrow (3)$$

adding (1) & (2) $\quad y_1(t) + y_2(t) = e^{x_1(t)} + e^{x_2(t)} \rightarrow (4)$

(3) become

$$y_1(t) + y_2(t) = e^{x_1(t)+x_2(t)} \rightarrow (5)$$

equation (4) $\neq$ equation (5)

Then it is non-linear

(v)

$$y(t) = x^2(t)$$
$$y_1(t) = x_1^2(t) \rightarrow (1)$$
$$y_2(t) = x_2^2(t) \rightarrow (2)$$
$$y_3(t) = x_3^2(t) \rightarrow (3)$$

adding (1) & (2)

$$y_1(t) + y_2(t) = x_1^2(t) + x_2^2(t) \rightarrow (4)$$

(3) becomes

$$y_1(t) + y_2(t) = \left[x_1(t) + x_2(t)\right]^2 \rightarrow (5)$$

equation (4) $\neq$ equation (5)

Then it is non – linear

(vi)

$$y(t) = t^2 x(t)$$
$$y_1(t) = t^2 x_1(t) \rightarrow (1)$$
$$y_2(t) = t^2 x_2(t) \rightarrow (2)$$
$$y_3(t) = t^2 x_3(t) \rightarrow (3)$$

adding (1) & (2)

$$y_1(t) + y_2(t) = t^2\left(x_1(t) + x_2(t)\right) \rightarrow (4)$$

(3) becomes

$$y_3(t) = t^2\left[x_1(t) + x_2(t)\right] \rightarrow (5)$$

equation (4) = equation (5)

Then, the system is linear

UNIT – II

Analysis of Continuous Time Signals

Part – A

1. Write down the exponential form of the Fourier series representation of a Periodic signal? [CO2-L1-May/June 2012]

$x(t) = a_k e^{jkwt}$

Where$a_k = \frac{1}{T} x(t) e^{-jkwt}$.

here the integration is taken from 0 to T.

The set of coefficients { ak} are often called the Fourier series coefficients or spectral coefficients. The coefficient ao is the dc or constant component of x(t).

2. Write short notes on dirichlets conditions for fourier series. [CO2-L1-May/June 2014]

x(t) must be absolutely integrable

The function x(t) should be single valued within the interval T.

The function x(t) should have finite number of discontinuities in any Finite interval of time T.

The function x(t) should have finite number of maxima &minima in the interval T.

3. State Time shifting property in relation to Fourier series. [CO2-L1-Nov/Dec 2012]

$x(t - t_0) = a_k e^{-jkwt}$

Time shifting property states that; when a periodic signal is shifted in time, the magnitudes of its Fourier series coefficients remain unaltered.

4. State convolution property in relation to Fourier transform. [CO2-L1-May/June 2011]

Y(t)= x(t)*h(t) $\xrightarrow{FT}$ Y(jω)= H(jω)X(jω)

Convolution property states that convolution in time domain corresponds to multiplication in the frequency domain.

5. State parseval's relation for continuous time Fourier transform. [CO2-L1-May/June 2012]

If x(t) and X(jω) are a fourier transform pair then

$$x(t)^2dt = \frac{1}{2\pi j}|X(j\omega|^2 d\omega$$

6. What is the use of Laplace transform? [CO2-L1-Nov/Dec 2009]

Laplace transform is another mathematical tool used for analysis of signals and systems. Laplace transform is used for analysis of unstable systems.

7. State the time shifting property for Laplace transforms. [CO2-L1-May/June 2012]

Let x(t) X(S) be a laplace transform pair.

If x(t) is delayed by time t_0,then its laplace transform is multiplied by e^{-st_0}

$$L.T\ x(t-t_0) = e^{-st_0}X(S)$$

8. State initial value theorem and final value theorem for Laplace transform.

[CO2-L1-May/June 2012]

If L[x(t)]=X(s), then initial value theorem states that x(0)=$\lim_{s--\infty} SX(S)$

If L[x(t)]=X(s), then final value theorem states that x(∞)=$\lim_{s--0} SX(S)$

9. What are the difference between Fourier series and Fourier transform? [CO2-L2-May/June 2010]

S.NO	Fourier Series	Fourier Transform
1	Fourier series is calculated for periodic signals.	Fourier Transform is calculated for non-periodic as well as periodic signals.
2	Expands the signals in time domain.	Represents the signal in frequency domain
3	Three types of Fourier series such as trigonometric, Polar and Complex Exponential	Fourier transform has no such types.

10. Find the Laplace transform of the signal $x(t) = e^{-at}u(t)$[CO2-L1-May/June 2013]

We know that,

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}\ dt = \int_{-\infty}^{\infty} e^{-at}\,\mathrm{u}(t)e^{-j\Omega t}\ dt = \int_{0}^{\infty} e^{-at}\, e^{-j\Omega t}\ dt$$

$$= \int_0^{\infty} e^{-(a+j\Omega)t}\ dt = [\frac{e^{-\infty}}{-(a+j\Omega)} - \frac{e^{0}}{-(a+j\Omega)}] = \frac{1}{a+j\Omega}$$

$$X(\Omega) = \frac{1}{a+j\Omega}$$

11. Define the Fourier transform pair for continuous time signal. (Or) Give synthesis and analysis equations of CT Fourier Transform. [CO2-L1-Nov/Dec 2012]

Fourier Transform: $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$

Inverse Fourier Transform: $\boldsymbol{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega}$

12. Find inverse Fourier transform of X(ω)=2πδ (ω). [CO2-L1-May/June 2015]

Inverse Fourier Transform: $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\omega)e^{j\omega t}\,d\omega$$

= 1 since $\delta(\omega)$= 1 for $\omega = 0$

0 for $\omega \neq 0$

13. State the time scaling property of Laplace Transform. [CO2-L1-May/June 2013]

It states that If LT[x(t)] =X(S) then,LT[x(at)] = $\frac{1}{|a|}X(\frac{S}{a})$

14.Define region of convergence of the Laplace Transform. [CO2-L1-Nov/Dec 2012]

For a given signal the range of values of S, for which the integral $\int_{-\infty}^{\infty}|x(t)|$ dt converges is called the region of convergence**. i.e** $\boldsymbol{\int_{-\infty}^{\infty}\left|x(t)e^{-j\Omega t}\right|dt < \infty}$

15. State the relationship between fourier transform and laplace transform. [CO2-L1-May/June 2015]

The Laplace Transform is given by $X(S) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$

The Fourier Transform is given by $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$

The Laplace Transform is same as Fourier Transform when $S = j\omega$

16. State any two properties of ROC of laplace transform X(s) of a signal x(t). [CO2-L1-Nov/Dec 2014]

Properties of ROC:

No poles lie in ROC.

ROC of the causal signal is right hand sided. It is of the form Re(s)>a.

ROC of the non causal signal is left hand sided. It is of the form Re(s) < a.

The system is stable if its ROC includes j ω axis of s-plane.

17. What is the condition to be satisfied for the existence of Fourier transform for CT periodic signals? [CO2-L2-Nov/Dec 2011]

The function x(t) should be absolutely integrable for the existence of Fourier transform. i.e$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$

18. Determine Fourier series coefficients for signal cosπt [CO2-L1-May/June 2012]

$$Cos\pi t = \frac{e^{j\pi t}+e^{-j\pi t}}{2}$$

Fourier series is given as,

$$x(t) = \sum_{k=-\infty}^{\infty} X(K)e^{jk\pi t}$$

19. State parseval's theorem for continuous time a periodic signal. [CO2-L1]

Let $x_1(t)$ and $x_2(t)$ be signals with Fourier transform $x_1(j\Omega)$ and $x_2(j\Omega)$ respectively.

Then we have

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\times(j\Omega)|^2\, d\Omega$$

20.State time shifting property and Frequency shifting property of Fourier transform. [CO2-L1-May/June 2015]

Time Shifting Property

If F[x(t)] = x(jΩ) then F[x(t-t_0)] = x(jΩ)$e^{-j\Omega to}$

Frequency shifting property

If F [x(t)] = x (jΩ) then

F [x(t) $e^{j\Omega ot}$] = x[Ω - Ω_0)]

21. State time differentiation and Time integration property of Fourier transform

Time Differentiation[CO2-L1]

If F[x (t)] = x (jΩ) = x (jΩ) then

$$F\left[\int_{-\infty}^{t} x(\tau)d\tau\right]$$
$$= \frac{1}{j\Omega}\times(j\Omega)+\pi X(0).S(\Omega)$$

Time integration

If F[x (t)] = x (jΩ) = x (jΩ) then

$$F\left[\int_{-\infty}^{t} x(\tau)d\tau\right]$$
$$= \frac{1}{j\Omega}\times(j\Omega)+\pi X(0).S(\Omega)$$

22. Define the Laplace and Inverse Laplace transform. [CO2-L1-Nov/Dec 2014]

The laplace transform of a signal x(t) is defined as

$$X(s)=\int_{-\infty}^{\infty} x(t)e^{-st}dt$$

where s is complex frequency denoted by s = σ+jΩ

$$x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds$$

23. What is the condition for convergence of the Laplace transform? [CO2-L2-May/June 2010]

The necessary condition for convergence of the Laplace transform is absolutely integral of x(t) $e^{\sigma t}$. That is, X(s) exist if

$$\int_{-\infty}^{\infty}\left|x(t)e^{-\sigma t}\right|<\infty$$

24. Obtain the Fourier transform of $x(t) = e^{-at}u(t), a > 0$[CO2-L1]

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

$$= \int_{-\infty}^{\infty} e^{-at} e^{-j2\pi ft} dt$$

$$= \frac{1}{a+j2\pi f}$$

25. State any two properties of continuous time Fourier transform. [CO2-L1]

Convolution In time domain

It states that,

$$x(t)*y(t) \overset{FT}{\leftrightarrow} X(j\Omega)Y(j\Omega)$$

Frequency shifting

It states that,

$$x(t)\, e^{j\Omega_0 t} \overset{FT}{\leftrightarrow} X(j\Omega\text{-}j\Omega_0)$$

Part – B

1.Find the inverse Laplace transform of F(s) =$\frac{S-2}{S(S+1)^3}$ [CO2-H1-May/June 2013]

Solution:

F(s) =$\frac{S-2}{S(S+1)^3}$

S-2=$\frac{A}{S} + \frac{B}{S+1} + \frac{C}{(S+1)^2} + \frac{D}{(S+1)^3}$

S-2=$A(S+1)^3 + B(S)(S+1)^2 + C(S)(S+1) = DS$

Put s=0 Put s=1

-2=A -3=-d

D=3

Equate the coffe of s^3

0=A+B

0=-2+B

B=2

Equate the coffe of s^2

0=A($S^3+1+3S^2+3S)+B[S(S^2+1+2S)]+C$

0=A($S^3+3S^2+3S+1)+B(S^3+S+2S^2)+C$

0=3A+2B+C

0=3(-2)+4+C

0=-6+4+C

C=2

X(S)=$\frac{-2}{S}+\frac{2}{S+1}+\frac{2}{(S+1)^2}$+$\frac{3}{((S+1)^3}$

X(t)= -2u(t)+2$e^{-t}u(t)+2te^{-t}$u(t)+$\frac{3t^2}{2!}e^{-t}u(t)$

2.State and derive the formula for Fourier series analysis of continuous – time periodic signals. [CO2-H1-May/June 2009]

A periodic signal is one which repeat it self periodically over - ∞ < t <∞. For example, a sinusoidal signal x (t) = A sin Ω_ot is a periodic signal with period T =$\frac{2\pi}{\Omega_o}$. Now let us consider a signal x (t) a sum of sine and cosine functions whose frequencies are integral multiple of Ω_o as shown below.

x (t) = $a_o + \sum_{n=1}^{k}[a_n\cos(n\Omega_o t) + b_n\sin(n\Omega_o t)]$

Where a_o, a_1 . . . . . . a_k and b_1, b_2 . . . . . . b_k are constant and Ω_0 is the fundamental frequency

x (t) = (t +T)

$$x(t+T) = a_0 + \sum_{n=1}^{k}[a_n \cos n\Omega_o(t+T) + b_n \sin n\Omega_o(t+T)]$$

$$x(t+T) = a_0 + \sum_{n=1}^{k}[a_n \cos(n\Omega_o t + 2n\pi) + b_n \sin(n\Omega_o t + 2\pi n)]$$

$$x(t+T) = a_0 + \sum_{n=1}^{k}[a_n \cos(n\Omega_o t) + b_n \sin(n\Omega_o t)]$$

$$x(t+T) = x(t)$$

This series of sine and cosine term is known as trigonometric Fourier series and can be written as.

$$X(t) = a_0 + \sum_{n=1}^{\infty}[a_n \cos(n\Omega_o t) + b_n \sin(n\Omega_o t)] \rightarrow (1)$$

Evaluation of Fourier coefficient

The constants a_0, a_1 . . . a_n and b_1, b_2. . . . ..b_n are called as Fourier coefficient.

To evaluate a_0 we shall integrate both sides of equation (1) over one period (t_0, t_0+T) of x (t) at an arbitrary time t_0.

Thus

$$\int_{t_0}^{t_0+T} x(t) = a_0 \int_{t_0}^{t_0+T} dt + a_1 \int_{t_0}^{t_0+T}\left[\sum_{n=1}^{\infty}[a_n \cos(n\Omega_o t) + b_n (\sin n\Omega_o t)]\right]$$

$$\int_{t_0}^{t_0+T} x(t)\,dt = a_0 T + \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_0+T} \cos(n\Omega_0 t)\,dt + \sum_{t_0}^{t_0+T} b_n \int_{t_0}^{t_0+T} \sin(n\Omega_0 t)\,dt \rightarrow (2)$$

Each of the integrals in the summation in equation (2) is zero since net areas of sinusoids over complete periods are zero to any non – zero integer n and any t_o.

Thus we obtain

$$\int_{t_0}^{t_0+T} x\left(t\right)dt = a_0 T \left(\begin{array}{l} \therefore \int_{t_0}^{t_0+T} \cos(n\Omega_0 t)\, dt = 0 \\ \therefore \int_{t_0}^{t_0+T} \sin(n\Omega_0 t)\, dt = 0 \end{array} \right.$$

Then

$$a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} x(t)\, dt$$

To find Fourier coefficient a_n, multiply equation (1) by cos ($n_1\ \Omega_0 t$) and integrate over one period. That is

$$\int_{t_0}^{t_0+T} x(t) \cos(m\Omega_0 t)\, dt = a_0 \ \therefore \int_{t_0}^{t_0+T} \cos(m\Omega_0 t)\, dt$$

$$+ \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_0+T} \cos(n\Omega_0 t) \cos(m\Omega_0 t)\, dt$$

$$+ \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_0+T} \sin(n\Omega_0 t) \cos(m\Omega_0 t)\, dt \ \rightarrow (3)$$

The first integral on the right – hand side of equation (3) is zero because we are integrating over are integer multiple of periods.

Then

$$\int_{t_0}^{t_0+T} x(t) \cos(m\Omega_0 t)\, dt = a_m \frac{T}{2}$$

Re arrange, we get

$$a_m = \frac{2}{T} \int_{t_0}^{t_0+T} x(t) \cos(m\Omega_0 t)\, dt$$

$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos(n\Omega_0 t)\,dt$ To find b_n multiply equation (1) by sin $(m\Omega_0 t)$ we obtain.

$$\int_{t_0}^{t_0+T} x(t)\sin(m\Omega_0 t)\,dt = \int_{t_0}^{t_0+T} a_0 \sin(m\Omega_0 t)\,dt$$

$$+\sum_{n=1}^{\infty} a_n \int_{t_0}^{t_0+T} \cos(n\Omega_0 t)\sin(m\Omega_0 t)\,dt$$

$$+\sum_{n=1}^{\infty} b_n \int_{t_0}^{t_0+T} \sin(n\Omega_0 t)\sin(m\Omega_0 t)\,dt$$

This time the only non zero integral is

$$b_m = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin(m\Omega_0 t)\,dt$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin(n\Omega_0 t)\,dt$$

Note: -

$$\int_{t_0}^{t_0+T} \cos(n\Omega_0 t)\ \cos(m\Omega_0 t)\,dt = \begin{cases} 0 & m \neq n \\ T/2 & m = n \neq 0 \end{cases}$$

$$\int_{t_0}^{t_0+T} \sin(n\Omega_0 t)\cos(m\,\Omega_0 t)\,dt = 0 \text{ for all } m, n$$

$$\int_{t_0}^{t_0+T} \sin(n\Omega_0 t)\sin(m\Omega_0 t)\,dt = \begin{cases} 0 & m \neq n \\ T/2 & m = n \neq 0 \end{cases}$$

3. Find the trigonometric Fourier series for the periodic x (t). [CO2-H1-Nov/Dec 2011]

For the given signal the period T = 4 for our convenience we choose one period of the signal from t = -1 to t =3 rather than from t =0 to t= 4 to reduce the number of integrals.

The fundamental frequency $\Omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$

And x (t) = $a_0 + \sum_{n=1}^{\infty}[a_n \cos\left(\frac{n\pi}{2}t\right) + b_n \sin\left(\frac{n\pi}{2}t\right)$

Given = x (t) = $\begin{cases} 1 \text{ for} -1 \le t \le 1 \\ -1 \text{ for } 1 < t \le 3 \end{cases}$

$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt = \frac{1}{4}\int_{-1}^{3} x(t)\,dt$

$a_0 = \frac{1}{4}\left[\int_{-1}^{1} dt + \int_{1}^{3}(-1)\,dt\right] = \frac{1}{4}[(1-(-1)] - (3-1)]$

$a_0 = \frac{1}{4}(2-2) = 0$

$a_0 = 0$

$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos(n\Omega_0 t)\,dt$

$a_n = \frac{1}{2}\int_{-1}^{3} x(t)\cos\left[\frac{n\pi}{2}t\right]dt$

$a_n = \frac{1}{2}\int_{-1}^{1}\cos\left(\frac{n\pi}{2}t\right)dt + \int_{1}^{3}(-1)\cos\left(\frac{n\pi}{2}t\right)dt$

$$a_n = \frac{1}{2}\left[\left(\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}t\right)\right)_{-1}^{1} - 1\left(\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}t\right)\right)_{1}^{3}\right]$$

$$a_n = \frac{1}{2}\left[\frac{8}{n\pi}\sin\left(\frac{n\pi}{2}\right)\right]$$

$$a_n = \frac{4}{n\pi}\sin\frac{n\pi}{2}$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin(n\Omega_0 t)\,dt$$

$$b_n = \frac{1}{2}\int_{-1}^{3} x(t)\sin\left(\frac{n\pi}{2}t\right)dt$$

$$b_n = \frac{1}{2}\left[\frac{-2}{n\pi}\left[\cos\left(\frac{n\pi}{2}t\right)\right]_{-1}^{1} + \frac{2}{n\pi}\left[\cos\left(\frac{n\pi}{2}t\right)\right]_{1}^{3}\right]$$

$$b_n = \frac{1}{2}\left[\frac{-2}{n\pi}\left[\cos\frac{n\pi}{2} - \cos\frac{n\pi}{2}\right] + \frac{2}{n\pi}\left(\cos\frac{3n\pi}{2} - \cos\frac{3n\pi}{2}\right)\right]$$

$$b_n = \frac{1}{2}\left[\frac{-2}{n\pi}[0] + \frac{2}{n\pi}[0]\right]$$

$$b_n = \frac{1}{2}[0]$$

$$b_n = 0$$

$$a_n = \frac{4}{n\pi}\sin\frac{n\pi}{2} = \begin{cases} 0 & n = \text{even} \\ \frac{4}{n\pi} & n = 1, 5, 9, 13 \\ \frac{-4}{n\pi} & n = 3, 7, 11, 15 \end{cases}$$

There fore

$$X(t) = \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{2} t$$

$$X(t) = \frac{4}{\pi} \cos\left(\frac{\pi}{2}t\right) - \frac{4}{3\pi} \cos\left(\frac{3\pi}{2}t\right) + \frac{4}{5\pi} \cos\left(\frac{5\pi}{2}t\right) \ldots\ldots$$

$$X(t) = \frac{4}{\pi}\left[\cos\left(\frac{\pi}{2}t\right) - \frac{1}{3}\cos\left(\frac{3\pi}{2}t\right) + \frac{1}{5}\cos\left(\frac{5\pi}{2}t\right)\ldots\ldots\right]$$

4. Obtain the inverse Laplace transform of x(s)=$\frac{1}{s^2+3s+2}$, ROC: -2<Re{s}<-1 [CO2-H1-Nov/Dec 2012]

Solution:

$$X(S) = \frac{1}{S^2+3S+2}$$

$$= \frac{1}{(S+1)(S+2)}$$

$$= \frac{1}{(S+1)(S+2)} = \frac{A}{S+1} + \frac{B}{S+2}$$

1=A(S+2)+B(S+1)

Put S=-1 Put S=-2

A=1 B=-1

$$X(S) = \frac{1}{S+1} - \frac{1}{S+2}$$

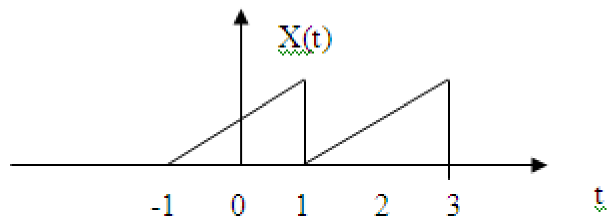
For ROC : -2<Re[s]<-1 Re[s]>-2

Re[s]<-1 $e^{-2t}u(t)$

-$e^{-t)}u(t)$

X(t)=$-e^{-t}u(-t) - e^{-2t}u(t)$

5. Find the trigonometric Fourier series for the periodic signal x (t) as shown in fig. [CO2-H1-May/June 2009]



From the figure we find that T =2 and $\Omega_0 = \frac{2\pi}{T} = \pi$. For our convenience we take the integration interval from t = -1 to t =1. During this interval x (t) = t

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(n\pi t) + b_n \sin(n\pi t)]$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt$$

$$a_0 = \frac{1}{2}\int_{-1}^{1} t.dt$$

$$a_0 = \frac{1}{2}\left[\frac{t^2}{2}\right]_{-1}^{1} = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{2}\right]$$

$$a_0 = 0$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos(n\Omega_0 t)\,dt$$

$$a_n = \frac{2}{2}\int_{-1}^{1}(t)\cos(n\Omega_0 t)\,dt$$

$$a_n = \frac{t}{n\pi}[\sin n\pi t]_{-1}^{1} + \frac{1}{n^2\pi^2}[\cos(n\pi t)]_{-1}^{1}$$

$$a_n = 0 + 0$$

$$a_n = 0$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin(n\Omega_0 t)\,dt$$

$$b_n = \frac{2}{2}\int_{-1}^{1} t\sin(n\pi t)\,dt$$

$$b_n = \frac{-t}{n\pi}\left[\cos(n\pi t)\right]_{-1}^{1} + \frac{1}{n^2\pi^2}\left[(\cos(n\pi t)\right]_{-1}^{1}$$

$$b_n = \frac{-2}{n\pi}\cos n\pi$$

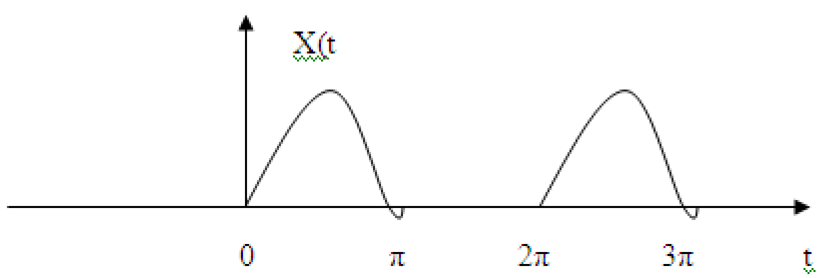
$$b_n = \frac{2}{\pi}\left[\frac{-1(-1)^n}{n}\right]$$

Substituting a_0, a_n and b_n values in general Fourier series we get.

$$x(t) = \sum_{n=1}^{\infty}\frac{2}{\pi}\left[\frac{-(-1)^n}{n}\right]\sin n\pi t$$

$$= \frac{2}{\pi}\left[\sin(\pi t)\ \frac{-1}{2}\sin(2\pi t) + \frac{1}{3}\sin(3\pi t) - \frac{1}{4}\sin(4\pi t) + \ldots\right]$$

6. Find cosine Fourier series of half rectified sine wave. [CO2-H1-May/June 2011]



The signal x (t) = $\begin{cases} A\ \sin\Omega_0\ t \text{ for } 0 \le t \le \pi \\ 0 \text{ for } \pi \le t \le 2\pi \end{cases}$

The time period of the wave form is given by

T = 2π and $\Omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\, dt$$

$$a_0 = \frac{1}{2\pi}\int_0^{\pi} A \sin t\, dt$$

$$a_0 = \frac{-A}{2\pi}\,[\cos t]_0^T$$

$$a_0 = \frac{-A}{2\pi}\,(\cos \pi - \cos 0)$$

$$a_0 = \frac{A}{T}$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t) \cos nt\, dt$$

$$a_n = \frac{2}{2\pi}\int_0^{T} A \sin t \cos nt\, dt$$

$$a_n = \frac{A}{\pi}\int_0^{\pi} \frac{[\sin (1+n)\, t + \sin (1-n)\, t]\, dt}{2}$$

$$a_n = \frac{A}{2\pi}\left[\left(-\frac{\cos (1+n)t}{(1+n)}\right)_0^{\pi} + \left(\frac{-\cos (1-n)t}{1-n}\right)_0^{\pi}\right]$$

$$a_n = \frac{A}{2\pi}\left[\frac{2}{1+n} + \frac{2}{1-n}\right] = \frac{2A}{\pi(1-n^2)} \text{ for n even}$$

$$a_n = \frac{2A}{\pi\,(1-n^2)} \text{ for n even}$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin nt\, dt$$

$$b_n = \frac{2}{2\pi}\int_0^{\pi} A \sin t \sin nt\, dt$$

$$b_n = \frac{A}{\pi}\int_0^{\pi} \frac{[\cos(1-n)t - \cos(1+n)t]}{2}\, dt$$

$$b_n = \frac{A}{2\pi}\left[\left(\frac{\sin(1-n)t}{1-n}\right)_0^{\pi} - \left(\frac{\sin(1+n)t}{1+n}\right)_0^{\pi}\right]$$

$$b_n = \frac{A}{2\pi}[0-0]$$

$$b_n = 0$$

For n =1

$$a_1 = \frac{2}{2\pi}\int_0^{\pi} A \sin t \cos t\, dt$$

$$a_1 = \frac{2A}{2\pi}\int_0^{\pi} \frac{\sin 2t}{2} dt$$

$$a_1 = \frac{-A}{2\pi}\left[\frac{\cos 2t}{2}\right]_0^{\pi}$$

$$a_1 = 0$$

For n =1

$$b_1 = \frac{2}{2\pi}\int_0^{\pi} A \sin t \sin t\, dt$$

$$= \frac{A}{\pi}\int_0^{\pi} \sin^2 t\, dt$$

$$= \frac{A}{\pi}\int_0^{\pi} \frac{(1-\cos 2t)}{2} dt$$

$$= \frac{A}{2\pi}\int_0^{\pi} dt - \int_0^{T} \cos 2t\, dt$$

$$= \frac{A}{2\pi}[\pi - 0]$$

$$b_1 = \frac{A}{2}$$

$$A_0 = a_0 = \frac{A}{\pi}M$$

$$A_1 = \sqrt{a_1^2 + b_1^2} = \frac{A}{2}$$

$$A_1 = \sqrt{a_n^2 + b_n^2} = \frac{2A}{\pi\ (1-n^2)}$$

$$\partial_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right) = \tan^{-1}\left(\frac{-0}{a_n}\right) = 0$$

$$x(t) = \frac{A}{\pi} + \frac{A}{2}\cos\left(nt - \frac{\pi}{2}\right) + \sum_{n=2}^{\infty} \frac{2A}{\pi\ (1-n^2)} \cos nt$$

7. Find the Laplace transform of the following signal. [CO2-H1-Nov/Dec2013]

(i) $x(t) = e^{-at}u(t)$

(ii) $x(t) = Sin\omega t u(t)$

(iii)$x(t) = Cos\omega t u(t)$

(i)

$$x(t) = e^{-at}u(t)$$

$$x(s)=\int_0^{\infty} x(t)e^{-st}dt$$

$$x(s) = \int_0^{\infty} e^{-at}u(t)e^{-st}dt$$

$$x(s) = \int_0^{\infty} e^{-(s+a)t}dt$$

$$x(s) = \frac{\left[e^{-(s+a)t}\right]_0^{\infty}}{-(s+a)}$$

$$x(s) = \frac{[0-1]}{-(s+a)}$$

$$\boxed{X(s) = \frac{1}{s+a}}$$

(ii)

$$x(t) = \sin\omega t\ u(t)$$

$$x(s)=\int_0^{\infty} x(t)e^{-st}dt$$

$$x(s) = \int_0^{\infty} \sin\omega t\ e^{-st}dt$$

$$x(s) = \int_0^{\infty} \frac{e^{j\omega t} - e^{-j\omega t}}{2j} e^{-st}dt$$

$$x(s) = \frac{1}{2j}\left[\int_0^{\infty} e^{j\omega t}e^{-st}dt - \int_0^{\infty} e^{-j\omega t}e^{-st}dt\right]$$

$$x(s) = \frac{1}{2j}\left[\int_0^{\infty} e^{-(s-j\omega)t}dt - \int_0^{\infty} e^{-(s+j\omega)t}dt\right]$$

$$x(s) = \frac{1}{2j}\left[\frac{\left[e^{-(s-j\omega)t}\right]_0^{\infty}}{-(s-\omega)} - \frac{\left[e^{-(s+j\omega)t}\right]_0^{\infty}}{-(s+\omega)}\right]$$

$$x(s) = \frac{1}{2j}\left[\left(\frac{0-1}{-(s-j\omega)}\right)-\left(\frac{0-1}{-(s+j\omega)}\right)\right]$$

$$x(s) = \frac{1}{2j}\left[\frac{1}{s-j\omega}-\frac{1}{s+j\omega}\right]$$

$$x(s) = \frac{1}{2j}\left[\frac{s+j\omega-(s-j\omega)}{(s^2+\omega^2)}\right]$$

$$x(s) = \frac{1}{2j}\frac{2j\omega}{(s^2+\omega^2)}$$

$$\boxed{x(s) = \frac{\omega}{s^2+\omega^2}}$$

(iii)

$$x(t) = \cos\omega t \; u(t)$$

$$x(s)=\int_0^\infty x(t)e^{-st}dt$$

$$x(s) = \int_0^\infty \cos\omega t \; e^{-st}dt$$

$$x(s) = \int_0^\infty \left(\frac{e^{j\omega t}-e^{-j\omega t}}{2}\right)e^{-st}dt$$

$$x(s) = \frac{1}{2}\left[\int_0^\infty e^{j\omega t}e^{-st}dt+\int_0^\infty e^{-j\omega t}e^{-st}dt\right]$$

$$x(s) = \frac{1}{2}\left[\int_0^\infty e^{-(s-j\omega)t}dt+\int_0^\infty e^{-(s+j\omega)t}dt\right]$$

$$x(s) = \frac{1}{2}\left[\frac{\left[e^{-(s-j\omega)t}\right]_0^\infty}{-(s-\omega)}+\frac{\left[e^{-(s+j\omega)t}\right]_0^\infty}{-(s+\omega)}\right]$$

$$x(s) = \frac{1}{2}\left[\frac{1}{s-j\omega}+\frac{1}{s+j\omega}\right]$$

$$x(s)=\frac{1}{2}\left[\frac{s+\omega+s-\omega}{\left(s^2+\omega^2\right)}\right]$$

$$x(s)=\frac{1}{2}\frac{2s}{\left(s^2+\omega^2\right)}$$

$$\boxed{x(s)=\frac{s}{s^2+\omega^2}}$$

8. Find Laplace transform of the following sequence. [CO2-H1-Nov/Dec2014]

(i) $x(t)=u(t)$

(ii) $x(t)=t^n$

(iii) $x(t)=e^{-at}sin\omega t$

(iv) $x(t)=e^{-at}Cos\omega t$

(i)

$$x(t)=1$$

$$x(s)=\int_0^\infty x(t)e^{-st}dt$$

$$x(s)=\int_0^\infty (1)e^{-st}dt$$

$$x(s)=\frac{\left[e^{-st}\right]_0^\infty}{-s}$$

$$x(s)=\frac{(0-1)}{-s}$$

$$\boxed{x(s)=\frac{1}{s}}$$

(ii)

$$x(t) = t^n$$

$$x(s) = \int_0^{\infty} x(t)e^{-st}dt$$

$$x(s) = \int_0^{\infty} t^n e^{-st}dt$$

$$x(s) = \left[\frac{t^n e^{-st}}{-s}\right]_0^{\infty} - \int_0^{\infty} \frac{e^{-st}}{-s} nt^{n-1}dt$$

$$x(s) = 0 + \frac{n}{s}\int_0^{\infty} e^{-st}t^{n-1}dt$$

$$x(s) = \frac{n}{s}\left[t^{n-1}\frac{e^{-st}}{-s}\right]_0^{\infty} + \frac{n}{s}\int_0^{\infty} \frac{e^{-st}}{-s}(n-1)t^{n-2}dt$$

$$x(s) = [0] + \ldots\ldots\ldots\ldots$$

Finally

$$\boxed{x(s) = \frac{n!}{s^{n+1}}}$$

(iii)

$$x(t) = e^{-at}\sin\omega t$$

$$L\left[e^{-at}\right] = \frac{1}{s+a}$$

$$L[\sin\omega t] = \frac{\omega}{s^2+\omega^2}$$

$$\boxed{L\left[e^{-at}\sin\omega t\right] = \frac{\omega}{(s+a)^2+\omega^2}}$$

(iv)

$$x(t) = e^{-at}\cos\omega t$$

$$L\left[e^{-at}\right] = \frac{1}{s+a}$$

$$L\left[\cos\omega t\right] = \frac{\omega}{s^2+\omega^2}$$

$$\boxed{L\left[e^{-at}\cos\omega t\right] = \frac{s+a}{(s+a)^2+\omega^2}}$$

9. Find the Fourier transform and sketch the magnitude and phase spectrum of $x(t) = e^{-at}$ u(t) [CO2-H1-May/Jun2014].

$$x(t) = e^{-at}u(t)$$

$$x(j\Omega) = \int_0^\infty x(t)e^{-j\Omega t}dt$$

$$x(j\Omega) = \int_0^\infty e^{-at}e^{-j\Omega t}dt$$

$$x(j\Omega) = \int_0^\infty e^{-(a+j\Omega)t}dt$$

$$x(j\Omega) = \frac{1}{-(a+j\Omega)}\left[e^{-(a+j\Omega)t}\right]_0^\infty$$

$$x(j\Omega) = \frac{1}{a+j\Omega}$$

$$\boxed{\left|x(j\Omega)\right| = \frac{1}{\left(a^2+\Omega^2\right)^{1/2}}} - \text{Magnitude}$$

$$\boxed{\underline{|x(j\Omega)} = -\tan^{-1}\left(\frac{\Omega}{a}\right)} - \text{Phase}$$

To sketch magnitude and phase spectrum, let us assume a=2; The values of $|x(j\Omega)|$ and $\underline{|x(j\Omega)}$ for various value of Ω are tabulated as shown below and plotted in Figure.

Ω	0	0.5	1	2	3	4	5	10	∞
$\|x(j\Omega)\|$	0.5	0.485	0.447	0.35	0.27	0.22	0.185	0.09	0

$\angle x(j\Omega)$	0	-14°	-26.5°	-45°	-56.3°	-63.43°	-68°	-78.7°	-90°
Ω	-∞	-10	-5	-4	-3	-2	-1	-0.5	0
$\lvert x(j\Omega)\rvert$	0	0.09	0.185	0.22	0.27	0.35	0.447	0.485	0.5
$\angle x(j\Omega)$	90°	78.7°	68°	63.43°	56.3°	45°	26,5°	14°	0°

10. Find the Fourier transform of an sketch the magnitude and phase spectrum of $x(t) = e^{-|t|}$[CO2-H1-Nov/Dec2015].

$$x(t) = e^{-|t|}$$

$$x(j\Omega) = \int_{-\infty}^{\infty} e^{-|t|} e^{j\Omega t} dt$$

$$x(j\Omega) = \int_{-\infty}^{0} e^{t} e^{-j\Omega t} dt + \int_{0}^{\infty} e^{-t} e^{-j\Omega t} dt$$

$$x(j\Omega) = \int_{0}^{\infty} e^{-t} e^{j\Omega t} dt + \int_{0}^{\infty} e^{-t} e^{-j\Omega t} dt$$

$$x(j\Omega) = \int_{0}^{\infty} e^{-(1-j\Omega)t} dt + \int_{0}^{\infty} e^{-(1+j\Omega)t} dt$$

$$x(j\Omega) = \frac{1}{-(1-j\Omega)} \left[e^{-(1+j\Omega)t} \right]_0^{\infty} + \frac{1}{-(1+j\Omega)} \left[e^{-(1+j\Omega)t} \right]_0^{\infty}$$

$$x(j\Omega) = \frac{1}{1-j\Omega} + \frac{1}{1+j\Omega}$$

$$x(j\Omega) = \frac{2}{1+\Omega^2}$$

$$\left| x(j\Omega) \right| = \frac{2}{1+\Omega^2} \text{ for all } \Omega$$

$$\angle x(j\Omega) = 0 \text{ for all } \Omega$$

Ω in	0	1	2	3	5	10	∞

(radian)							
$\|x(j\Omega)\|$	2	1	0.4	0.2	0.077	0.02	0

For negative values of Ω

Ω in (radian)	$-\infty$	-10	-5	-3	-2	-1
$\|x(j\Omega)\|$	0	0.02	0.077	0.2	0.4	1

UNIT – III

Linear Time Invariant- Continuous Time Systems

Part – A

1. Define convolution integral. [CO3-L1-Nov/Dec2015]

The convolution of of two signals is given by y(t)= x(t)*h(t)

The convolution integral is given as,y(t)= x(t)*h(t)=$\int_{-\infty}^{\infty} x(t)h(t-\tau).dt$

This is known as convolution integral.

2. List the properties of convolution integral. [CO3-L1-May/Jun2011]

a. commutative property

b.distributive property

c. associative property

d.shift property

e. convolution with an impulse

f.width property

3. When the LTI-CT system is said to be stable and causal? [CO3-L1-May/Jun2014]

A LTI-CT system is said to be stable if the impulse response of the system is absolutely integrable.

$$\int_{-\infty}^{\infty} h(t)dt < \infty$$

An LTI continuous time system is causal if and only if its impulse response is zero for negative values of t.

if *h*(*t*)=0 when *t*<0

4. Define LTI-CT systems. what are the tools used for analysis of LTI-CT systems? [CO3-L1-Nov/Dec2013]

In a continuous time system if the time shift in the input signal results in the corresponding time shift in the output, then it is called the LTI-CT system

The tools used for the analysis of the LTI-CT system are

Fourier transform

Laplace transform

5. What is meant by impulse response of any system? [CO3-L1-May/Jun2014]

When the unit impulse function is applied as input to the system, the output is nothing but impulse response h(t). The impulse response is used to study various properties of the system such as causality, stability, dynamicity etc.

6. State and prove Time scaling properties of Laplace transform[CO3-L1-May/Jun2013]

$$[x(t)] \overset{LT}{\leftrightarrow} X(S) \text{ then}$$

$$x(at) \overset{LT}{\leftrightarrow} \frac{1}{a} X(S/a)$$

Proof

$$LT x(t) = \int_0^{\infty} x(t) e^{-st}\, dt$$

$$= \int_0^{\infty} x(at) e^{-st}\, dt$$

put $\tau = at$; $t = \tau/a$; $at = \tau$;$adt = d\tau$; $dt = 1/a d\tau$

$$= \int_0^{\infty} x(\tau) e^{-s\left(\frac{\tau}{a}\right)} \frac{1}{a} d\tau$$

$$= \frac{1}{a} \int_0^{\infty} x(\tau) e^{-\left(\frac{s}{a}\right)\tau}\, d\tau$$

$$LT[x(at)] = \frac{1}{a} X(S/a)$$

7. What is the overall impulse response h(t) when two systems with impulse response h_1(t)and h_2(t) are in parallel and in series? [CO3-L1-May/Jun2011] For parallel connection, $h(t) = h1(t) + h2(t)$

For series connection, $h(t) = h1(t) * h2(t)$

8. Check whether the causal system with transfer function H(s) = 1/(s-2) is stable [CO3-H2-Nov/Dec2014]

Here the pole lies at s = 2. Since the pole of causal system does not lie on the left side

of j**ω** axis, the system is not stable.

9. The impulse response of the LTI – CT system is given as$h(t) = e^{-at}u(t)$. Determine transfer function and check whether the system is causal and stable. [CO3-L1-Nov/Dec2014]

$h(t) = e^{-at}u(t)$

Taking laplace transform,

H(s) = 1/(s+1)

Here the pole lies at s = -1, i.e. located in left half of s-plane. Hence this system is causal and stable.

10. What are the conditions for a system to be LTI system? [CO3-L1-May/Jun2014]

Input and output of an LTI system are related by,$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

i.e. convolution

11. What is the impulse response of two LTI systems connected in parallel? [CO3-L1-May/Jun2014]

If the systems are connected in parallel, having responses $h1(t)$ and $h2(t)$then their overall response is given as, $h(t) = h1(t) + h2(t)$

12. Write Nth order differential equation. [CO3-L1]

The Nth order differential equation can be written as

$$\sum_{0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

Here N$\geq M$

13.Determine the response of the system with impulse response h(t)= t u(t) for the input x(t)= u(t) [CO3-L1]

The response is given as, $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$

$$= \int_{-\infty}^{\infty} \tau u(\tau)u(t-\tau)d\tau$$

Here $u(\tau)u(t-\tau)$ =1 for 0 to t. hence above equation will be,

$$\int_{0}^{t} \tau d\tau = \frac{1}{2}t^2$$

14. What are the three elementary operations in block diagram representation of Dontinuous time system? [CO3-L1-May/Jun2013]

Scalar Multiplication

Adder

Delay Element

15. Determine the Laplace transform of a signal x(t)=u(t-3) [CO3-L1]

$$LT[x(t)] = \int_0^{\infty} x(t)e^{-st}\, dt$$

$$= \int_0^{\infty} u(t-3)e^{-st}\, dt$$

$$= \int_3^{\infty} (1)e^{-st}\, dt$$

$$= \int_3^{\infty} e^{-st}\, dt$$

$$=\frac{-1}{s}[e^{-\infty s}-e^{-3s}]$$

$$=\frac{-1}{s}[0-e^{-3s}]$$

$$X(s)=\frac{e^{-3s}}{s}$$

16. Determine the Initial value for the given signals $X(s)=\frac{s+2}{s^2+5s+6}$ [CO3-H1]

Initial Value: $X(0)=\lim_{s=\infty} sX(s)$

$$=\lim_{s=\infty} s\frac{s+2}{s^2+5s+6}$$

$$X(0)=\lim_{s=\infty} s\frac{s(1+\frac{2}{s})}{s^2(1+\frac{5}{s}+\frac{6}{s^2})}$$

$$X(0)=\lim_{s=\infty}\frac{(1+\frac{2}{s})}{(1+\frac{5}{s}+\frac{6}{s^2})}$$

$$=\frac{1+\frac{2}{\infty}}{1+\frac{5}{\infty}+\frac{6}{\infty^2}}=1$$

PART – B

1.The LTI System is characterized by impulse response function given by H(S) $=\frac{1}{S+10}$, ROC :Re>-10 Determine the output of a system when it is excited by the input x(t)= -2$e^{-2t}u(-t)-3e^{-3t}u(t)$ [CO3-H1-May/Jun2012]

Solution:

Gn : H(S) $=\frac{1}{S+10}$,ROC: Re>-10

X(t)=-2$e^{-2t}u(-t)-3e^{-3t}$u(t)

X(S) = $\frac{2}{S+2}-\frac{3}{S+3}$

H(S)=$\frac{Y(S)}{X(S)}$

Y(S)=H(S).X(S)

$= \frac{1}{S+10}.(\frac{2}{S+2}-\frac{3}{S+3})$

Y(S)=$\frac{2}{(S+10)(S+2)}-\frac{3}{(S+10)(S+3)}$

Let $Y_1(S)=\frac{2}{(S+10)(S+2)}$

2=A(S+2)+B(S+10)

Put S=-2 Put S=-10

2=8B 2=-8A

B=$\frac{1}{4}$ A=$-\frac{1}{4}$

$Y_1(S)=-\frac{1}{4}\left(\frac{1}{S+10}\right)+\frac{1}{4}(\frac{1}{S+2})$

$Y_1(t)$=$-\frac{1}{4}[e^{-10t}u(t)]+\frac{1}{4}[e^{-2t}u(t)]$

$y_2(s)=\frac{3}{(S+10)(S+3)}$

3=A(S+3)+B(S+10)

Put S=-3 Put s=-10

3=7B 3=-7a

B=$\frac{3}{7}$ A= $-\frac{3}{7}$

$y_2(S)=\frac{-3}{7}\left(\frac{1}{S+10}\right)+\frac{3}{7}(\frac{1}{S+3})$

$y_2(t)=-\frac{3}{7}e^{-10t}u(t)+\frac{3}{7}e^{-3t}u(t)$

$y(t)=y_1(n)-y_2(n)$

$$=-\frac{1}{4}e^{-10t}u(t)+\frac{1}{4}e^{-2t}u(t)+\frac{3}{7}e^{-10t}u(t)-\frac{3}{7}e^{-3t}u(t)$$

$$y(t)=\frac{1}{4}[e^{-2t}u(t)-e^{-10t}u(t)]+\frac{3}{7}[e^{-10t}u(t)-e^{-3t}u(t)$$

2. Using Laplace transform, solve the differential equation [CO3-H1-Nov/Dec2014]

$$\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+2y(t)=\frac{d}{dt}x(t)$$

if $y(0^-)=2;$

$$\frac{dy(0^-)}{dt}=1 \text{ and } x(t)=e^{-t}u(t)$$

Given

$$\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+2y(t)=\frac{d}{dt}x(t)$$

Taking Laplace transform on both side we get

$$\left[s^2y(s)-s\,y(0^-)-\frac{d}{dt}y(0^-)\right]+3\left[sy(s)-y(0^-)\right]+2y(s)$$

$$=s\times(s)\text{-}x(0^-)$$

$$s^2y(s)-2s-1+3\left[sy(s)-2\right]+2y(s)=sy(s)$$

$$y(s)\left[s^2+3s+2\right]=2s+7+s\times(s) \qquad \rightarrow(1)$$

Given

$$x(t)=e^{-t}u(t)$$

then

$$x(s)=\frac{1}{s+1} \qquad \rightarrow(2)$$

(2) in (1)

$$Y(s)\left[s^2+3s+2\right]=2s+7+\frac{s}{s+1}$$

$$Y(s)=\frac{2s+7}{s^2+3s+2}+\frac{s}{(s+1)\left(s^2+3s+2\right)}$$

$$Y(s)=\frac{(2s+7)(s+1)+s}{(s+1)s^2+3s+2}$$

$$Y(s)=\frac{2s^2+10s+7}{(s+1)(s^2+3s+2)}$$

$$Y(s)=\frac{2s^2+10s+7}{(s+1)^2(s+2)}$$

$$Y(s)=\frac{A}{s+1}+\frac{B}{(s+1)^2}+\frac{C}{s+2}$$

$$A=\frac{d}{ds}\left[\frac{(s+1)^2\left(2s^2+10s+7\right)}{(s+1)^2(s+2)}\right]\Big/_{\text{at } s=-1}$$

$$A=\frac{(s+2)(45+10)-\left(2s^2+10s+7\right)}{(s+2)^2}\Big/_{\text{at } s=-1}$$

$$\boxed{A=7}$$

$$B=(s+1)^2\frac{2s^2+10s+7}{(s+1)^2(s+2)}\Big/_{\text{at }\quad s=-1}$$

$$B=\frac{2s^2+10s+7}{s+2}\Big/_{\text{at }\quad s=-1}$$

$$\boxed{B=-1}$$

$$C = (s+2)\left.\frac{2s^2+10s+7}{(s+1)^2(s+2)}\right/_{\text{at}} \quad s=-2$$

$$\boxed{C=-5}$$

$$Y(s) = \frac{7}{s+1} - \frac{1}{(s+1)^2} - \frac{5}{s+2}$$

Taking inverse Laplace transform on both sides we get

$$\boxed{y(t) = 7e^{-t}u(t) - te^{-t}u(t) - 5e^{-2t}u(t)}$$

3. Using Laplace, transform, solve the following differential equation

$$\mathbf{\frac{d^3y(t)}{dt^3} + 7\frac{d^2y(t)}{dt^2} + 16\frac{d}{dt}y(t) + 12y(t) = x(t)}$$

Initial condition are zero and x (t) = δ(t). [CO3-H1-May/Jun2014]

Given

$$\frac{d^3y(t)}{dt^3} + 7\frac{d^2y(t)}{dt^2} + 16\frac{d}{dt}y(t) + 12y(t) = x(t)$$

Taking Laplace transform on both side we get

$$S^3y(s) + 7s^2y(s) + 16s\ y(s) + 12y(s) = x(s)$$

$$Y(s) = \left[s^3 + 7s^2 + 16s + 12\right] = X(s) \rightarrow (1)$$

Given

$$x(t) = \delta(t)$$

There

$$x(s) = 1 \qquad \rightarrow (2)$$

(2) in (1)

$$Y(s)=\left[s^3+7s^2+16s+12\right]=1$$

$$Y(s)=\frac{1}{s^3+7s^2+16s+12}$$

Synthetic division method for get roots of

$s^3+7s^2+16s+12$

$$\begin{array}{r|rrrr} -3 & 1 & 7 & 16 & 12 \\ & 0 & -3 & -12 & -12 \\ \hline & 1 & 4 & 4 & \underline{|0} \end{array}$$

$$\Rightarrow s^3+7s^2+16s+12=(s+3)(s^2+4s+4)$$

$$Y(s)=\frac{1}{s^3+7s^2+16s+12}$$

$$Y(s)=\frac{1}{(s+3)(s^2+4s+4)}$$

$$Y(s)=\frac{1}{(s+3)(s+2)^2}$$

$$Y(s)=\frac{A}{s+2}+\frac{B}{(s+2)^2}+\frac{C}{(s+3)}$$

$$A=\frac{d}{ds}\left[(s+2)^2\frac{1}{(s+3)(s+2)^2}\right]\Big/_{\text{at } s=-2}$$

$$A=\frac{d}{ds}\left[\frac{1}{s+3}\right]\Big/_{\text{at } s=-2}$$

$$A = \left.\frac{(s+3)(0)-1}{(s+3)^2}\right/_{\text{at } s=-2}$$

$$A = \left.\frac{-1}{(s+3)^2}\right/_{\text{at } s=-2}$$

$$\boxed{A=-1}$$

$$B = \left.(s+2)^2\frac{1}{(s+3)(s+2)^2}\right/_{s=-2}$$

$$B = \left.\frac{1}{s+3}\right/_{s=-2}$$

$$\boxed{B=1}$$

$$C = \left.(s+3)\frac{1}{(s+3)(s+2)^2}\right/_{s=-3}$$

$$\boxed{C=1}$$

$$Y(s) = \frac{-1}{s+2} + \frac{1}{(s+2)^2} + \frac{1}{s+3}$$

$$\Rightarrow \boxed{y(t) = -e^{-2t}u(t) + te^{-2t}u(t) + e^{-3t}u(t)}$$

4. A system is described by the following differential equation [CO3-H1-May/Jun2013]

$\mathbf{\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 12y(t) = x(t)}$ **Determine the response of the system to a unit step applied at t=0. The initial Conditions are** $\mathbf{y(0^-) = -2}$ $\mathbf{\frac{dy}{dt}(0^-) = 0}$

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 12y(t) = x(t)$$

Applying Laplace – transform on both sides

$$\left[s^2y(s) - sy(0^-) - \frac{d}{dt}y(0^-)\right] + 7\left[sy(s) - y(0^-)\right] + 12y(s) = x(s)$$

Substituting

$Y(0^{-}) = -2$

$$\frac{dy}{dt}(0^{-}) = 0, \quad \text{we obtain}$$

$$s^2 y(s) + 2s + 7sy(s) + 14 + 12y(s) = y(s)$$

$$\left(s^2 + 7s + 12\right) y(s) + 2s + 14 = x(s) \qquad \rightarrow (1)$$

For a unit step input

$x(t) = u(t)$

Then

$$x(s) = \frac{1}{s} \qquad \rightarrow (2)$$

(2) in (1)

$$\left(s^2 + 7s + 12\right) y(s) + 2s + 14 = \frac{1}{s}$$

$$\left(s^2 + 7s + 12\right) y(s) = \frac{1}{s} - (2s + 14)$$

$$Y(s) = \frac{1}{s\left(s^2 + 7s + 12\right)} - \frac{(2s + 14)}{\left(s^2 + 7s + 12\right)}$$

$$Y(s) = \frac{1 - 2s^2 - 14s}{s\left(s^2 + 7s + 12\right)}$$

$$Y(s) = \frac{1 - 14s - 2s^2}{s(s+3)(s+4)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$A = \left. s\frac{1 - 2s^2 - 14s}{s(s+3)(s+4)} \right/_{s=0}$$

$$\boxed{A = \frac{1}{12}}$$

$$B = (s+3)\frac{1-2s^2-143}{s(s+3)(s+4)}\Big/_{s=-3}$$

$$\boxed{B=-\frac{25}{3}}$$

$$C = (s+4)\frac{1-2s^2-14s}{s(s+3)(s+4)}\Big/_{\text{at } s=-4}$$

$$\boxed{C=\frac{25}{4}}$$

$$Y(s) = \frac{1}{12s} - \frac{25}{3(s+3)} + \frac{25}{4(s+4)}$$

Taking inverse Laplace transform

$$\boxed{y(t)=\frac{1}{12} - \frac{25}{3}e^{-3t}y(t) + \frac{25}{4}e^{-4t}u(t)}$$

5. For a system with transfer function $H(s) = \frac{s+5}{s^2+5s+6}$ Find the zero-state response if the input x (t) is $e^{-3t}u$ (t) [CO3-H1-Nov/Dec2011]

Given

$$H(s) = \frac{s+5}{s^2+5s+6}$$

$$\frac{Y(s)}{X(s)} = \frac{s+5}{s^2+5s+6}$$

$$\frac{Y(s)}{X(s)} = \frac{s+5}{(s+2)(s+3)}$$

$$Y(s) = \frac{s+5}{(s+2)(s+3)}.X(s) \qquad \rightarrow (1)$$

given

$$x(t)=e^{-3t}u(t)$$

then

$$x(s)=\frac{1}{s+3} \qquad \rightarrow (2)$$

(2) in (1)

$$Y(s) = \frac{s+5}{(s+2)(s+3)}.\left[\frac{1}{(s+3)}\right]$$

$$Y(s) = \frac{s+5}{(s+2)(s+3)^2}$$

$$Y(s) = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{(s+3)^2}$$

$$A = (s+2)\frac{s+5}{(s+2)(s+3)^2}\Bigg/_{s=-2}$$

$$A = \frac{s+5}{(s+3)^2}\Bigg/_{s=-2}$$

$$A = \frac{-2+5}{(-2+3)^2}$$

$$\boxed{A=3}$$

$$B = \frac{d}{ds}\left[(s+3)^2\ \frac{s+5}{(s+2)(s+3)^2}\right]\Bigg/_{s=-3}$$

$$B = \frac{d}{ds}\left(\frac{s+5}{s+2}\right)\Bigg/_{s=-3}$$

$$B = \frac{s+2-(s+5)}{(s+2)^2}\Bigg/_{\text{at } s=-3}$$

$$B = \left.\frac{-3}{(s+2)^2}\right|_{s=-3}$$

$$B = \frac{-3}{(-3+2)^2}$$

$$\boxed{B=-3}$$

$$C = (s+3)^2 \left.\frac{s+5}{(s+2)(s+3)^2}\right|_{s=-3}$$

$$C = \left.\frac{s+5}{s+2}\right|_{s=-3}$$

$$C = \frac{-3+5}{-3+2} = -2$$

$$\boxed{C=-2}$$

$$Y(s) = \frac{3}{s+2} - \frac{3}{s+3} - \frac{2}{(S+3)^2}$$

Taking inverse Laplace transform

$$\boxed{y(t) = 3e^{-2t}u(t) - 3e^{-3t}u(t) - 2te^{-2t}u(t)}$$

6. Relaize the following in direct form[CO3-H1-May/Jun2014]

$$\frac{d^3y(t)}{dt^3}+\frac{4d^2y}{dt^2}+\frac{7dy(t)}{dt}+sy(t)+\frac{5d^2x(t)}{dt^2}+\frac{4dx(t)}{dt}+7x(t)$$

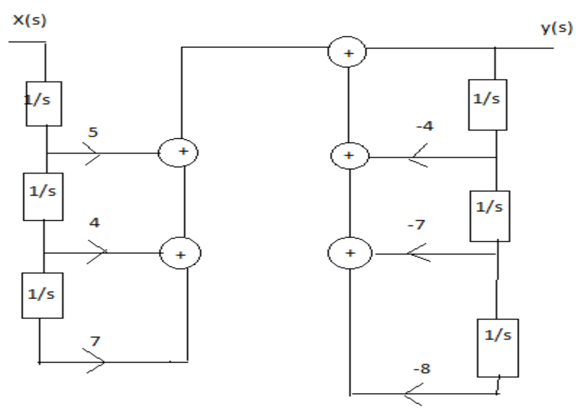
SOLUTION:

$s^3y(s)+4s^2y(s)+7sy(s)+8y(s)=5s^2x(s)+4sx(s)+7x(s)$

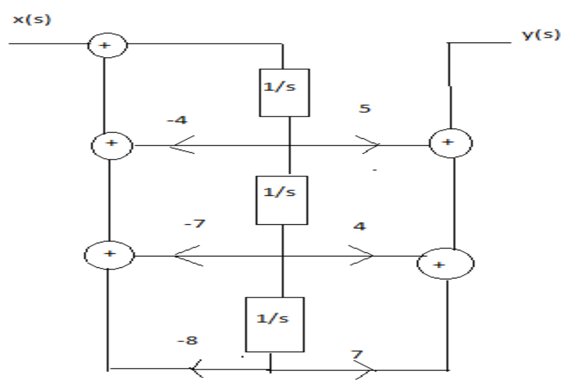
$s^3y(s)-5s^2x(s)+4sx(s)+7x(s)-4s^2y(s)-7sy(s)-8y(s)$

$$y(s)=\frac{5x(s)}{s}+\frac{4x(s)}{s^2}+\frac{7x(s)}{s^3}-\frac{4y(s)}{s}-\frac{7y(s)}{s^2}-\frac{8y(s)}{s^3}$$

Direct form -I



Direct form-II



7 .The systems is described by the input Output relation

$\frac{d^2y(t)}{dt^2}+\frac{dy(t)}{dt}+3y(t)=\frac{dx(t)}{dt}+2x(t)$ **Find the system transfer function ,frequency response and impulse response [CO3-H1-Nov/Dec2015]**

Solution:

To obtain system transfer function: frequency response and impulse response

Taking Laplace transform of differential equation (with zero initial conditions)

$$s^2y(s)+4sy\,(s)+3y(s)=sx(s)+2x(s)$$

$$y(s)[s^2+4s+3]=x(s)[s+2]$$

$$H(s)=\frac{y(s)}{x(s)}$$

$$H(s)=\frac{s+2}{s^2+4s+3}$$

To obtain frequency response:

S=jω

$$H((j\omega)=\frac{j\omega+2}{(jw)^2+4(j\omega)+3}$$
,

To obtain impulse response:

$$H(s)=\frac{s+2}{s^2+4s+3}=\frac{s+2}{(s+3)(s+1)}$$

Using partial fraction method

$$H(s)=\frac{A}{S+3}+\frac{B}{S+1}$$

$$A=(S+3)\frac{(S+2}{(S+3)(S+1)}\Bigg|_{S=-3}$$

$=\frac{-3+2}{-3+1}=\frac{1}{2}$

$A=\frac{1}{2}$

$B=(S+1)\frac{(S+2}{(S+3)(S+1)}\Big|$ S=-3

$=\frac{-1+2}{-1+3}=\frac{1}{2}$

H(S)=$\frac{\frac{1}{2}}{s+3}+\frac{\frac{1}{2}}{s+1}$

Tacking inverse Laplace transform,

$h(t)=[\frac{1}{2}e^{-3t}+\frac{1}{2}e^{-t}]u(t)$

8. Find the impulse and the step response of the following system [CO3-H1-May/Jun2013]

$$\mathbf{H(s)=\frac{10}{s^2+6s+10}}$$

$$H(s)=\frac{10}{s^2+6s+10}$$

$$\frac{Y(s)}{X(s)}=\frac{10}{s^2+6s+10}$$

$$Y(s)=\frac{10}{s^2+6s+10}.X(s) \qquad \rightarrow (1)$$

For impulse response x(t) = δ(t)

Then x(s) = 1 $\rightarrow$(2)

(2) in (1) we get

$$Y(s) = \frac{10}{s^2 + 6s + 10}(1)$$

$$Y(s) = \frac{10}{(s+3)^2 + 1^2}$$

Taking inverse Laplace transforms impulse response

$$\rightarrow \boxed{y(t) = 10e^{-3t}\sin t}$$

For a unit step input

X(t) = u (t)

$$X(s) = \frac{1}{s} \qquad \rightarrow (3)$$

Equation (3) in (1) we get

$$Y(s) = \frac{10}{s^2 + 6s + 10} \cdot \left(\frac{1}{s}\right)$$

$$Y(s) = \frac{10}{s\left(s^2 + 6s + 10\right)}$$

$$Y(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 6s + 10}$$

$$\Rightarrow \frac{10}{s\left(s^2 + 6s + 10\right)} = \frac{A\left(s^2 + 6s + 10\right) + s\left(Bs + C\right)}{s\left(s^2 + 6s + 10\right)}$$

$$\left(A + B\right)s^2 + \left(6A + C\right)s + 10A = 10$$

Computing coefficient S^2 and S and constant we get

A	+	B	=	0
6A	+	C	=	0
10A			=	10

$$\boxed{A = 1}$$

1 + B = 0

$\boxed{B = -1}$ and $\boxed{C=-6}$

$$Y(s) = \frac{1}{s} + \frac{-s-6}{s^2+6s+10}$$
$$Y(s) = \frac{1}{s} - \frac{s+6}{s^2+6s+10}$$
$$Y(s) = \frac{1}{s} - \frac{s+6}{(s+3)^2+1}$$
$$Y(s) = \frac{1}{s} - \left[\frac{s+3}{(s+3)^2+1} + \frac{3}{(s+3)^2+1}\right]$$

Taking inverse Laplace transform

$$\boxed{y(t) = u(t) - \left\{e^{-3t}\cos t + 3e^{-3t}\sin t\right\}}$$

9. Find the impulse and the step response of the following system. [CO3-H1-Nov/Dec2011]

$$\mathbf{H(s) = \frac{s+2}{s^2+5s+4}}$$

$$H(s) = \frac{s+2}{s+5s+4}$$
$$\frac{Y(s)}{X(s)} = \frac{s+2}{(s+1)(s+4)}$$
$$Y(s) = \frac{s+2}{(s+1)(s+4)}.X(s) \qquad \rightarrow (1)$$

For an impulse x(t) = $\delta(t)$

X(s) = 1 $\rightarrow$(2)

$$Y(s) = \frac{s+2}{(s+1)(s+4)} . (1)$$

(2) in (1) we get $$Y(s) = \frac{A}{s+1} + \frac{B}{s+4}$$

$$A = (s+1)\frac{s+2}{(s+1)(s+4)} \Big/ \text{at } s=-1$$

$$\boxed{A=\frac{1}{3}}$$

$$B = (s+4)\frac{s+2}{(s+1)(s+4)} \Big/ \text{at } \ s=-4$$

$$\boxed{B=\frac{2}{3}}$$

$$Y(s) = \frac{1}{3(s+1)} + \frac{2}{3(s+4)}$$

Taking inverse Laplace transforms impulse response →

$$\boxed{y(t) = \frac{1}{3}e^{-t}u(t) + \frac{2}{3}e^{-4t}u(t)}$$

For step input x(t) = u (t)

$$X(s) = \frac{1}{s} \qquad \rightarrow (3)$$

(3) in (1) we get

$$Y(s) = \frac{s+2}{(s+1)(s+4)} \cdot \left(\frac{1}{s}\right)$$

$$Y(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$A = s\frac{s+2}{s(s+1)(s+4)}\Bigg/_{s=0}$$

$$\boxed{A=\frac{1}{2}}$$

$$B = (s+1)\frac{s+2}{s(s+1)(s+4)}\Bigg/_{s=-1}$$

$$\boxed{B=-\frac{1}{3}}$$

$$C = (s+4)\frac{s+2}{s(s+1)(s+4)}\Bigg/_{s=-4}$$

$$\boxed{C=-\frac{1}{6}}$$

$$Y(s) = \frac{1}{2s} - \frac{1}{3(s+1)} - \frac{1}{6(s+4)}$$

Taking inverse Laplace transform

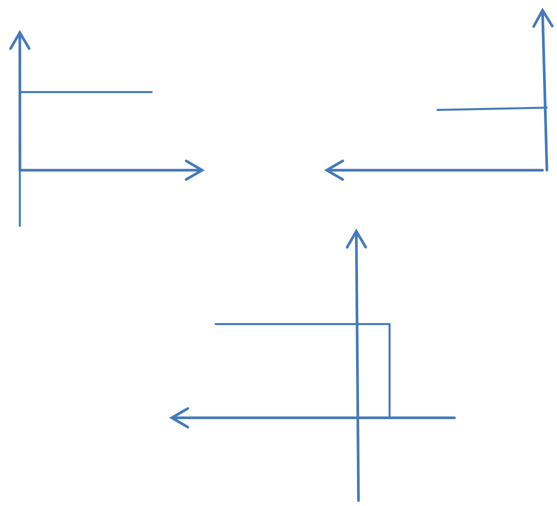
$$\boxed{y(t)=\frac{1}{2}u(t) - \frac{1}{3}e^{-t}u(t) - \frac{1}{6}e^{-4t}u(t)}$$

10.Find the convolution integral of the given signal x(t)=e^{-at}u(t), h(t)=u(t). [CO3-H1-May/Jun2015] [May2015]

Solution:

Convolution integral is given by,

$$(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)$$



x(τ)=e^{-at}u(τ) , h(τ)= u(τ)
Time axis:x(τ)=0 to ∞
h(τ) $= 0$ to ∞
Hence the time limit is (0, ∞)

$$y(t) =\int_0^t x(\tau)h(t-\tau)d\,\tau$$

$$=\int_0^t e^{-a\tau}(1)d\,\tau$$

$$=\left[\frac{e^{-a\tau}}{-a}\right]_0^t$$

$$=\frac{-1}{a}\left[e^{-at}-e^{0}\right]$$

$$\boxed{Y(t) \;=\frac{1}{a}\left[\,1-e^{-at}\right] ,0<=t<=\infty}$$

11. Find the inverse Laplace transform of x(s)=$\frac{3s+7}{s^2-2s-3}$. [CO3-H1-May/Jun2014]

(i) ROC Re(s)>3
(ii) ROC Re(s)<-1
(iii)ROC -1<Re(s)<3

Solution:

x(s)=$\frac{3s+7}{s^2-2s-3} = \frac{3s+7}{(s+1)(s-3)}$

Take partial fraction of the given signal.

x(s)=$\frac{3s+7}{s^2-2s-3} \longrightarrow \frac{3s+7}{(s+1)(s-3)} = \frac{A}{s+1} + \frac{B}{s-3}$

3s+7=A(s-3) + B(s+1)

If s=3, we get $\boxed{B=4}$

If s=-1, we get $\boxed{A=-1}$

Therefore, $\boxed{x(s) = \frac{-1}{s+1} + \frac{4}{s-3}}$

i. Re(s)>3

x(t)=-e^{-t}u(t) + 4e^{3t}u(t)

ii. Re(s)<-1

x(t)= e^{-t}u(t) - 4e^{3t}u(t)

iii. -1 <Re(s)<3

i.e., s>-1, s<3

x(t)=- e^{-t}u(t) - 4e^{3t}u(t)

UNIT – IV

Analysis Of Discrete Time Signals

Part – A

1. Define impulse response of a DT system. [CO4-L1-May/Jun2011]

The impulse response is the output produced by DT system when unit impulse is applied at the input. The impulse response is denoted by h(n). The impulse response h(n) is obtained by taking inverse Z transform from the transfer function H(z).

2. State the significance of block diagram representation. [CO4-L1-May/Jun2009]

The LTI systems are represented with the help of block diagrams. The block diagrams are more effective way of system description. Block Diagrams Indicate how individual calculations are performed. Various blocks are used for block diagram representation.

3. What is the condition for causality if H(z) is given. [CO4-L1-Nov/Dec2015]

A discrete LTI system with rational system function H(z) is causal if and only if

I.The ROC is the exterior of the circle outside the outermost pole.

Ii.When H(z) is expressed as a ratio of polynomials in z , the order of the numerator can not be greater than the order of the denominator.

4. What is the condition for stability if H(z) is given. [CO4-L1- Nov/Dec2010]

A discrete LTI system with rational system function H(z) is stable if and only if all of the poles H(z) lies inside the unit circle. That is they must all have magnitude smaller than 1.

5. What is the relation between Z transform and Fourier transform of discrete time signal. [CO4-L1-May/Jun2010]

$X(Z) = x(n)Z^{-n}$

$X(\omega) = x(n)e^{-j\omega n}$

$X(Z)$ at Z=$e^{-j\omega}$ is $X(\omega)$

When z- transform is evaluated on unit circle (ie.| z |= 1) then it becomes Fourier transform.

6. Define region of convergence with respect to Z transform. [CO4-L1-May/Jun2015]

Region of convergence (ROC) is the area in Z plane where Z transform convergence .In other word, it is possible to calculate the X(z) in ROC.

7. State the initial value theorem of Z transforms. [CO4-L1-May/Jun2010]

The initial value of the sequence is given as, $X(0) = \lim_{Z=\infty} X(Z)$

8. What is meant by aliasing? [CO4-L1-May/Jun2015]

When the high frequency interferes with low frequency and appears as low then the phenomenon is called aliasing.

9. Define Nyquist rate and Nyquist interval. [CO4-L1-May/Jun2010]

When the sampling rate becomes exactly equal to ‘2W’ samples/sec,for a give bandwidth of W hertz, then it is called Nyquist rate .’

Nyquist interval is the time interval between any two adjacent samples. Nyquist rate =2W Hz & Nyquist interval=1/2W seconds.

10. Define unilateral Z-Transform or one sided Z-transform [CO4-L1-Nov/Dec2011]

The unilateral Z-Transform of signal x(t) is given as,

$$X(Z) = \sum_{n=0}^{\infty} x(n)Z^{-n}$$

The unilateral and bilateral Z-Transforms are same for causal signals.

11. State the final value theorem for z-transform. [CO4-L1- Nov/Dec2012]

The final value of a sequence is given as,

$$X(\infty) = \lim_{Z=0}(1 - Z^{-1})X(Z)$$

12. Define DTFT pair. [CO4-L1- Nov/Dec2014]

$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ Analysis Equation

$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n}\, d\omega$ Synthesis Equation

13. State the sampling theorem. [CO4-L1- May/Jun2014]

A bandwidth signal of finite energy, which has no frequency components higher than W hertz, is completely described by specifying the values of the signal at instants of time separated by 1/2W seconds.

A band limited signal of finite energy, which has no frequency components higher than W hertz, may be completely recovered from the knowledge of its samples taken at the rate of 2W samples per second.

14. What are the Properties of ROC? [CO4-L1-May/Jun2013]

The ROC of a finite duration sequence includes the entire z- plane, except z= 0 and |z|= 1.

ROC does not contain any poles.

ROC is the ring in the z-plane cantered about origin.

ROC of causal sequence (right handed sequence) is of the form |z| > r. v. ROC of left handed sequence is of the form |z| < r.

ROC of two sided sequence is the concentric ring in the z plane.

15.State convolution property of Z transforms. [CO4-L1-May/Jun2011]

The convolution property states that if

X1[n] $\overset{ZT}{\leftrightarrow}$X1(Z)

x2[n] $\overset{ZT}{\leftrightarrow}$X2(Z) then

x1[n] *x2[n] X1(Z) X2(Z)

That is convolution of two sequences in time domain is equivalent to multiplication of their Z transforms.

16. State the methods to find inverse Z transform. [CO4-L1]

Partial fraction expansion

Contour integration

Power series expansion

Convolution method

17. State the condition for existence of DTFT? [CO4-L1]

The conditions are

If x(n)is absolutely summable then
|x(n)|<∞
If x(n) is not absolutely summable then it should have finite energy for
DTFT to exit.

18. State parseval's relation for discrete – time Periodic and Aperiodic signals. [CO4-L1]

The perseval's relation for discrete – time periodic signal is given by

$$C_k = \frac{1}{N}\sum_{n=0}^{N-1} X(n)e^{\frac{-j2\pi kn}{N}}$$

The parseval's energy theorem for discrete time Aperiodic signal is given by

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega$$

19. What is mean by Aliasing[CO4-L1-Nov/Dec2014]

Aliasing is a phenomenon where the high frequency components of the sampled signal interfere with each other because of inadequate sampling fs < 2ωm. Aliasing Aliasing leads to distortion in recovered signal. This is the reason why sampling frequency should be at least twice the bandwidth of the signal.

20. What are the properties of frequency response H($e^{j\omega}$) of an LTI system? [CO4-L1]

H ($e^{j\omega}$) is a continuous function of ω
The frequence response is periodic with period 2π
The Magnitude response $|H(e^{j\omega}|$ is even function of ω
The phase response angle $|H(e^{j\omega}|$ is odd function of ω

21 State time shifting and frequency shifting properties of discrete time Fourier transform. [CO4-L1]

Time Shifting If DTFT $[x(n)] = X(e^{j\omega})$

Then

DTFT $[x(n-K)] = e^{-j\omega k} X(e^{j\omega})$

Where k is an integer

Frequency Shifting

If DTFT $[x(n)] = X(e^{j\omega})$

Then

DTFT $\left[x(n)e^{jn\omega_0}\right] = X[e^{(\omega-\omega_0)}]$

22. Find the Discrete time Fourier transform of unit impulse sequence. [CO4-L1-May/Jun2015]

Unit impulse sequence $\delta(n) = \begin{matrix} 1; when\ n = 0 \\ 0; otherwise \end{matrix}$

DTFT $[\delta(n)] = \sum_{n=-\infty}^{\infty} \delta(n) e^{-j\omega n} = 1$

DTFT $[\delta(n)]$ =1

23. Find the discrete – time Fourier transform of unit step sequence. [CO4-L1-Nov/Dec2013]

Unit step sequence $u(n) = \begin{matrix} 1; when\ n \geq 0 \\ 0; otherwise \end{matrix}$

$$\text{DTFT}[u(n)] = \sum_{k=-\infty}^{\infty} u(n)e^{-j\omega k}$$

$$= \sum_{k=0}^{\infty} 1e^{-j\omega k}$$

$$= 1 + e^{-j\omega} + e^{-2j\omega} + \cdots.$$

$$\text{DTFT}[u(n)] = \frac{1}{1-e^{-j\omega}}$$

24.Find the discrete – time Fourier transform of δ(n-k). [CO4-L1-May/Jun2011]

$\delta(n-k) = \begin{matrix} 1; when\ n = k \\ 0; otherwise \end{matrix}$

$$\text{DTFT}[\delta(n-k)] = \sum_{n=-\infty}^{\infty} \delta(n-k)e^{-j\omega n}$$

$$\text{DTFT}[\delta(n-k) = (1)e^{-j\omega k}$$

25. Find the discrete – time Fourier transform of the following x (n) = {1,-1, 2, and 2}.[CO4-L1]

x (n) = {1, -1, -2, 2}

The sequence values are x(0) = 1; x(1) = -1; x(2) = 2; x(3) = 2

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} X(n)e^{-j\omega n}$$

$$= 1 - e^{-j\omega} + 2e^{-j2\omega} + 2e^{-j3\omega}$$

Part – B

1.Find the inverse Z-transform of x(z)=$\frac{z^2}{(z-0.5)(z-1)^2}$ |Z|>1 [CO4-H1-May/Jun2014]

Solution:

x(z)=$\frac{z^2}{(z-0.5)(z-1)^2}$ |Z|>1

Using partial fraction expansion

$$\frac{X(Z)}{Z}=\frac{z}{(z-0.5)(z-1)^2}$$

$$\frac{x(z)}{z}=\frac{A}{(Z-0.5)}+\frac{B_1}{(Z-1)}+\frac{B_2}{(Z-1)^2}$$

$$A=(Z-0.5)\frac{X(Z)}{Z}\bigg|_{Z=0.5}$$

$$=(Z-0.5)\frac{Z}{(Z-0.5)(Z-1)^2}\bigg|_{Z=0.5}$$

$$=\frac{Z}{(Z-1)^2}\bigg|_{Z=0.5}$$

$$=\frac{0.5}{(0.5-1)^2}=\frac{0.5}{0.25}$$

A=2

$$B_2=(Z-1)^2\frac{X(Z)}{Z}\bigg|_{Z=1}$$

$= (Z-1)^2 \frac{Z}{(Z-0.5)(Z-1)^2} \Big|$ Z=1

$= \frac{Z}{Z-0.5} \Big|$ Z=1

$= \frac{1}{1-0.5}$

$B_2 = 2$

$B_1 = \frac{d}{dz}(z-1)^2 \frac{x(z)}{z} \Big|$ z=1

$= \frac{d}{dz}(z-1)^2 \frac{z}{(z-0.5)(z-1)^2} \Big|$ z=1

$= \frac{d}{dz} \frac{z}{(z-0.5)} \Big|$ z=1

$=\frac{(z-0.5)(1)^{-z}}{(z-0.5)(2)} \Big| z = 1$

B= -2

$$\frac{x(z)}{z} = \frac{-2}{z-0.5} - \frac{2}{(z-1)} + \frac{2}{(z-1)^2}$$

$$x(z) = \frac{0.5z}{z-0.5} + \frac{0.5z}{(z-1)} + \frac{2z}{(z-1)^2}$$

Tacking inverse z-transform on both side

$$x(n) = 2(0.5)^n u(n) + 2(1)^n u(n) + 2(n)^n u(n)$$

2. Find the DT Fourier transform of $x(n) = \sin\left(\frac{\pi n}{2}\right).u(n)$ [CO4-H1-May/Jun2011]

$$x(n) = \sin\left(\frac{\pi n}{2}\right).u(n)$$

$$x\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$$x\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} \sin\left(\frac{\pi n}{2}\right)u(n)e^{-j\omega n}$$

$$x\left(e^{j\omega}\right) = \sum_{n=0}^{\infty}\left[\frac{e^{\frac{j\pi n}{2}} - e^{-\frac{j\pi n}{2}}}{2j}\right]e^{-j\omega n}$$

$$x\left(e^{j\omega}\right) = \frac{1}{2j}\left[\sum_{n=0}^{\infty} e^{\frac{j\pi n}{2}} e^{-j\omega n} - \sum_{n=0}^{\infty} e^{-\frac{j\pi n}{2}} e^{-j\omega n}\right]$$

$$x\left(e^{j\omega}\right) = \frac{1}{2j}\left[\sum_{n=0}^{\infty} e^{\left(j\frac{\pi}{2}-j\omega\right)n} - \sum_{n=0}^{\infty} e^{\left(-j\frac{\pi}{2}-j\omega\right)n}\right]$$

$$x\left(e^{j\omega}\right) = \frac{1}{2j}\left[\frac{1}{1-e^{j\frac{\pi}{2}}e^{-j\omega}} - \frac{1}{1-e^{-j\frac{\pi}{2}}e^{-j\omega}}\right]$$

$$x\left(e^{j\omega}\right)=\frac{1}{2j}\left[\frac{1-e^{-j\frac{\pi}{2}}e^{-j\omega}-1+e^{j\frac{\pi}{2}}e^{-j\omega}}{\left(1-e^{j\frac{\pi}{2}}e^{-j\omega}\right)\left(1-e^{-j\frac{\pi}{2}}e^{-j\omega}\right)}\right]$$

$$x\left(e^{j\omega}\right)=\frac{1}{2j}\left[\frac{e^{-j\omega}\left(e^{j\frac{\pi}{2}}-e^{-j\frac{\pi}{2}}\right)}{1-e^{-j\omega}\left(e^{j\frac{\pi}{2}}+e^{-j\frac{\pi}{2}}\right)+e^{-j2\omega}}\right]$$

$$x\left(e^{j\omega}\right)=\frac{1}{2j}\left[\frac{e^{-j\omega}\left(e^{j\frac{\pi}{2}}-e^{-j\frac{\pi}{2}}\right)}{1-2\cos\frac{\pi}{2}e^{-j\omega}+e^{-j2\omega}}\right]$$

$$x\left(e^{j\omega}\right)=\frac{e^{-j\omega}\sin\frac{\pi}{2}}{1+e^{-j2\omega}}$$

$$\boxed{x\left(e^{j\omega}\right)=\frac{e^{-j\omega}}{1+e^{-j2\omega}}}$$

3. Find the DT Fourier transform of $x(n)=\cos\left(\frac{\pi n}{3}\right).u(n)$ [CO4-H1-Nov/Dec2014]

$$x(n)=\cos\left(\frac{\pi n}{3}\right).u(n)$$

$$x\left(e^{j\omega}\right)=\sum_{n=-\infty}^{\infty}x(n)e^{-j\omega n}$$

$$x\left(e^{j\omega}\right)=\sum_{n=-\infty}^{\infty}\cos\left(\frac{\pi n}{3}\right)u(n)e^{-j\omega n}$$

$$x\left(e^{j\omega}\right)=\sum_{n=0}^{\infty}\cos\left(\frac{\pi n}{3}\right)e^{-j\omega n}$$

$$x\left(e^{j\omega}\right)=\sum_{n=0}^{\infty}\left[\frac{e^{\frac{j\pi n}{3}}+e^{-\frac{j\pi n}{3}}}{2}\right]e^{-j\omega n}$$

$$x\left(e^{j\omega}\right)=\frac{1}{2}\left[\sum_{n=0}^{\infty}e^{\frac{j\pi n}{3}}e^{-j\omega n}+\sum_{n=0}^{\infty}e^{-\frac{j\pi n}{3}}e^{-i\omega n}\right]$$

$$x\left(e^{j\omega}\right)=\frac{1}{2}\left[\sum_{n=0}^{\infty}e^{\left(j\frac{\pi}{3}-j\omega\right)n}+\sum_{n=0}^{\infty}e^{\left(-\frac{j\pi}{3}-j\omega\right)n}\right]$$

$$x\left(e^{j\omega}\right)=\frac{1}{2}\left[\frac{1}{1-e^{j\frac{\pi}{3}}e^{-j\omega}}+\frac{1}{1-e^{-j\frac{\pi}{3}}e^{-j\omega}}\right]$$

$$x\left(e^{j\omega}\right)=\frac{1}{2}\left[\frac{1-e^{-j\frac{\pi}{3}}e^{-j\omega}+1-e^{j\frac{\pi}{3}}e^{-j\omega}}{\left(1-e^{-j\omega}\left(e^{j\frac{\pi}{3}}+e^{-j\frac{\pi}{3}}\right)+e^{-j2\omega}\right)}\right]$$

$$x\left(e^{j\omega}\right)=\frac{1}{2}\left[\frac{2-e^{-j\omega}2\left(\cos\frac{\pi}{3}\right)}{1-e^{-j\omega}2\cos\frac{\pi}{3}+e^{-j2\omega}}\right]$$

$$x\left(e^{j\omega}\right)=\frac{1-e^{-j\omega}\cos\frac{\pi}{3}}{1-e^{-j\omega}2\cos\frac{\pi}{3}+e^{-j2\omega}}$$

$$\boxed{x\left(e^{j\omega}\right)=\frac{1-0.5e^{-j\omega}}{1-e^{-j2\omega}+e^{-j2\omega}}}$$

4 .State and prove the properties in DTFT. [CO4-L1-Nov/Dec2015]

(i) Linearity

(ii) Time shifting

(iii) Frequency shifting

(iv) Time reversal

(i) Linearity:

State:

$$\mathrm{DTFT}\left[x_1(n)\right] = x_1\left(e^{j\omega}\right)$$
$$\mathrm{DTFT}\left[x_2(n)\right] = x_2\left(e^{j\omega}\right)$$
$$\mathrm{DTFT}\left[a_1x_1(n) + a_2x_2(n)\right] = a_1x_1\left(e^{j\omega}\right) + a_2x_2\left(e^{j\omega}\right)$$

Proof:

$$\mathrm{DTFT}\left[a_1x_1(n) + a_2x_2(n)\right] = \sum_{n=0}^{\infty}\left[a_1x_1(n) + a_2x_2(n)\right]e^{-j\omega n}$$
$$= a_1\sum_{n=0}^{\infty}x_1(n)e^{-j\omega n} + a_2\sum_{n=0}^{\infty}x_2(n)e^{-j\omega n}$$
$$= a_1x_1\left(e^{j\omega}\right) + a_2x_2\left(e^{j\omega}\right)$$

(ii) Time shifting:

State:

If DTFT [x (n)] = x ($e^{j\omega}$)

Then

DTFT [x(n-k)] = $e^{-j\omega k}$ x($e^{j\omega}$)

Proof:

Put p = n-k then

$$\text{DTFT}\left[x(n-k)\right] = \sum_{n=-\infty}^{\infty} x(n-k)e^{-j\omega n}$$

$$\text{DTFT}\left[x(n-k)\right] = \sum_{p=-\infty}^{\infty} x(p)e^{-j\omega}(p+k)$$

$$=e^{-j\omega k}\sum_{p=-\infty}^{\infty} x(p)e^{-j\omega p}$$

$$\text{DTFT}\left[x(n-k)\right] = e^{-j\omega k}x\left(e^{j\omega}\right)$$

Frequency shifting:

State:

If DTFT [x (n)] = x ($e^{j\omega}$)

DTFT [x(n)$e^{j\omega_0 n}$] = x[$e^{j(\omega-\omega_0)}$]

Proof:

$$\text{DTFT}\left[x(n)e^{j\omega_0 n}\right] = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega_0 n}e^{-j\omega_0 n}$$

$$= \sum_{p=-\infty}^{\infty} x(n)e^{-j(\omega-\omega_0)n}$$

$$=x\left(e^{j(\omega-\omega_0)}\right)$$

Time reversal:

State:

$$\text{If DTFT }\left[x(n)\right] = x\left(e^{j\omega}\right)$$

$$\text{DTFT }\left[x(-n)\right] = x\left(e^{-j\omega}\right)$$

Proof:

$$\text{DTFT } x(-n) = \sum_{n=-\infty}^{\infty} x(-n)e^{-j\omega n}$$

Let m=-n

$$\text{DTFT } \left[x(-n)\right] = \sum_{-m=-\infty}^{\infty} x(m)e^{j\omega m}$$

$$= \sum_{m=\infty}^{-\infty} x(m)e^{-(-j\omega m)}$$

$$\text{DTFT}\left[x(-n)\right] = x\left(e^{-j\omega}\right)$$

5 .State and prove the properties in DTFT[CO4-L1-May/Jun2015]

(i) Differentiation in frequency

(ii) Time convolution

(iii) Frequency convolution

(iv) Parseval's theorem.

(i) Differentiation in Frequency:

State: If

$$\text{DTFT}\left[x(n)\right] = x\left(e^{j\omega}\right)$$

$$\text{DTFT}\left[nx(n)\right] = j\frac{d}{d\omega}x\left(e^{j\omega}\right)$$

Proof:

$$x\left(e^{j\omega}\right) = \text{DTFT}\left[x(n)\right] = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

Differentiate with r.t. ω m both side

$$\frac{dx\left(e^{j\omega}\right)}{d\omega} = \sum_{n=-\infty}^{\infty} (-jn)x(n)e^{-j\omega n}$$

$$\frac{dx\left(e^{j\omega}\right)}{d\omega} = j\sum_{n=-\infty}^{\infty} nx(n)\,e^{-j\omega n}$$

$$\frac{dx\left(e^{j\omega}\right)}{d\omega} = -j\mathrm{DTFT}\left[nx(n)\right]$$

Time convolution:

State:

If $\mathrm{DTFT}\left[x_1(n)\right] = x_1\left(e^{j\omega}\right)$ and $\mathrm{DTFT}\left[x_2(n)\right] = x_2\left(e^{j\omega}\right)$

Then

$$\mathrm{DTFT}\left[x_1(n) * x_2(n)\right] = x_1\left(e^{j\omega}\right).x_2\left(e^{j\omega}\right)$$

Proof:

$$x_1(n) * x_2(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$\mathrm{DTFT}\left[x_1(n) * x_2(n)\right] = \sum_{n=-\infty}^{\infty}\left[\sum_{k=-\infty}^{\infty} x(k)h(n-k)\right]e^{-j\omega n}$$

Interchanging the order of summation we get

$$\mathrm{DTFT}\left[x_1(n) * x_2(n)\right] = \sum_{k=\infty}^{\infty} x(k)\sum_{n=-\infty}^{\infty} h(n-k)e^{-j\omega n}$$

put n-k=p, then

$$\mathrm{DTFT}\left[x_1(n) * x_2(n)\right] = \sum_{k=-\infty}^{\infty} x(k)\sum_{p=-\infty}^{\infty} h(p)e^{-j\omega p}e^{-j\omega k}$$

$$= \sum_{k=-\infty}^{\infty} x(k)H\left(e^{j\omega}\right).e^{-j\omega k}$$

$$\mathrm{DTFT}\left[x_1(n) * x_2(n)\right] = H\left(e^{j\omega}\right)\sum_{k=-\infty}^{\infty} x(k)e^{-j\omega n}$$

$$\boxed{\mathrm{DTFT}\left[x_1(n) * x_2(n)\right] = H\left(e^{j\omega}\right)x\left(e^{j\omega}\right)}$$

Frequency convolution:

State:

If $\mathrm{DTFT}\left[x_1(n)\right]=x_1\left(e^{j\omega}\right)$ and $\mathrm{DTFT}\left[x_2(n)\right]=x_2\left(e^{j\omega}\right)$

then $\mathrm{DTFT}\left[x_1(n).x_2(n)\right]=\dfrac{1}{2\pi}\int\limits_{2\pi} x_1\left(e^{j\theta}\right)x_2 e^{j(\omega-\theta)}d\theta$

Proof:

$$\mathrm{DTFT}\left[x_1(n)x_2(n)\right]=\sum_{n=-\infty}^{\infty}x_1(n)x_2(n)e^{-j\omega n}$$

$$=\sum_{n=-\infty}^{\infty}x_2(n)\left[\frac{1}{2\pi}\int_{2\pi}x_1\left(e^{j\theta}\right)e^{j\theta n}d\theta\right]$$

Enter changing the order of summation and integral, we obtain

$$\mathrm{DTFT}\left[x_1(n).x_2(n)\right]=\frac{1}{2\pi}\int_{2\pi}x_1\left(e^{j\theta}\right)\left[\sum_{n=-\infty}^{\infty}x_2(n)e^{-j(\omega-\theta)n}\right]d\theta$$

$$=\frac{1}{2\pi}\int_{2\pi}x_1\left(e^{j\theta}\right).x_2e^{j(\omega-\theta)}d\theta$$

Parseval's theorem:

State:

If

$\mathrm{DTFT}\left[x(n)\right]=x\left(e^{j\omega}\right)$

then

$$E=\sum_{n=-\infty}^{\infty}\left|x(n)\right|^2=\frac{1}{2\pi}\left|x\left(e^{j\omega}\right)\right|^2d\omega$$

Proof:

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} x(n)x*(n)$$

$$=\sum_{n=-\infty}^{\infty} x(n)\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} x(e^{j\omega})e^{j\omega n}\right]^* d\omega$$

Inter change the order of summation and integration yields.

$$E = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^*(e^{j\omega})\left[\sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}\right]d\omega$$

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^*(e^{j\omega})x(e^{j\omega})d\omega = \frac{1}{2\pi}|x(e^{j\omega})|^2 d\omega$$

6. Verify Parseval's theorem for DT sequence [CO4-H2-May/Jun2011]

[May2011]

$$\boldsymbol{\sum_{n=-\infty}^{\infty} x(n)x^*(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(e^{j\omega})x^*(e^{j\omega})d\omega}$$

$$\textbf{for} \quad \boldsymbol{x(n) = \left(\frac{1}{2}\right)^n u(n)}$$

Given

$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

L.H.S

$$= \sum_{n=-\infty}^{\infty} x(n)\, x^{*}(n)$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{2n} u(n)$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^{n}$$

$$= \frac{1}{1-\frac{1}{4}}$$

$$\boxed{\text{L.H.S} = \frac{4}{3}}$$

For the given

$$x(n) = \left(\frac{1}{2}\right)^{n} u(n)$$

$$x\left(e^{j\omega}\right) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

$$x\left(e^{j\omega}\right) = \frac{1}{1-0.5\cos\omega + j0.5\sin\omega}$$

$$x^{*}(j\omega) = \frac{1}{1-0.5\cos\omega - j0.5\sin\omega}$$

$$x\left(e^{j\omega}\right).x^{*}\left(e^{j\omega}\right) = \frac{1}{(1-0.5\cos\omega)^{2} + (0.5\sin\omega)^{2}}$$

$$x\left(e^{j\omega}\right).x^{*}\left(e^{j\omega}\right) = \frac{1}{1.25-\cos\omega}$$

R.H.S

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{1.25 - \cos\omega}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{1.25 - \dfrac{1-\tan^2\dfrac{\omega}{2}}{1+\tan^2\dfrac{\omega}{2}}}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1+\tan^2\dfrac{\omega}{2}}{0.25 + 2.25\tan^2\dfrac{\omega}{2}}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1+t^2}{0.25+2.25t^2}\,\frac{2dt}{1+t^2}$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{2.25\left(t^2+\frac{1}{9}\right)}dt$$

$$= \frac{1}{\pi(2.25)}\int_{-\infty}^{\infty}\frac{1}{t^2+\left(\frac{1}{3}\right)^2}dt$$

$$= \frac{3}{2.25\pi}\left[\tan^{-1}3t\right]_{-\infty}^{\infty}$$

$$= \frac{3}{2.25\pi}\left[\frac{\pi}{2}-\left(\frac{-\pi}{2}\right)\right]$$

$$= \frac{3}{2.25\pi}\left[\frac{\pi}{2}+\frac{\pi}{2}\right]$$

$$= \frac{3}{2.25\pi}[\pi]$$

$$= \frac{3}{2.25}$$

$$\boxed{R.H.S = \frac{4}{3}}$$

$L.H.S = R.H.S$ proved

7. Find the convolution of the signal given below using Fourier transform

$\mathbf{x_1(n) = \left(\frac{1}{2}\right)^n u(n);\ x_2(n) = \left(\frac{1}{3}\right)^n u(n)}$ **[CO4-H1-Nov/Dec2014]**

Given

$$x_1(n) = \left(\frac{1}{2}\right)^n u(n);\ x_2(n) = \left(\frac{1}{3}\right)^n u(n)$$

$$x_1\left(e^{j\omega}\right)=\frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

$$x_2\left(e^{j\omega}\right)=\frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$\text{DTFT}\left[x_1(n)*x_2(n)\right]=x_1\left(e^{j\omega}\right).x_2\left(e^{j\omega}\right)$$

$$\text{DTFT}\left[x_1(n)*x_2(n)\right]=\frac{1}{1-\frac{1}{2}e^{-j\omega}}.\frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$\left[x_1(n)*x_2(n)\right]=\text{DTFT}^{-1}\left[\frac{1}{\left(1-\frac{1}{2}e^{-j\omega}\right)}.\frac{1}{\left(1-\frac{1}{3}e^{-j\omega}\right)}\right]$$

$$\left[x_1(n)*x_2(n)\right]=\text{DTFT}^{-1}\left[\frac{e^{j\omega}}{\left(e^{j\omega}-1/2\right)}.\frac{e^{j\omega}}{\left(e^{j\omega}-1/3\right)}\right]$$

$$=\text{DTFT}^{-1}\left[Y\left(e^{j\omega}\right)\right]$$

$$Y\left(e^{j\omega}\right)=\frac{e^{j\omega}.e^{j\omega}}{\left(e^{j\omega}-1/2\right)\left(e^{j\omega}-1/3\right)}$$

$$\frac{Y\left(e^{j\omega}\right)}{e^{j\omega}}=\frac{e^{j\omega}}{\left(e^{j\omega}-1/2\right)\left(e^{j\omega}-1/3\right)}$$

$$\frac{Y\left(e^{j\omega}\right)}{e^{j\omega}}=\frac{A}{\left(e^{j\omega}-1/2\right)}+\frac{B}{\left(e^{j\omega}-1/2\right)}$$

$$Y\left(e^{j\omega}\right)=\frac{3e^{j\omega}}{e^{j\omega}-1/2}-\frac{2e^{j\omega}}{e^{j\omega}-1/3}$$

Taking inverse Fourier transform

$$y(n) = 3\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{3}\right)^n u(n)$$

7.Find the Z- transform of X(n)=n u(n) and x(n) = $a^{|n|}$[CO4-H1-Nov/Dec2013]

Solution:

nx(n) $\longleftrightarrow$ -Z $\frac{d}{dz}$x(z)

Z-transform of u(n)=$\frac{1}{1-z^{-1}}$ =$\frac{z}{z-1}$

From the multiplication by n property,(or)By differentiation property,

Z[nx(n)]-Z $\frac{d}{dz}$x(z)

Similarly,

Z[nu(n)] = -Z$\frac{d}{dz}$ ($\frac{z}{z-1}$)

= -Z $\left[\frac{(Z-1)(1)-Z(1)}{(Z-1)^2}\right]$

= -Z $\left[\frac{(Z-1-Z)}{(Z-1)^2}\right]$

$$\boxed{Z[nu(n)] = \frac{(Z)}{(Z-1)^2}}$$

(ii) x(n) = $a^{|n|}$

Solution:

If 'n'ispositive x(n)=a^n **,n<0**

If 'n' is negative x(n)=a^{-n} **,n>0**

It is infinite duration and non -casual signal.

WKT. $X(z)=\sum_{n=-\infty}^{\infty} x(n)z^{-n}$

$= \sum_{n=-\infty}^{-1} a^{-n}z^{-n} + \sum_{n=0}^{\infty} a^{n}z^{-n}$

$= \sum_{n=-\infty}^{-1} (a^{-1}z^{-1})^{n} + \sum_{n=0}^{\infty} (az^{-1})^{n}$

$= \sum_{n=-1}^{-\infty} (a^{-1}z^{-1})^{-n} + \sum_{n=0}^{\infty} (az^{-1})^{n}$

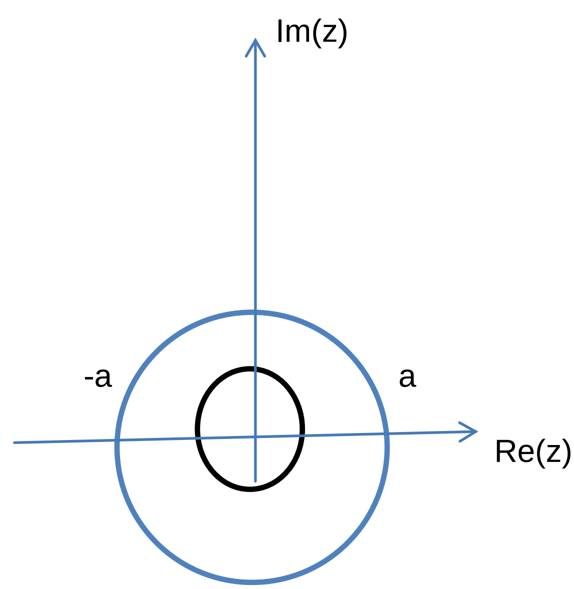
$= \sum_{n=1}^{\infty} (az)^{n} + \sum_{n=0}^{\infty} (az^{-1})^{n}$

$= \frac{az}{1-az} + \frac{az^{-1}}{1-az^{-1}}$ (since, $\sum_{n=1}^{\infty} a^{n} = \frac{a}{1-a}$ and , $\sum_{n=0}^{\infty} a^{n} = \frac{1}{1-a}$)

$$\boxed{X(Z) = \frac{az}{1-az} + \frac{a/z}{1-a/z}}$$

Region of convergence is:

$\lvert az\rvert<1$	$\lvert a/z\rvert<1$
$\lvert z\rvert<\frac{1}{\lvert a\rvert}$	$\lvert z\rvert>\lvert a\rvert$



8. Find inverse Z-transform of x(z) = $\frac{1}{(1+z^{-1})(1-z^{-1})^2}$ **using partial fraction method.[[CO4-H1-May/Jun2011]**

Solution:

Step1: Multiply &divide by z^3

$$X(z) = \frac{z^3}{z^3[(1+z^{-1})(1-z^{-1})^2]}$$

$$= \frac{z^3}{[z(1+z^{-1})][z\ (1-z^{-1})\, z\,(1-z^{-1})]}$$

$$= \frac{z^3}{(z+1)\,(z-1)(z-1)}$$

$$X(z) = \frac{z^3}{(z+1)\,(z-1)^2}$$

Step2:

$$\frac{x(z)}{z} = \frac{z^2}{(z+1)\,(z-1)^2} \longrightarrow \text{Eq.1}$$

Step3:Resolve into partial fraction

$$\frac{z^2}{(z+1)\,(z-1)^2} = \frac{A}{(z+1)} + \frac{B}{(z-1)} + \frac{C}{(z-1)^2}$$

z^2=A$(z-1)^2$ + B (z-1)(z+1) + C(z+1)

Put z=1,1=c(1+1) $\longrightarrow$ $\boxed{C=1/2}$

Put z=-1, 1=A$(-1-1)^2$ $\longrightarrow$ $\boxed{A=1/4}$

Put z=0, 0=A-B+C

$\boxed{B=3/4}$

B=A+C $\longrightarrow$ B=1/2+1/4 $\longrightarrow$

Step4: Therefore Eq.1 becomes,

$$\frac{x(z)}{z} = \frac{1/4}{(z+1)} + \frac{3/4}{(z-1)} + \frac{1/2}{(z-1)^2}$$

$$x(z) = \frac{z(\frac{1}{4})}{(z+1)} + \frac{z(\frac{3}{4})}{(z-1)} + \frac{z(\frac{1}{2})}{(z-1)^2}$$

$$x(z) = \frac{1/4}{1+z^{-1}} + \frac{3/4}{1-z^{-1}} + \frac{1/2}{(1-z^{-1})^2}$$

Step5: Taking inverse Z-transform, we get

$$\boxed{x(n)=\frac{1}{4}(1)^n u(n)+\frac{3}{4}(1)^n u(n)+\frac{1}{2}(1)^n n\, u(n)}$$

UNIT – V

Linear Time Invariant-Discrete Time Systems

Part – A

1. What is recursive and Non recursive system? [CO5-L1-May/Jun2011]

If the present output is dependent upon the present and past value of input then the system is said to be recursive system

If the present output is dependent upon the present and past value of input and past value of output then the system is said to be non recursive system.

2. What are the different types of structure realization? [CO5-L1]

i. Direct form I

ii. Direct form II

iii. Cascade form

iv. Parallel Form.

3.List the steps involved in finding convolution sum? [CO5-L1-May/Jun2013]

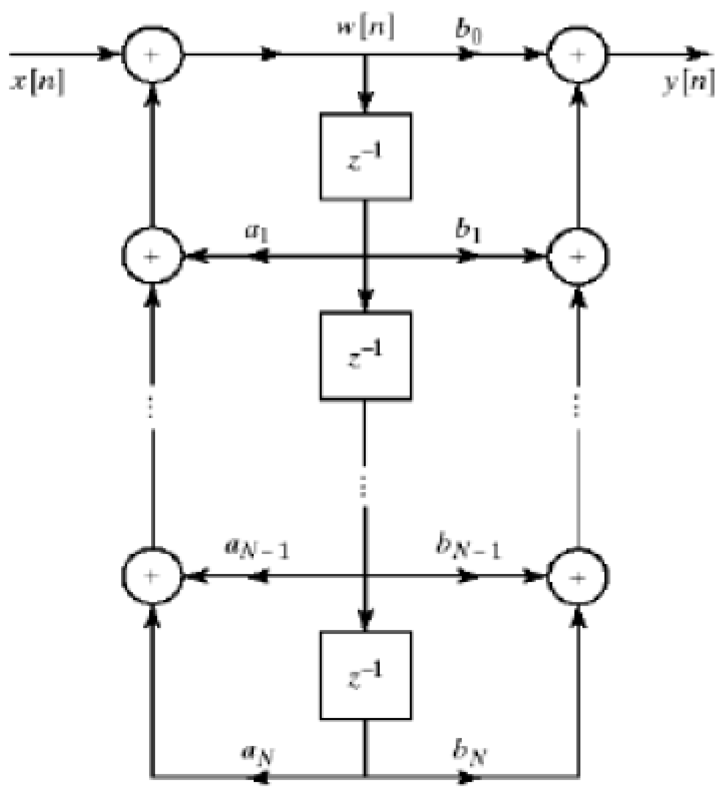
v. Folding

vi. Shifting

vii. Multiplication

viii. Summation

4. Give the direct form II structure. [CO5-L1-May/Jun2012]



5. Define LTI system is said to be causal and stable system? [CO5-L1-Nov/Dec2015]

A LTI system is causal if and only if ,h(n) = 0 for n<0.This is the sufficient and necessary condition for causality of the system.

The bounded input x(n) produces bounded output y(n) in the LTI system only if, $\sum_{k=-\infty}^{\infty} h(k) \leq \infty$. When this condition is satisfied, the system will be stable.

6. States the properties of convolution. **[CO5-L1-May/Jun2009]**

Commutative property of convolution

x(n) * h(n) = h(n) * x(n) = y(n)

Associative property of convolution

[x(n) * h1(n)] * h2(n) = x(n) * [h1(n) * h2(n)]

Distributive property of convolution

x(n) * [h1(n) + h2(n)] = x(n) * h1(n) + x(n) * h2(n)

7. If x(n) and y(n) are discrete variable functions, what is its convolution sum. [CO5-L1-May/Jun2013]

The convolution sum is, $\sum_{k=-\infty}^{\infty} x(k)h(n-k)$

8. Determine the system function of the discrete time system described by the difference equation. Y(n) = 0.5y(n-1)+x(n) [CO5-L1-May/Jun2012]

Taking z-transform of both sides,

Y(z) = 0.5Z^{-1}Y(z)+X(z)

H(z) = Y(z)/X(z) = 1/(1 – 0.5Z^{-1})

9. A causal LTI system has impulse response h(n), for which the z-transform is H(z) = (1+Z^{-1})/(1-0.5Z^{-1})(1+0.25Z^{-1}). Is the system stable? Explain. [CO5-L1-May/Jun2010]

H(z) can be written in terms of positive powers of z as follows:

H(z) = $Z(Z+1)/(Z-0.5)(Z+0.25)$

Poles are at p1 =0.5 and p2 = -0.25. Since both the poles are inside unit circle. This system is stable.

10. Check whether the system with system function H(Z) = (1/1-0.5Z^{-1})+(1/1-2Z^{-1}) with ROC |z| < 0.5 is causal and stable? [CO5-L1-Nov/Dec2013]

H(z) = $Z/(Z-0.5) + Z/(Z-2)$.

Poles of this system are located at z = 0.5 and z = 2. This system is not causal and stable, since all poles are not located inside unit circle.

11. Is the discrete time system described by the difference equation y(n) = x(-n) is causal? [CO5-L1-May/Jun2013]

Here y(-2) = x(-(-2)) = x(2). This means output at n=-2 depends upon future inputs. Hence this system is not causal.

12. Consider a system whose impulse is h(t) = e-|t|. Is this system is causal or non causal? [CO5-L1-Nov/Dec2011]

Here h(t) =$e^{|-t|}$

= e^{-t} for t $\geq$ 0

= e^{t} for t $<$ 0

Since h(t) is not equal to zero for t < 0, the system is non causal.

13. Find the step response of the system if the impulse response [CO5-H1-May/Jun2011]

$$\boldsymbol{h(n) = \delta(n-2) - \delta(n-1)}$$

Sol/n:

$$Y(n) = h(n) * u(n), since\ x(n) = u(n), step\ input$$

$$= \delta(n-2) * u(n) - \delta(n-1) * u(n)$$

$$u(n-2) - u(n-1)$$

14. Obtain the convolution of a) $\boldsymbol{x(n) * \delta(n)}$ b) x(n) $*$[$\boldsymbol{h_1(n) + h_2(n)}$] [CO5-L1]

Sol/n

a) $x(n) * \delta(n) = \delta(n)$
b) x(n) $*[h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$

15. **Consider an LTI system with impulse response h(n)= δ(n-n_0) for an input x(n), find $\boldsymbol{Ye^{j\omega}}$ [[CO5-L1]**

Here is the spectrum of output. By convolution theorem we can write,

$$Ye^{j\omega} = H(e^{j\omega})\, X(e^{j\omega})$$

Here $H(e^{j\omega}) = DTFT[\delta(n-n_0)] = e^{-j\omega n_0}$

$Ye^{j\omega} = e^{-j\omega n_0}\, X(e^{j\omega})$

16. Define Transfer function in Block diagram? [CO5-L1-May/Jun2010]

The z – transform of the output is product of Transfer function and z – transform of input which we conveniently represented by

Y (z) = H (z). X (z)

17. Define poles and zeros of a transfer function. [CO5-L1-Nov/Dec2011]

The transfer function of a system is the ratio of two polynomials. The not of the numerator polynomial are called the zeros of the transfer function and the root of the denominator polynomial are called pole's of the transfer function.

18. What is the over all impulse response h(n) when two system whose impulse responses $h_1(n)$ and $h_2(n)$ are in parallel and series? [CO5-L1-May/Jun2012] [May

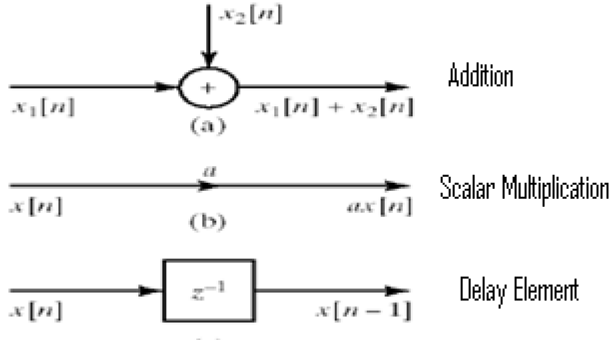
If the two system whose impulse responses $h_1(n)$ and $h_2(n)$ are in parallel, then overall impulse response is

$h(n) = h_1(n) + h_2(n)$

If the two system whose impulse responses $h_1(n)$ and $h_2(n)$ are in series, then overall impulse response is

$h(n) = h_1(n) * h_2(n)$

19. What are the elements generally used in block diagram?[CO5-L1-May/Jun2013]



20. Define FIR and IIR system? [CO5-L1]

The systems for which unit step response h(n) has finite number of terms, they are called Finite Impulse Response (FIR) systems.

The systems for which unit step response h(n) has infinite number of terms, they are called Infinite Impulse Response (IIR) systems.

21.Convolve the two sequences x(n)={1,2,3} and h(n)={5,4,6,2}[CO5-L2-Nov/Dec2014]

	1	2	3
5	5	10	15
4	4	8	12
6	6	12	18
2	2	4	12

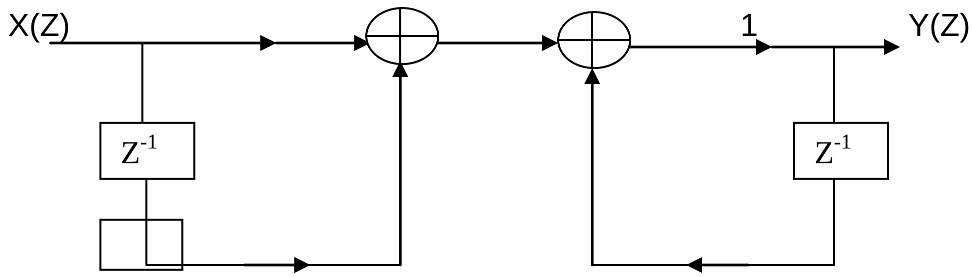
Y(n)={5,14,29,26,12}

22. Write expression for General Transfer function of system realization. [CO5-L1]

$$H(z) = \frac{y(z)}{x(z)} = \frac{\sum_{n=0}^{N} b_n z^{-n}}{1 - \sum_{n=1}^{M} a_n z^{-n}}$$

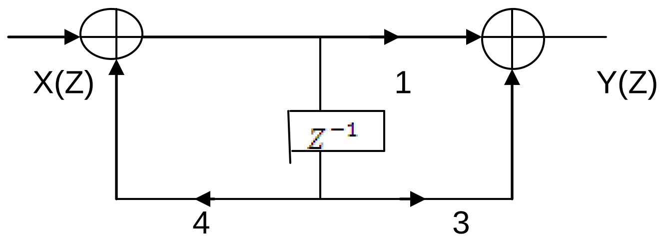
23. Draw the Direct form I realization for the Transfer function[CO5-L1]

$$H(z) = \frac{1 - z^{-1}}{1 + 2z^{-1}}$$



24. Draw direct form II (non-canonic) structure for transfer function [CO5-L1]

$$H(z) = \frac{1 + 3z^{-1}}{1 - 4z^{-1}}$$



25.How many number of addition, multiplications and memory locations are required to realize Direct Form-I and Direct form-II realizations. [CO5-L1-May/Jun2014]

Direct Form-I	Direct form-II
Addition: M+N	M+N
Multiplications:M+N+1	M+N+1
Delay:M+N	Max{M,N}

Part – B

1.Realize Parellel form block diagram of the follow system function [CO5-H1-May/Jun2011]

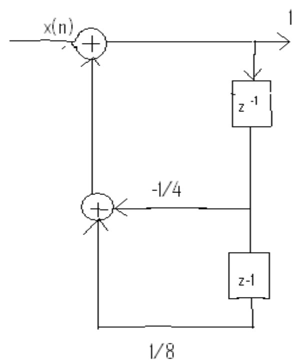
H(z)=$\frac{1}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$

Solution:

1.Direct form relalization

H(z)=$\frac{1}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$

H(z)=$\frac{1}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$

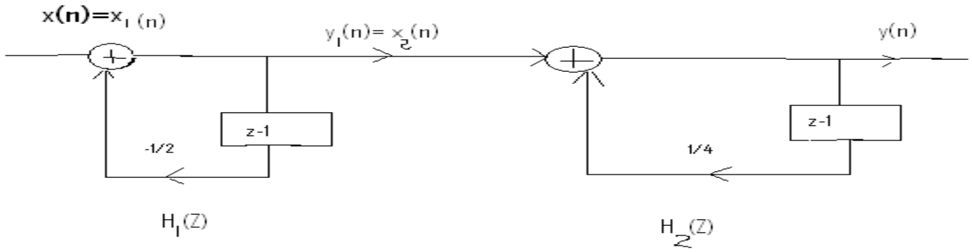


2.cascade form realization

$$H(Z)=\frac{1}{\left(1+\frac{1}{2}z^{-1}\right)}\cdot\frac{1}{\left(1-\frac{1}{4}z^{-1}\right)}$$

$$H_1(Z)=\frac{1}{\left(1+\frac{1}{2}z^{-1}\right)}\ ,\ H_2(Z)=\frac{1}{\left(1-\frac{1}{4}z^{-1}\right)}$$

H(Z)=$H_1(Z).H_2(Z)$



H(Z)= $\frac{1}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$

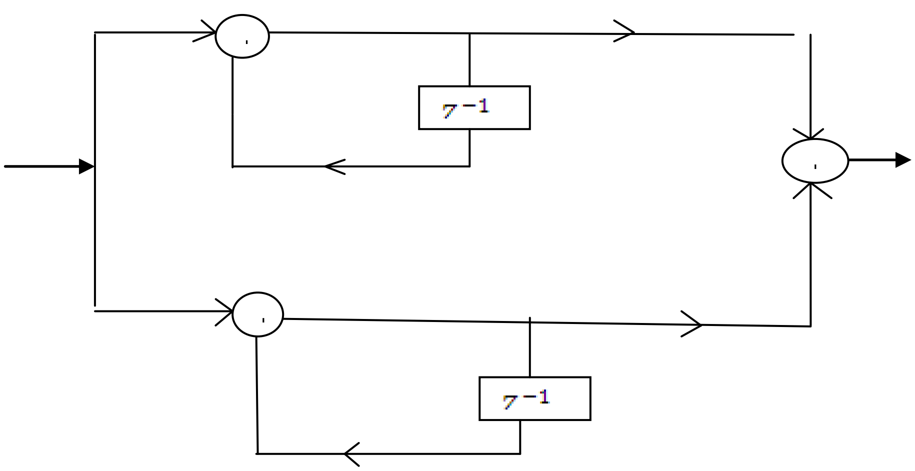
$$=\frac{z^2}{\left(z+\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$$

$$\frac{H(Z)}{Z}=\frac{Z}{\left(Z+\frac{1}{2}\right)\left(Z-\frac{1}{4}\right)}$$

$$=\frac{\frac{2}{3}}{z+\frac{1}{2}}+\frac{\frac{1}{3}}{z-\frac{1}{4}}$$

H(Z)= $=\frac{\frac{2}{3}}{1+\frac{1}{2}Z^{-1}}+\frac{\frac{1}{3}}{1-\frac{1}{4}Z^{-1}}$

H(Z)= $H_1(Z)+H_2(Z)$



2. Determine the response y(n),n>=0 of the system describe by the second order difference equation Y(n)-3(n-1)-4y(n-2)=x(n)+x(n-1) when the input sequence is X(n)=$4^n u(n)$ [CO5-H1-Nov/Dec2014]

SOLUTION:

Taking as a z transform

Y(z)[1-4$z^{-1}+4z^{-2}] = x(z)$[1-z^{-1}]

$=\frac{1}{(1-4z^{-1})}$(1-$z^{-1})$

Y(z)=$\frac{1-z^{-1}}{(1-4z^{-1})(1-4z^{-1}+4z^{-2})}$

$$\frac{y(z)}{z} = \frac{z(z-1)}{(z-4)(z^2-4z+4)}$$

$\frac{y(z)}{z}=\frac{z(z-1)}{(z-4)(z-2)^2}$

$\frac{y(z)}{z} = \frac{A}{(Z-4)} + \frac{B}{Z-2} + \frac{C}{(Z-2)^2}$

A=$\frac{Y(Z)}{Z}$. $Z-4$ $\Big|$ Z=4

$=\frac{4(3)}{(4-2)^2}$

A=3

B=$\frac{d}{dz}\left[\left(\frac{y(z)}{z}\right)(z-2)^2\right]\Big|$ z=2

$=\frac{(2z-1)(z-4)(z^2-z)}{(z-4)^2}\Big|$ z=2

B=$\frac{z^2-8z+4}{(z-4)^2}$ z=2

B=-2

C =$\frac{y(z)(z-2)^2}{z}\Bigg|_{z=2}$

C=-1

$$\frac{y(z)}{z}=\frac{3}{(Z-4)}+\frac{-2}{Z-2}+\frac{-1}{(Z-2)^2}$$

$$y(z)=\frac{3z}{(Z-4)}-\frac{2z}{Z-2}+\frac{-z}{(Z-2)^2}$$

Taking inverse z transform

Y(n)=[$3(4)^n-2(2)^n-\frac{1}{2}n(2)^n$]u(n)

Y(n)=[$3(4)^n-2(2)^n-n(2)^{n-1}$]u(n)

3. Find the impulse response and step response for the following systems. $y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=x(n)$ [CO5-H1-May/Jun2015]

Given

$$y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=x(n)$$

Taking z – transform on both side, we get

$$y(z)-\frac{3}{4}z^{-1}y(z)+\frac{1}{8}z^{-2}y(z)=x(z)$$

$$\frac{y(z)}{x(z)}=\frac{1}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} \qquad \rightarrow(1)$$

For impulse response

x(n) = δ(n)

x(z) = 1 → (2)

(2) in (1) we get

$$y(z) = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

$$y(z) = \frac{z^2}{z^2 - \frac{3}{4}z + \frac{1}{8}}$$

$$\frac{y(z)}{z} = \frac{z}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)}$$

$$\frac{y(z)}{z} = \frac{A}{z - \frac{1}{2}} + \frac{B}{z - \frac{1}{4}}$$

$$A = \left(z - \frac{1}{2}\right) \frac{z}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)} \Bigg/ z = \frac{1}{2}$$

$$\boxed{A = 2}$$

$$B = \left(z - \frac{1}{4}\right) \frac{z}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)} \Bigg/ z = \frac{1}{4}$$

$$\boxed{B = -1}$$

$$\frac{y(z)}{z} = \frac{2}{z - \frac{1}{2}} - \frac{1}{z - \frac{1}{4}}$$

$$y(z) = \frac{2z}{z-\frac{1}{2}} - \frac{z}{z-\frac{1}{4}}$$

Taking inverse z- transform we get

$$y(n) = 2\left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{4}\right)^n u(n)$$

Step response

$$x(n) = x(n)$$

$$x(z) = \frac{z}{z-1} \qquad \rightarrow (3)$$

(3) in (1) we get

$$y(z) = \frac{1}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} \cdot \left(\frac{z}{z-1}\right)$$

$$y(z) = \frac{z^2}{\left(z^2-\frac{3}{4}z+\frac{1}{8}\right)} \cdot \left(\frac{z}{z-1}\right)$$

$$\frac{y(z)}{z} = \frac{z^2}{(z-1)\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$$

$$\frac{y(z)}{z} = \frac{A}{z-1} + \frac{B}{z-\frac{1}{2}} + \frac{C}{z-\frac{1}{4}}$$

$$A = (z-1)\frac{z^2}{(z-1)\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\Bigg/_{z=1} = \frac{8}{3}$$

$$B = \left(z - \frac{1}{2}\right) \frac{z^2}{(z-1)\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} \Bigg/_{z=\frac{1}{2}} = -2$$

$$C = \left(z - \frac{1}{4}\right) \frac{z^2}{(z-1)\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} \Bigg/_{z=\frac{1}{4}} = \frac{1}{3}$$

$$\frac{y(z)}{z} = \frac{8}{3(z-1)} - \frac{2}{z-\frac{1}{2}} + \frac{1}{3\left(z-\frac{1}{4}\right)}$$

$$y(z) = \frac{8z}{3(z-1)} - \frac{2z}{z-\frac{1}{2}} + \frac{z}{3\left(z-\frac{1}{4}\right)}$$

$$\boxed{y(n) = \frac{8}{3}u(n) - 2\left(\frac{1}{2}\right)^n u(n) + \frac{1}{3}\left(\frac{1}{4}\right)^n u(n)}$$

4. Find the output y(n) of a linear time invariant discrete time system specified by the equation. $\mathbf{y(n) - \frac{3}{2}y(n-1) + \frac{1}{2}y(n-2) = 2x(n) + \frac{3}{2}x(n-1)}$ **If the initial condition are y(-1) = 0, y(-2) = 1 and input** $\mathbf{x(n) = \left(\frac{1}{4}\right)^n u(n)}$ **[CO5-H1-Nov/Dec2011]**

Given

$$y(n) - \frac{3}{2}y(n-1) + \frac{1}{2}y(n-2) = 2x(n) + \frac{3}{2}x(n-1)$$

Taking z – transform on both sides

$$y(z)-\frac{3}{2}\left[z^{-1}y(z)+y(-1)\right]+\frac{1}{2}\left[z^{-2}y(z)+z^{-1}y(-1)+y(-2)\right]$$
$$=2x(z)+\frac{3}{2}\left[z^{-1}x(z)+x(-1)\right]$$

we have

y(-1) = 0; y(-2) = 1 and x(-1) =0

$$y(z)-\frac{3}{2}z^{-1}y(z)+\frac{1}{2}\left[z^{-2}y(z)+1\right]=x(z)\left[2+\frac{3}{2}z^{-1}\right]\rightarrow(1)$$

For input

$$x(n)=\left(\frac{1}{4}\right)^{n}u(n)$$
$$x(z)=\frac{1}{1-\frac{1}{4}z^{-1}}\rightarrow(2)$$

(2) in (1) we get

$$y(z)\left[1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}\right]=\frac{-1}{2}+\frac{1}{\left(1-\frac{1}{4}z^{-1}\right)}\left(2+\frac{3}{2}z^{-1}\right)$$

$$y(z)=\frac{-1}{2\left(1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}\right)}+\frac{\left[2+\left(\frac{3}{2}\right)z^{-1}\right]}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}\right)}$$

$$y(z)=\frac{-z^{2}}{2\left(z^{2}-\frac{3}{2}z+\frac{1}{2}\right)}+\frac{z^{2}\left(2z+\frac{3}{2}\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}\right)}$$

$$y(z)=\frac{-z^{2}}{2(z-1)\left(z-\frac{1}{2}\right)}+\frac{z^{2}\left(2z+\frac{3}{2}\right)}{\left(z-\frac{1}{4}\right)(z-1)\left(z-\frac{1}{2}\right)}$$

$$y(z) = y_1(z) + y_2(z)$$

$$y_1(z) = \frac{-z^2}{2(z-1)\left(z-\frac{1}{2}\right)}$$

$$\frac{y_1(z)}{z} = \frac{-z}{2(z-1)\left(z-\frac{1}{2}\right)}$$

$$\frac{y_1(z)}{z} = \frac{A}{(z-1)} + \frac{B}{\left(z=\frac{1}{2}\right)}$$

$$y_2(z) = \frac{z^2\left(2z+\frac{3}{2}\right)}{\left(z-\frac{1}{4}\right)(z-1)\left(z-\frac{1}{2}\right)}$$

$$\frac{y_2(z)}{z} = \frac{z\left(2z+\frac{3}{2}\right)}{\left(z-\frac{1}{4}\right)(z-1)\left(z-\frac{1}{2}\right)}$$

$$\frac{y_2(z)}{z} = \frac{A_1}{z-\frac{1}{4}} + \frac{B_1}{z-1} + \frac{C_1}{z-\frac{1}{2}}$$

$$A = (z-1).\left.\frac{-z}{2(z-1)\left(z-\frac{1}{2}\right)}\right/_{z=1}$$

$$A = \left.\frac{-z}{2\left(z-\frac{1}{2}\right)}\right/_{z=1}$$

$$\boxed{A=-1}$$

$$A_1 = \left(z-\frac{1}{4}\right)\frac{z\left(2z+\frac{3}{2}\right)}{\left(z-\frac{1}{4}\right)(z-1)\left(z-\frac{1}{2}\right)}\Bigg/_{z=\frac{1}{4}}$$

$$\boxed{A_1 = \frac{8}{3}}$$

$$B = \left(z-\frac{1}{2}\right)\frac{-z}{2(z-1)\left(z-\frac{1}{2}\right)}\Bigg/_{z=\frac{1}{2}}$$

$$B = \frac{-z}{2(z-1)}\Bigg/_{z=\frac{1}{2}}$$

$$\boxed{B = \frac{1}{2}}$$

$$B_1 = (z-1)\frac{z\left(2z+\frac{3}{2}\right)}{(z-1)\left(z-\frac{1}{4}\right)\left(z-\frac{1}{2}\right)}\Bigg/_{z=1}$$

$$\boxed{B_1 = \frac{28}{3}}$$

$$C_1 = \left(z-\frac{1}{2}\right)\frac{z\left(2z+\frac{3}{2}\right)}{(z-1)\left(z-\frac{1}{4}\right)\left(z-\frac{1}{2}\right)}\Bigg/_{z=\frac{1}{2}}$$

$$\boxed{C_1 = -10}$$

$$y_1(z) = \frac{-z}{z-1} + \frac{z}{2\left(z-\frac{1}{2}\right)}$$

$$y_1(n) = -u(n) + 0.5\left(\frac{1}{2}\right)^n u(n)$$

$$\frac{y_2(z)}{z} = \frac{8}{3\left(z-\frac{1}{4}\right)} + \frac{28}{3(z-1)} - \frac{10}{\left(z-\frac{1}{2}\right)}$$

$$y_2(z) = \frac{8z}{3\left(z-\frac{1}{4}\right)} + \frac{28z}{3(z-1)} - \frac{10z}{\left(z-\frac{1}{2}\right)}$$

$$y_2(n) = \frac{8}{3}\left(\frac{1}{4}\right)^n u(n) + \frac{28}{3}u(n) - 10\left(\frac{1}{2}\right)^n u(n)$$

$$\boxed{y(n) = \frac{25}{3}u(n) + \frac{8}{3}\left(\frac{1}{4}\right)^n u(n) - \frac{19}{2}\left(\frac{1}{2}\right)^n u(n)}$$

5. Using z – transfer determine the response of the linear time-invariant system with difference equation. $y(n) - 2r\cos\theta y(n-1) + r^2 y(n-2) = x(n)$ to an excitation x(n) = a^n u(n). [CO5-H1-May/Jun2010]

Given

$$y(n) - 2r\cos\theta y(n-1) + r^2 y(n-2) = x(n)$$

Taking z – transform and both side and applying initial conditions to zero yields.

$$y(z) - 2r\cos\theta z^{-1} y(z) + r^2 z^{-2} y(z) = x(z)$$

$$\Rightarrow \frac{y(z)}{x(z)} = \frac{1}{1-2r\cos\theta z^{-1} + r^2 z^{-2}} \qquad \to (1)$$

For

$$x(n) = a^n u(n)$$

$$x(z) = \frac{1}{1-az^{-1}} \qquad \to (2)$$

(2) in (1)

$$y(z)=\frac{1}{\left(1-az^{-1}\right)\left(1-2r\cos\theta z^{-1}+r^2z^{-2}\right)}$$

$$y(z)=\frac{1}{\left(1-az^{-1}\right)\left(1-re^{j\theta}z^{-1}\right)\left(1-re^{-j\theta}z^{-1}\right)}$$

$$y(z)=\frac{z^3}{(z-a)\left(z-re^{j\theta}\right)\left(z-re^{-j\theta}\right)}$$

$$\frac{y(z)}{z}=\frac{z^2}{(z-a)\left(z-re^{j\theta}\right)\left(z-re^{-j\theta}\right)}$$

$$\frac{y(z)}{z}=\frac{A}{(z-a)}+\frac{B}{\left(z-re^{j\theta}\right)}+\frac{C}{\left(z-re^{-j\theta}\right)}$$

$$A=(z-a)\left.\frac{z^2}{(z-a)\left(z-re^{j\theta}\right)\left(z-re^{-j\theta}\right)}\right/_{z=a}$$

$$A=\frac{a^2}{a^2-2ar\cos\theta+r^2}$$

$$B=\left(z-re^{j\theta}\right)\left.\frac{z^2}{(z-a)\left(z-re^{j\theta}\right)\left(z-re^{-j\theta}\right)}\right/_{z=re^{j\theta}}$$

$$B=\frac{r^2e^{j2\theta}}{\left(re^{j\theta}-a\right)\left(re^{j\theta}-re^{-j\theta}\right)}$$

$$C=B^*=\frac{r^2e^{-j2\theta}}{\left(re^{-j\theta}-a\right)\left(re^{-j\theta}-re^{j\theta}\right)}$$

$$y(z)=\frac{a^2}{a^2-2ar\cos\theta+r^2}\left(\frac{z}{z-a}\right)+\qquad\frac{r^2e^{j2\theta}}{\left(re^{j\theta}-a\right)\left(re^{j\theta}-re^{-j\theta}\right)}\left(\frac{z}{z-re^{j\theta}}\right)+$$

$$\frac{r^2 e^{-j2\theta}}{(re^{-j\theta}-a)(re^{-j\theta}-re^{j\theta})}\left(\frac{z}{z-re^{-j\theta}}\right)$$

$$y(n)=\frac{a^2}{a^2-2ar\cos\theta+r^2}a^n u(n)+\frac{r^{n+1}}{\sin\theta}\left[\frac{r\sin(n+1)\theta-a\sin(n+2)\theta}{a^2-2ar\cos\theta+r^2}\right]u(n)$$

6. Find the impulse response of the difference equation Y(n)-4y(n-1)+3y(n-2)=x(n)+2x(n-1) [CO5-H1-May/Jun2015]

Solution:

Given difference equation

Y(n)-4y(n-1)+3y(n-2)=x(n)+2x(n-1)

Taking z transform of difference equation

Y(z)$-4z^{-1}y(z)+3z^{-2}y(z)=x(z)+2z^{-1}x(z)$

$y(z)[1-4z^{-1}+3z^{-2}]=x(z)[1+2z^{-1}]$

H(Z)=$\frac{Y(Z)}{X(Z)}=\frac{1+2Z^{-1}}{1-4Z^{-1}+3Z^{-2}}$

$$\frac{H(Z)}{Z}=\frac{Z+2}{Z^2-4Z+3}$$

$$\frac{H(Z)}{Z}=\frac{Z+2}{(Z-3)(Z-1)}$$

Using partial fraction method

$$\frac{H(Z)}{Z}=\frac{A}{Z-3}+\frac{B}{Z-1}$$

A=$\frac{Z+2}{(Z-3)(Z-1)}(Z-3)\Big|_{Z=3}$

=$\frac{3+2}{3-1}=\frac{5}{2}$

A=$\frac{5}{2}$

$B = \frac{z+2}{(z-3)(z-1)}(Z-1)\Big|_{Z=1}$

$= \frac{1+2}{1-3} = -\frac{3}{2}$

$B = -\frac{3}{2}$

$H(Z) = \frac{\frac{5}{2}}{1-3z^{-1}} - \frac{\frac{3}{2}}{1-z^{-1}}$

Taking inverse z transform

$h(n) = -\frac{5}{2}(3)^n u(n) - \frac{3}{2}u(n)$

7. Find the response of y(n) + y(n-1) – 2y(n-2) = u(n-1) +2u(n-2) due to y(-1) = 0.5; y(-2) = 0.25. [CO5-H1-Nov/Dec2013]

Given

Y(n) + y (n-1) – 2 y(n-2) = u (n-1) +2u(n-2)

Take z – transform on both sides

$$y(z) + \left[z^{-1}y(z) + y(-1)\right] - 2\left[z^{-2}y(z) + z^{-1}y(-1) + y(-2)\right]$$

$$= \frac{z^{-1}}{1-z^{-1}} + \frac{2z^{-2}}{1-z^{-1}}$$

$$y(z)\left[1+z^{-1}-2z^{-2}\right] = z^{-1} + \frac{z^{-1}}{1-z^{-1}} + \frac{2z^{-2}}{1-z^{-1}} = \frac{z}{(z+2)(z-1)} + \frac{z(z+2)}{(z-1)(z^2+z-2)}$$

$$y(z) = \frac{z^{-1}}{1+z^{-1}-2z^{-2}} + \frac{z^{-1}+2z^{-2}}{(z-1)(z^2+z-2)}$$

$$y(z) = y_1(z) + y_2(z)$$

$$y_1(z) = \frac{z}{(z+2)(z-1)}$$

$$\frac{y_1(z)}{z} = \frac{1}{(z+2)(z-1)}$$

$$\frac{y_1(z)}{z}=\frac{A}{z+2}+\frac{B}{z-1}$$

$$A=(z+2)\left.\frac{1}{(z+2)(z-1)}\right/_{z=-2}$$

$$A=\left.\frac{1}{z-1}\right/_{z=-2}$$

$$\boxed{A=-\frac{1}{3}}$$

$$B=(z-1)\left.\frac{1}{(z+2)(z-1)}\right/_{z=1}$$

$$B=\left.\frac{1}{z+2}\right/_{z=1}$$

$$\boxed{B=\frac{1}{3}}$$

$$\frac{y_1(z)}{z}=\frac{-1}{3(z+2)}+\frac{1}{3(z-1)}$$

$$y_1(z)=\frac{-z}{3(z+2)}+\frac{z}{3(z-1)}$$

$$y_1(n)=\frac{-1}{3}(-2)^n u(n)+\frac{1}{3}u(n)$$

$$y_2(z)=\frac{z(z+2)}{(z-1)(z^2+z-2)}$$

$$y_2(z)=\frac{z(z+2)}{(z-1)(z-1)(z+2)}$$

$$y_2(z)=\frac{z}{(z-1)^2}$$

$$y_2(n) = nu(n)$$

$$\boxed{y(n) = \frac{1}{3}u(n) + nu(n) - \frac{1}{3}(-2)^n u(n)}$$

8. Solve the following difference equation for y (n) using z – transform and the specified initial condition. [CO5-H1-Nov/Dec2011]

$$\mathbf{y(n) - y(n-1) + \frac{1}{4}y(n-2) = x(n) \quad n \geq 0}$$

$$\mathbf{\text{where } x(n) = 2\left(\frac{1}{8}\right)^n ; y(-1) = 2 \text{ and } y(-2) = 4}$$

Given

$$y(n) - y(n-1) + \frac{1}{4}y(n-2) = x(n)$$

Taking z – transform on both sides.

$$y(z) - \left[z^{-1}y(z) + y(-1)\right] + \frac{1}{4}\left[z^{-2}y(z) + z^{-1}y(-1) + y(-2)\right] = x(z)$$

$$y(z)\left[1 - z^{-1} + \frac{1}{4}z^{-2}\right] - 2 + \frac{1}{2}z^{-1} + 1 = x(z)$$

$$y(z)\left[1 - z^{-1} + \frac{1}{4}z^{-2}\right] = 1 - 0.5z^{-1} + x(z) \qquad \rightarrow (1)$$

Given

$$x(n) = 2\left(\frac{1}{8}\right)^n u(n)$$

$$x(z) = 2\left(\frac{1}{1 - \frac{1}{8}z^{-1}}\right) \qquad \rightarrow (2)$$

(2) in (1)

$$y(z)\left[1-z^{-1}+\frac{1}{4}z^{-2}\right]=1-0.5z^{-1}+\frac{2}{1-\frac{1}{8}z^{-1}}$$

$$y(z)=\frac{1-0.5z^{-1}}{1-z^{-1}+\frac{1}{4}z^{-2}}+\frac{2}{\left(1-z^{-1}+\frac{1}{4}z^{-2}\right)\left(1-\frac{1}{8}z^{-1}\right)}$$

$$y(z)=\frac{z(z-0.5)}{z^2-z+\frac{1}{4}}+\frac{2z^3}{\left(z^2-z+\frac{1}{4}\right)\left(z-\frac{1}{8}\right)}$$

$$y(z)=\frac{z(z-0.5)}{\left(z-\frac{1}{2}\right)^2}+\frac{2z^3}{\left(z-\frac{1}{2}\right)^2\left(z-\frac{1}{8}\right)}$$

$$y(z)=\frac{z}{\left(z-\frac{1}{2}\right)}+\frac{2z^3}{\left(z-\frac{1}{2}\right)^2\left(z-\frac{1}{8}\right)}$$

$$y(z)=y_1(z)+y_2(z)$$

$$y_1(z)=\frac{z}{z-\frac{1}{2}}$$

$$y_1(n)=\left(\frac{1}{2}\right)^n u(n)$$

$$y_2(n)=\frac{2z^3}{\left(z-\frac{1}{2}\right)^2\left(z-\frac{1}{8}\right)}$$

$$\frac{y_2(n)}{z}=\frac{2z^3}{\left(z-\frac{1}{2}\right)^2\left(z-\frac{1}{8}\right)}$$

$$\frac{y_2(n)}{z}=\frac{A}{z-\frac{1}{2}}+\frac{B}{\left(z-\frac{1}{2}\right)^2}+\frac{C}{\left(z-\frac{1}{8}\right)}$$

$$A = \frac{d}{dz}\left(z-\frac{1}{2}\right)^2 \frac{2z^2}{\left(z-\frac{1}{2}\right)^2.\left(z-\frac{1}{8}\right)} \Bigg/ z=\frac{1}{2}$$

$$A = \frac{d}{dz} \frac{2z^2}{\left(z-\frac{1}{8}\right)} \Bigg/ z=\frac{1}{2}$$

$$A = \frac{\left(z-\frac{1}{8}\right)4z-2z^2}{\left(z-\frac{1}{8}\right)^2} \Bigg/ z=\frac{1}{2}$$

$$\boxed{A=\frac{16}{9}}$$

$$B = \left(z-\frac{1}{2}\right)^2 \frac{2z^2}{\left(z-\frac{1}{2}\right)^2\left(z-\frac{1}{8}\right)} \Bigg/ z=\frac{1}{2} \quad = \frac{4}{3}$$

$$C = \left(z-\frac{1}{8}\right) \frac{2z^2}{\left(z-\frac{1}{2}\right)^2\left(z-\frac{1}{8}\right)} \Bigg/ z=\frac{1}{8} \quad = \frac{2}{9}$$

$$Y_2(z) = \frac{16}{9}\frac{z}{z-\frac{1}{2}} + \frac{4}{3}\frac{z}{\left(z-\frac{1}{2}\right)^2} + \frac{2}{9}\frac{z}{\left(z-\frac{1}{8}\right)}$$

$$y_2(n) = \frac{16}{9}\left(\frac{1}{2}\right)^n u(n) + \left(\frac{4}{3}\right)\left(\frac{1}{2}\right)^n u(n) + \frac{2}{9}\left(\frac{1}{8}\right)^n u(n)$$

$$\boxed{y(n) = y_1(n) + y_2(n) = \frac{25}{9}\left(\frac{1}{2}\right)^n u(n) + \frac{8}{3}n\left(\frac{1}{2}\right)^n u(n) + \frac{2}{9}\left(\frac{1}{8}\right)^n u(n)}$$

9. Solve the following difference equation for y (n) using z- transform and the specified initial condition [CO5-H1-May/Jun2014]

$$\mathbf{y(n) = 0.5y(n-1) + x(n)}$$

$$\mathbf{x(n) = \cos\left(\frac{\pi}{3}n\right).u(n);\ y(-1) = 0}$$

$$y(n) = 0.5y(n-1) + x(n) \qquad n \geq 0$$

Applying z – transform on both side and substituting y (-1) = 0 yields.

$$y(z) = 0.5z^{-1}y(z) + x(z)$$

$$y(z)\left[1 - 0.5z^{-1}\right] = x(z)$$

$$\frac{y(z)}{x(z)} = \frac{1}{\left(1 - 0.5z^{-1}\right)} \qquad \rightarrow (1)$$

For

$$x(n) = \cos\frac{\pi}{3}nu(n)$$

$$x(z) = \frac{z^2 - 0.5z}{z^2 - z + 1} \qquad \rightarrow (2)$$

(2) in (1) we get

$$y(z)=\frac{z^2-0.5z}{(z^2-z+1)(1-0.5z^{-1})}$$

$$y(z)=\frac{z^2(z-0.5)}{(z^2-z+1)(z-0.5)}$$

$$y(z)=\frac{z^2}{z^2-z+1}$$

$$\frac{y(z)}{z}=\frac{z}{z^2-z+1}$$

$$\frac{y(z)}{z}=\frac{z}{\left[z-(0.5+j0.866)\right]\left[z-(0.5-j0.866)\right]}$$

$$\frac{y(z)}{z}=\frac{A}{\left[z-(0.5+j0.866)\right]}+\frac{A*}{\left[z-(0.5-j0.866)\right]}$$

$$A=\left[z-(0.5+j0.866)\right]\frac{z}{\left[z-(0.5+j0.866)\right]\left[z-(0.5-j0.866)\right]}$$

$$A=0.5-j0.288$$

IIIly B=0.5+j0.288

$$\frac{y(z)}{z}=\frac{0.5-j0.288}{\left[z-(0.5+j0.866)\right]}+\frac{[0.5+j0.288]}{\left[z-(0.5-j0.866)\right]}$$

$$y(z)=\frac{(0.5-j0.288)z}{z-(0.5+j0.866)}+\frac{(0.5+j0.288)z}{\left[z-(0.5-j0.866)\right]}$$

$$y(n)=(0.5-j0.288)(0.5+j0.866)^n u(n)+$$
$$(0.5+j0.288)(0.5-j0.866)^n u(n)$$

$$\boxed{y(n)=1.15\sin\left[\frac{\pi(n+1)}{3}\right]u(n)}$$

10.A Causal system is represented by the following difference equation $\mathbf{y(n)+\frac{1}{4}y(n-1)=x(n)+\frac{1}{2}x(n-1)}$ **[CO5-H1-May/Jun2013]**

Find the system function H(z) and give the corresponding ROC.
Find unit sample response of the system
Find the frequency response $H(e^{j\omega})$ and determine its magnitude and phase.
(a) Given

$$y(n)+\frac{1}{4}y(n-1)=x(n)+\frac{1}{2}x(n-1)$$

Applying z – transform on both sides and assuming initial condition to zero we get

$$y(z)+\frac{1}{4}z^{-1}y(z)=x(z)+\frac{1}{2}z^{-1}x(z)$$

$$y(z)\left[1+\frac{1}{4}z^{-1}\right]=x(z)\left[1+\frac{1}{2}z^{-1}\right]$$

$$\frac{y(z)}{x(z)}=\frac{1+\frac{1}{2}z^{-1}}{1+\frac{1}{4}z^{-1}}=\frac{z+\frac{1}{2}}{z+\frac{1}{4}}$$

$$H(z)=\frac{z+\frac{1}{2}}{z+\frac{1}{4}}$$

As the system is causal the ROC is $|z| > \frac{1}{4}$

(b)

For unit sample sequence x (n) = δ(n)

X(z) =1

$$H(z) = \frac{y(z)}{(1)} = \frac{z + \frac{1}{2}}{z + \frac{1}{4}}$$

$$H(z) = y(z) = \frac{z}{z + \frac{1}{4}} + \frac{\frac{1}{2}}{z + \frac{1}{4}}$$

$$y(z) = \frac{z}{z + \frac{1}{4}} + \frac{1}{2} z^{-1} \frac{z}{z + \frac{1}{4}}$$

Taking z – transform on both side we get

$$y(n) = \left(-\frac{1}{4}\right)^{n} u(n) + \frac{1}{2}\left(-\frac{1}{4}\right)^{n-1} u(n-1)$$

(c)

The pole z = - ¼ is inside the unit circle, therefore the system is stable and the Fourier transform $H(e^{j\omega})$ can be found form H(z)

$$H(e^{j\omega}) = H(z)\Big/_{z = e^{j\omega}}$$

$$H(e^{j\omega}) = \frac{z + \frac{1}{2}}{z + \frac{1}{4}}\Bigg/_{z = e^{j\omega}}$$

$$H(e^{j\omega}) = \frac{e^{j\omega} + \frac{1}{2}}{e^{j\omega} + \frac{1}{4}}$$

$$H(e^{j\omega}) = \frac{\cos\omega + j\sin\omega + \frac{1}{2}}{\cos\omega + j\sin\omega + \frac{1}{4}}$$

$$\left|H\left(e^{j\omega}\right)\right| = \left[\frac{\left(\cos\omega + \frac{1}{2}\right)^2 + \sin^2\omega}{\left(\cos\omega + \frac{1}{4}\right)^2 + \sin^2\omega}\right]^{1/2}$$

$$\left|H\left(e^{j\omega}\right)\right| = \left[\frac{\cos^2\omega + \cos\omega + \frac{1}{4} + \sin^2\omega}{\cos^2\omega + \frac{1}{2}\cos\omega + \frac{1}{16} + \sin^2\omega}\right]^{1/2}$$

$$\left|H\left(e^{j\omega}\right)\right| = \left[\frac{\frac{5}{4} + \cos\omega}{\frac{17}{16} + 0.5\cos\omega}\right]^{1/2}$$

$$\underline{H\left(e^{j\omega}\right)} = \tan^{-1}\frac{\sin\omega}{0.5 + \cos\omega} - \tan^{-1}\frac{\sin\omega}{0.25 + \cos\omega}$$

11. Find the output of the system whose input and output are related by y (n) = 7 y (n-1) – 12y (n-2) +2x (n) –x (n-2) for input x (n) = u (n). [CO5-H1-May/Jun2015] [May2015]

y (n) = 7 y (n-1) – 12y (n-2) +2x (n) –x (n-2)

Taking z – transform on both sides with zero initial condition, we get

$$y(z) = 7z^{-1}y(z) - 12z^{-1}y(z) + 2x(z) - z^{-2}x(z)$$

$$y(z)\left[1 - 7z^{-1} + 12z^{-2}\right] = x(z)\left[2 - z^{-2}\right]$$

$$H(z) = \frac{y(z)}{x(z)} = \frac{2 - z^{-2}}{1 - 7z^{-1} + 12z^{-2}}$$

$$\frac{y(z)}{x(z)} = \frac{2z^2 - 1}{z^2 - 7z + 12} \qquad \rightarrow (1)$$

For input x(n) = u(n)

$$\Rightarrow x(z) = \frac{z}{z-1} \qquad \rightarrow (2)$$

(2) in (1) we get

$$y(z) = \frac{z\left(2z^2-1\right)}{\left(z^2-7z+12\right)(z-1)}$$

$$\frac{y(z)}{z} = \frac{2z^2-1}{(z-1)(z-4)(z-3)}$$

$$= \frac{A}{z-1} + \frac{B}{z-4} + \frac{C}{z-3}$$

$$A = (z-1)\left.\frac{\left(2z^2-1\right)}{(z-1)(z-4)(z-3)}\right/_{z=1} = \frac{1}{6}$$

$$B = (z-4)\left.\frac{\left(2z^2-1\right)}{(z-1)(z-4)(z-3)}\right/_{z=4} = \frac{31}{3}$$

$$C = (z-3)\left.\frac{\left(2z^2-1\right)}{(z-1)(z-4)(z-3)}\right/_{z=3} = \frac{-17}{2}$$

$$y(z) = \frac{1}{6}\left(\frac{z}{z-1}\right) + \frac{31}{3}\left(\frac{z}{z-4}\right) - \frac{17}{2}\left(\frac{z}{z-3}\right)$$

$$\boxed{y(n) = \frac{1}{6}u(n) + \frac{31}{3}(4)^n u(n) - \frac{17}{2}(3)^n u(n)}$$

12.Find the impulse response of the system whose input and output relation is given by y (n) – 4 y (n-1) +3 y (n-2) = x (n) +2x (n-1). [CO5-H1-Nov/Dec2014]

Given

y (n) – 4 y (n-1) +3 y (n-2) = x (n) +2x (n-1)

Taking z – transform an both sides

$$y(z)-4z^{-1}y(z)+3z^{-2}y(z)=x(z)+2z^{-1}x(z)$$

$$\frac{y(z)}{x(z)}=\frac{1+2z^{-1}}{1-4z^{-1}+3z^{2}}$$

$$\frac{y(z)}{x(z)}=\frac{z(z+2)}{\left(z^{2}-4z+3\right)}$$

$$\frac{y(z)}{x(z)}=\frac{z(z+2)}{(z-1)(z-3)} \qquad \rightarrow (1)$$

For a impulse

$$X(n) = \delta(n)$$

$$X(z)=1 \qquad \rightarrow (2)$$

(2) in (1)

$$y(z)=\frac{z(z+2)}{(z-1)(z-3)}$$

$$\frac{y(z)}{z}=\frac{z+2}{(z-2)(z-3)}$$

$$\frac{y(z)}{z}=\frac{A}{z-1}+\frac{B}{z-3}$$

$$\frac{y(z)}{z}=\frac{-3}{2(z-1)}+\frac{5}{2(z-3)}$$

$$y(z)=\frac{-3z}{2(z-1)}+\frac{5z}{2(z-3)}$$

$$\boxed{y(n)=-\frac{3}{2}u(n)+\frac{5}{2}(3)^{n}u(n)}$$

13. Consider a causal and stable. LTI system whose input x(n) and output y(n) are related through the second – order difference equation. [CO5-H1-May/Jun2013]

$$\mathbf{y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)}$$

(a) Determine the frequency response $H(e^{j\omega})$ for system
(b) Determine the impulse response h(n) for the system.

Given

$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)$$

Taking Fourier transform on both side

$$Y(e^{j\omega}) - \frac{1}{6}e^{-j\omega}y(e^{j\omega}) - \frac{1}{6}e^{-j2\omega}y(e^{j\omega}) = x(e^{j\omega})$$

$$Y(e^{j\omega})\left[1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-j2\omega}\right] = x(e^{j\omega})$$

The frequency response

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-j\omega 2}}$$

$$H(e^{j\omega}) = \frac{e^{j2\omega}}{e^{j2\omega} - \frac{1}{6}e^{j\omega} - \frac{1}{6}}$$

$$\frac{H(e^{j\omega})}{e^{j\omega}} = \frac{e^{j\omega}}{\left(e^{j\omega} - \frac{1}{2}\right)\left(e^{j\omega} + \frac{1}{3}\right)}$$

$$= \frac{A}{\left(e^{j\omega} - \frac{1}{2}\right)} + \frac{B}{\left(e^{j\omega} + \frac{1}{3}\right)} \qquad = \frac{3}{5}.\frac{1}{e^{j\omega} - \frac{1}{2}} + \frac{2}{5}.\frac{1}{e^{j\omega} + \frac{1}{3}}$$

$$H(e^{j\omega}) = \frac{3}{5}\frac{e^{j\omega}}{e^{j\omega} - \frac{1}{2}} + \frac{2}{5}\frac{e^{j\omega}}{e^{j\omega} + \frac{1}{3}}$$

impulse response $h(n) = DTFT^{-1}\left[H(e^{j\omega})\right]$

$$\boxed{h(n) = \frac{3}{5}\left(\frac{1}{2}\right)^n u(n) + \frac{2}{5}\left(-\frac{1}{3}\right)^n u(n)}$$

14. Obtain the cascade form realization of the system described by the difference equation. $\mathbf{y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + 3x(n-1) + 2x(n-2)}$ **[CO5-H1-Nov/Dec2013]**

Given

$$y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + 3x(n-1) + 2x(n-2)$$

Taking z – transform on both sides we get

$$y(z) - \frac{1}{4}z^{-1}y(z) - \frac{1}{8}z^{-2}y(z) = x(z) + 3z^{-1}x(z) + 2z^{-1}x(z)$$

$$y(z)\left[1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}\right] = x(z)\left[1 + 3z^{-1} + 2z^{-2}\right]$$

$$\frac{y(z)}{x(z)} = \frac{1 + 3z^{-1} + 2z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

$$H(z) = \frac{\left(1 + z^{-1}\right)\left(1 + 2z^{-1}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$$

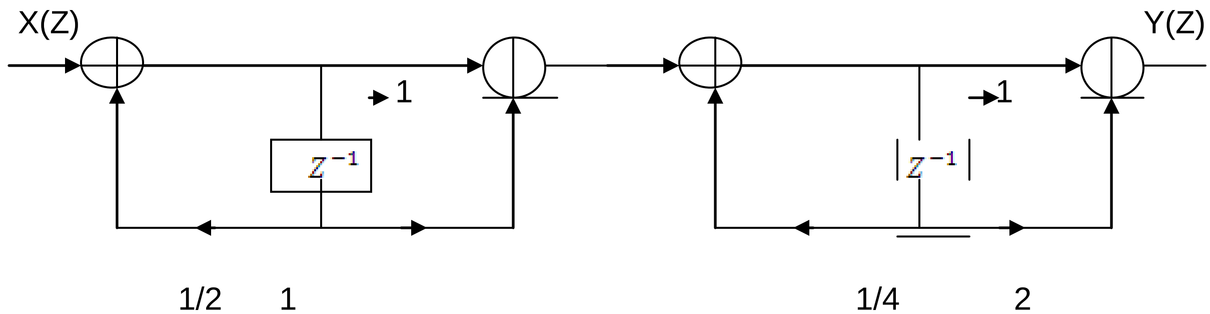
$$H(z) = H_1(z).H_2(z)$$

where

$$H_1(z) = \frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}};$$

$$H_2(z) = \frac{1+2z^{-1}}{1+\frac{1}{4}z^{-1}}$$

Now the cascade realization of H(z) can be obtained by cascading Fig (1) and Fig (2)



15. Obtain parallel form realization of the system described by the difference equation. [CO5-H1-Nov/Dec2015]

$$\mathbf{y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}(n-2) = x(n) + 3x(n-1) + 2x(n-2)}$$

$$y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + 3x(n-1) + 2x(n-2)$$

Taking z – transform on both sides we get

$$y(z) - \frac{1}{4}z^{-1}y(z) - \frac{1}{8}z^{-2}y(z) = x(z) + 3z^{-1}x(z) + 2z^{-2}x(z)$$

$$\frac{y(z)}{x(z)} = \frac{1+3z^{-1}+2z^{-2}}{1-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$

$$\begin{array}{r|l} & 16 \\ \hline \frac{1}{8}z^{-2}-\frac{1}{4}z^{-1}+1 & 2z^{-2}+3z^{-1}+1 \\ & 2z^{-2}-4z^{-1}+16 \\ & (-)\quad(+)\quad(-) \\ \hline & \quad 7z^{-1}-15 \end{array}$$

$$H(z)=16+\frac{-15+7z^{-1}}{1+3z^{-1}+2z^{-2}}$$

$$H(z)=16+\frac{-15+7z^{-1}}{\left(1-z^{-1}\right)\left(1+2z^{-1}\right)}$$

$$H(z)=16+\frac{22}{1+z^{-1}}-\frac{37}{1+2z^{-1}}\rightarrow(1)$$