SKP Engineering College

Tiruvannamalai – 606611

A Course Material

on

Communication Theory

By

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Quality Certificate

This is to Certify that the Electronic Study Material

Subject Code: EC6402

Subject Name: Communication Theory

Year/Sem: II/IV

Being prepared by me and it meets the knowledge requirement of the University curriculum.

Signature of the Author

Name: K.Vijayalakshmi

Designation: Assistant Professor

This is to certify that the course material being prepared by Ms. K.Vijayalakshmi is of the adequate quality. He has referred more than five books and one among them is from abroad author.

Signature of HD

Name:

Seal:

Signature of the Principal

Name: Dr.V.Subramania Bharathi

Seal:

EC6402 COMMUNICATION THEORY LTPC 3 0 0 3

UNIT I AMPLITUDEMODULATION 9

Generation and detection of AM wave-spectra-DSBSC,HilbertTransform,Pre-envelope &complex envelope -SSB and VSB –comparison -Superheterodyne Receiver.

UNIT II ANGLE MODULATION 9

Phase and frequencymodulation-NarrowBandandWindbandFM-Spectrum-FM modulation anddemodulation– FMDiscriminator- PLLas FM Demodulator - Transmission bandwidth.

UNIT III RANDOMPROCESS 9

Randomvariables,CentrallimitTheorem,RandomProcess,StationaryProcesses,Mean,Correlation&Covariance functions,PowerSpectralDensity,Ergodic Processes,GaussianProcess,Transmission of a RandomProcess Through a LTI filter.

UNIT IV NOISE CHARACTERIZATION 9

Noise sources and types–Noise figure and noise temperature– Noise in cascaded systems.Narrowband noise–PSD of in-phase and quadraturenoise–Noise performance in AM systems–Noise performance in FM systems–Pre-emphasis and de-emphasis–Capture effect,thresholdeffect.

UNIT V INFORMATION THEORY 9

Entropy-Discrete Memoryless channels-ChannelCapacity-Hartley-Shannonlaw-Source coding theorem-Huffman &Shannon-Fanocodes

TOTAL: 45 PERIODS

OUTCOMES:Atthe end ofthe course,the students would

- □ Designamcommunicationsystems.
- □ Design Angle Modulatedcommunication Systems
- □ Applythe Concepts Ofrandomprocess To The Design Of Communicationsystems
- □ Analyze The Noise Performance Ofam Andfmsystems

TEXT BOOKS:

1. J.G.Proakis,M.Salehi,—FundamentalsofCommunicationSystems‖,PearsonEducation

2006.

2. S.Haykin,—DigitalCommunications‖,JohnWiley,2005.

REFERENCES:

1. B.P.Lathi,—ModernDigitalandAnalog CommunicationSystems,3rdEdition, Oxford UniversityPress, 2007.

2. B.Sklar,—DigitalCommunications FundamentalsandApplications,2ndEditionPearson Education 2007

3. HPHsu,SchaumOutlineSeries-—AnalogandDigitalCommunicationsTMH2006

4. Couch.L.,"ModernCommunicationSystems",Pearson, 2001.

CONTENTS

Unit – I
Amplitude Modulation Systems
PART – A

1. Define modulation?[CO1-L1-Nov/Dec2010]

Modulation is a process by which some characteristics of high frequency carrier signal is varied
in accordance with the instantaneous value of the modulating signal.

2. What are the types of analog modulation? [CO1-L1-Nov/Dec2012]

Amplitude modulation.
Angle Modulation
Frequency modulation
Phase modulation.

3. Define depth of modulation. [CO1-L1]

It is defined as the ratio between message amplitude to that of carrier amplitude.
m=Em/Ec

4. What are the degrees of modulation?

- Under modulation. m<1
- Critical modulation m=1
- Over modulation m>1

5. What is the need for modulation?[CO1-L1-May/June2010]

- Needs for modulation:
- Ease of transmission

- Multiplexing
- Reduced noise
- Narrow bandwidth
- Frequency assignment
- Reduce the equipments limitations

6. What are the types of AM modulators? [CO1-L2-Nov/Dec2009]

There are two types of AM modulators. They are

- Linear modulators
- Non-linear modulators

Linear modulators are classified as follows

- Transistor modulator

There are three types of transistor modulator.

- Collector modulator
- Emitter modulator
- Base modulator
- Switching modulators

Non-linear modulators are classified as follows

- Square law modulator
- Product modulator
- Balanced modulator

7. Compare the difference between high level and low level modulation? [CO1-L2-Nov/Dec2011]

In high level modulation, the modulator amplifier operates at high power levels and delivers power directly to the antenna. In low level modulation, the modulator amplifier performs modulation at relatively low power levels. The modulated signal is then amplified to high power level by class B power amplifier. The amplifier feeds power to antenna.

8. Define Detection (or) Demodulation. [CO1-L1-Nov/Dec2012]

Detection is the process of extracting modulating signal from the modulated carrier. Different types of detectors are used for different types of modulations.

9. Define Amplitude Modulation. [CO1-L1-May/June2013]

In amplitude modulation, the amplitude of a carrier signal is varied according to variations in amplitude of modulating signal.

The AM signal can be represented mathematically as, eAM = (Ec + Em sinωmt) sinωct and the modulation index is given as,m = Em /EC (or) Vm/Vc

10. What is Super Heterodyne Receiver? [CO1-L1-Nov/Dec2010]

The super heterodyne receiver converts all incoming RF frequencies to a fixed lower frequency, called intermediate frequency (IF). This IF is then amplitude and detected to get the original signal.

11. What is single tone and multi tone modulation? [CO1-L1-May/June2013]

If modulation is performed for a message signal with more than one frequency component then the modulation is called multi tone modulation.

If modulation is performed for a message signal with one frequency component then the modulation is called single tone modulation.

12. Compare AM with DSB-SC and SSB-SC. [CO1-L2-Nov/Dec2014]

S.No	AM signal	DSB	SC SSB
1	Bandwidth = 2fm	Bandwidth = 2fm	Bandwidth = fm
2	Contains USB,LSB,Carrier	Contains USB,LSB	Contains USB or LSB
3	More Power is required for transmission	Power required is less than that of AM.	Power required is less than AM &DSB-SC

13. What are the advantages of VSB-AM? [CO1-L1-Nov/Dec2013]

1. It has bandwidth greater than SSB but less than DSB system.
2. Power transmission greater than DSB but less than SSB system.
3. No low frequency component lost. Hence it avoids phase distortion.

14. How will you generating DSBSC-AM?

There are two ways of generating DSBSC-AM such as

a).Balancedmodulator

b).Ring modulators.

15. What are advantages of ring modulator?

a).Its output is stable.

b). It requires no external power source to activate the diodes.

c).Virtually no maintenance.

d). Long life.

14. How will you generating DSBSC-AM? [CO1-L2]

There are two ways of generating DSBSC-AM such as

a).Balanced modulator

b).Ring modulators.

15. What are advantages of ring modulator? [CO1-L1-Nov/Dec2010]

a).Its output is stable.

b). It requires no external power source to activate the diodes.

c).Virtually no maintenance.

d). Long life.

16. Define Demodulation. [CO1-L1]

Demodulation or detection is the process by which modulating voltage is recovered from the modulated signal. It is the reverse process of modulation. The devices used for demodulation or detection are called demodulators or detectors. For amplitude modulation, detectors or demodulators are categorized as,

a) Square-law detectors

b) Envelope detectors

17. Define Multiplexing. [CO1-L1]

Multiplexing is defined as the process of transmitting several message signals Simultaneously over a single channel.

18. Define Frequency Division Multiplexing. [CO1-L1]

Frequency division multiplexing is defined as many signals are transmitted simultaneously with each signal occupying a different frequency slot within a common bandwidth.

19. Define Guard Band. [CO1-L1]

Guard Bands are introduced in the spectrum of FDM in order to avoid any interference between the adjacent channels. Wider the guard bands, Smaller the interference.

20. Define SSB-SC. [CO1-L1]

(i) SSB-SC stands for Single Side Band Suppressed Carrier

(ii) When only one sideband is transmitted, the modulation is referred to as Single side band modulation. It is also called as SSB or SSB-SC.

21. Define DSB-SC. [CO1-L1]

After modulation, the process of transmitting the sidebands (USB, LSB) alone and suppressing the carrier is called as Double Side Band-Suppressed Carrier.

22. What are the disadvantages of DSB-FC? [CO1-L1]

(i) Power wastage takes place in DSB-FC

(ii) DSB-FC is bandwidth inefficient system.

23. Define Coherent Detection. [CO1-L1-Nov/Dec2015]

During Demodulation carrier is exactly coherent or synchronized in both the frequency and phase, with the original carrier wave used to generate the DSB-SC wave.This method of detection is called as coherent detection or synchronous detection.

24. What is Vestigial Side Band Modulation? [CO1-L1-May/June2014]

Vestigial Sideband Modulation is defined as a modulation in which one of the sideband is partially suppressed and the vestige of the other sideband is transmitted to compensate for that suppression.

25. What are the advantages of signal sideband transmission? [CO1-L1-Nov/Dec2014]

a) Power consumption

b) Bandwidth conservation c) Noise reduction

26. What are the disadvantages of single side band transmission? [CO1-L1]

a) Complex receivers: Single side band systems require more complex and expensive receivers in conventional AM transmission.

b) Tuning difficulties: Single side band receivers require more complex and precise tunig than conventional AM receivers.

27. Compare linear and non-linear modulators? [CO1-L2-Nov/Dec2012]

S.No	Linear Modulators	Non Linear Modulators
1	Heavy filtering is not required.	Heavy filtering is required.

2	These modulators are used in high level modulation.	These modulators are used in low level modulation.
3	The carrier voltage is very much greater than modulating signal voltage.	The modulating signal voltage is very much greater than the carrier signal voltage.

28. What is frequency translation? [CO1-L1-Nov/Dec2015]

Suppose that a signal is band limited to the frequency range extending from a frequency f1 to a frequency f2. The process of frequency translation is one in which the original signal is replaced with a new signal whose spectral range extends from f1‘ and f2‘ and which new signal bears, in recoverable form the same information as was borne by the original signal.

29. Determine the two situations identified in frequency translations? [CO1-L1-Nov/Dec2013]

a) Up Conversion: In this case the translated carrier frequency is greater than the incoming carrier

b) Down Conversion: In this case the translated carrier frequency is smaller than the increasing carrier frequency.

Thus, a narrowband FM signal requires essentially the same transmission bandwidth as the AM signal.

30. What is BW for AM wave? [CO1-L1]

The difference between these two extreme frequencies is equal to the bandwidth of the AM wave.

Therefore, Bandwidth, B = (fc + fm) - (fc - fm) B = 2fm

31. What is the BW of DSB-SC signal? [CO1-L1]

Bandwidth, B = (fc + fm) - (fc - fm) B = 2f

It is obvious that the bandwidth of DSB-SC modulation is same as that of general AM waves.

32. What are the demodulation methods for DSB-SC signals? [CO1-L1]

The DSB-SC signal may be demodulated by following two methods:

(i) Synchronous detection method.

(ii)Using envelope detector after carrier reinsertion.

33. Write the applications of Hilbert transform? [CO1-L1-May/June2015]

(i) For generation of SSB signals,

(ii) For designing of minimum phase type filters,

(iii) For representation of band pass signals.

34. What are the methods for generating SSB-SC signal? [CO1-L1]

SSB-SC signals may be generated by two methods as under:

(i)Frequency discrimination method or filter method. (ii)Phase discrimination method or phase-shift method.

PART – B

1. Explain the generation of AM signals using Square Law Modulator. [CO1-L2-Nov/Dec2012]

The circuit that generates the AM waves is called as amplitude modulator

1. Square Law Modulator

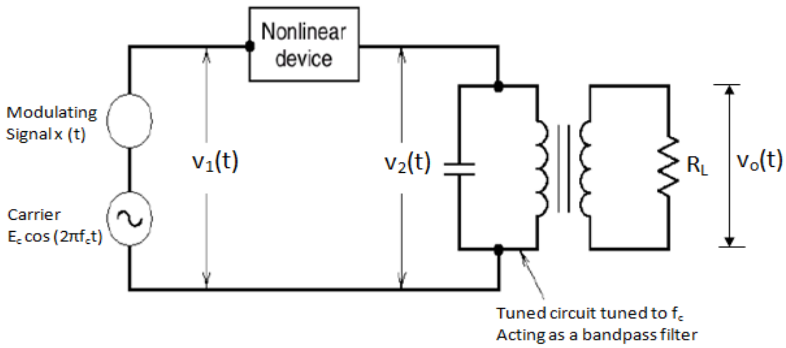
These circuits use a non-linear elements such as a diode for their implementation . Both these modulators are low power modulator circuits . Generation of AM Waves using the square law modulator could be understood in a better way by observing the square law modulator circuit shown in fig.

It consists of the following :A non-linear device

1. A bandpass filter

2. A carrier source and modulating signal

The modulating signal and carrier are connected in series with each other and their sum $V_1(t)$ is applied at the input of the non-linear device, such as diode, transistor etc.



Thus,

$$v_1(t) = x(t) + E_c \cos(2\pi f_c t) \quad \text{.............................(1)}$$

The input output relation for non-linear device is as under :

$$v_2(t) = a v_1(t) + b v_1^{\,2}(t) \quad \text{................................(2)}$$

where a and b are constants.

Now, substituting the expression (1) in (2), we get

$$v_2(t) = a[x(t) + E_c \cos(2\pi f_c t)] + +b[x(t) + E_c \cos(2\pi f_c t)]^2$$

Or,

$$v_2(t) = ax(t) + aE_c \cos(2\pi f_c t) + +b[x^2(t) + 2x(t)\cos(2\pi f_c t) + E_c^{\,2} cos^2(2\pi f_c t)]$$

Or,

$$v_2(t) = \underbrace{ax(t)}_{(1)} + \underbrace{aE_c \cos(2\pi f_c t)}_{(2)} + +\underbrace{bx^2(t)}_{(3)} + \underbrace{2bx(t)\cos(2\pi f_c t)}_{(4)} + \underbrace{bE_c^{\,2} cos^2(2\pi f_c t)}_{(5)}$$

The five terms in the expression for $V_2(t)$ are as under :

Term 1: ax(t) : Modulating Signal

Term 2 : a $E_c \cos(2\pi f_c t)$: Carrier Signal

Term 3 : b $x^2(t)$: Squared modulating Signal

Term 4 : 2 b x(t) cos ($2\pi f_c t$) : AM wave with only sidebands

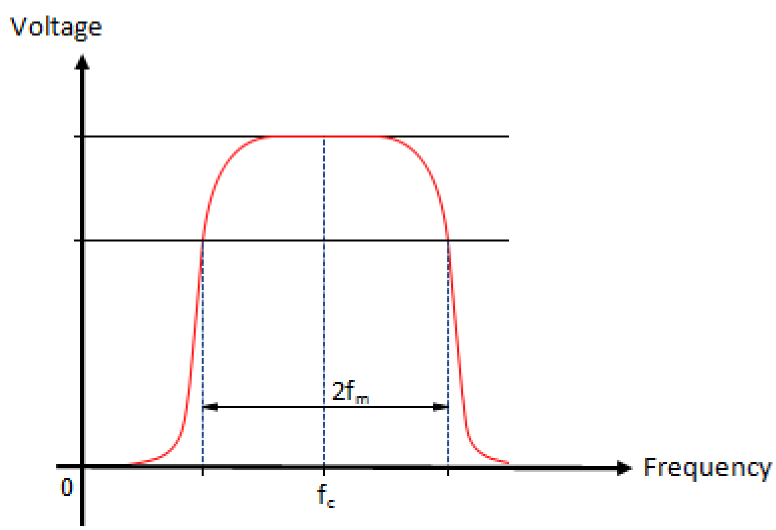
Term 5 : b $E_c^2 \cos^2 (2\pi f_c t)$: Squared Carrier

Out of these five terms, terms 2 and 4 are useful whereas the remaining terms are not useful .

Let us club terms 2, 4 and 1, 3, 5 as follows to get ,

$$v_2(t) = \underbrace{ax(t) + bx^2(t) + bE_c{}^2 cos^2(2\pi f_c t)}_{\text{Unuseful Terms}} + \underbrace{aE_c \cos(2\pi f_c t) + 2bx(t)E_c \cos(2\pi f_c t)}_{\text{Useful Terms}}$$

The LC tuned circuit acts as a bandpass filter . Its frequency responce is shown in fig 2 which shows that the circuit is tuned to frequency f_c and its bandwidth is equal to $2f_m$. This bandpass filter eliminates the unuseful terms from the equation of $v_2(t)$.



Hence the output voltage $v_o(t)$ contains only the useful terms .

$$V_o(t) = aE_c \cos(2\pi f_c t) + 2bx(t)E_c \cos(2\pi f_c t)$$

Or,

$$V_o(t) = [aE_c + 2bx(t)E_c] \cos(2\pi f_c t)$$

Therefore ,

$$V_o(t) = aE_c[1 + \frac{2b}{a}x(t)]\cos(2\pi f_c t)$$ ………………………….(3)

Comparing this with the expression for standard AM wave i.e.

$$s(t) = E_c[1 + mx(t)]\cos(2\pi f_c t)$$,

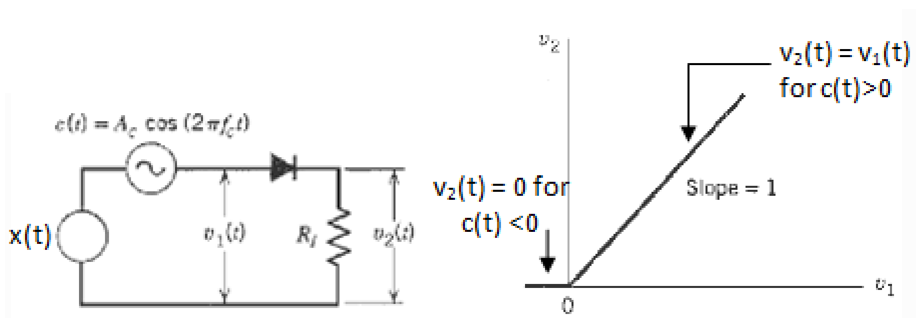
We find that the expression for $V_o(t)$ of equation (3) represents an AM wave with m = (2b/a) .

Hence, the square law modulator produces an AM wave .

2. Explain the generation of AM signals using Switching Modulator. [CO1-L2-May/June 2010]

Switching Modulator

Generation of AM Waves using the switching modulator could be understood in a better way by observing the switching modulator diagram. The switching modulator using a diode has been shown in fig



This diode is assumed to be operating as a switch .

The modulating signal x(t) and the sinusoidal carrier signal c(t) are connected in series with each other. Therefore, the input voltage to the diode is given by :

$$v_1(t) = c(t) + x(t) = E_c\cos(2\pi f_c t) + x(t)$$

The amplitude of carrier is much larger than that of x(t) and c(t) decides the status of the diode (ON or OFF) .

Working Operation and Analysis

Let us assume that the diode acts as an ideal switch . Hence, it acts as a closed switch when it is forward biased in the positive half cycle of the carrier and offers zero impedance . Whereas it acts as an open switch when it is reverse biased in the negative half cycle of the carrier and offers an infinite impedance .

Therefore, the output voltage $v_2(t) = v_1(t)$ in the positive half cycle of c(t) and $v_2(t) = 0$ in the negative half cycle of c(t) .

Hence , $v_2(t) = v_1(t)$ for $c(t) > 0$

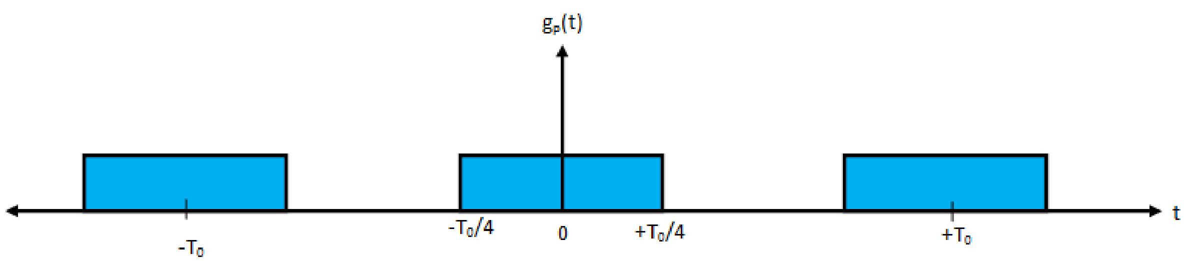
$v_2(t) = 0$ for $c(t) < 0$

In other words , the load voltage $v_2(t)$ varies periodically between the values $v_1(t)$ and zero at the rate equal to carrier frequency f_c .

We can express $v_2(t)$ mathematically as under :

$$v_2(t) = v_1(t)\,.\,g_p(t) = [x(t) + E_c\cos(2\pi f_c t)]\,g_p(t) \quad \text{..............................(4)}$$

where, $g_p(t)$ is a periodic pulse train of duty cycle equal to one half cycle period i.e. $T_0/2$ (where $T_0 = 1/f_c$) .



Let us express $g_p(t)$ with the help of Fourier series as under :

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos\left[2\pi f_c t(2n-1)\right] \quad \text{..................................(5)}$$

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + odd\ harmonic\ components \quad \text{.................................(6)}$$

Substituting $g_p(t)$ into equation (4), we get

$$v_2(t) = [x(t) + E_c \cos((2\pi f_c t)]\left\{\frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_c t(2n-1)]\right\}$$

Therefore,

$$v_2(t) = [x(t) + E_c \cos((2\pi f_c t)]\left\{\frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + odd\ harmonics\right\}$$

……………………….(7)

The odd harmonics in this expression are unwanted, and therefore, are assumed to be eliminated .

Hence,

$$v_2(t) = \underbrace{\frac{1}{2}x(t)}_{\text{Modulating Signal}} + \underbrace{\frac{1}{2}E_c \cos(2\pi f_c t) + \frac{2}{\pi}x(t)\cos(2\pi f_c t)}_{\text{AM Wave}} + \underbrace{\frac{2E_c}{\pi}cos^2(2\pi f_c t)}_{\text{Second harmonic of carrier}}$$

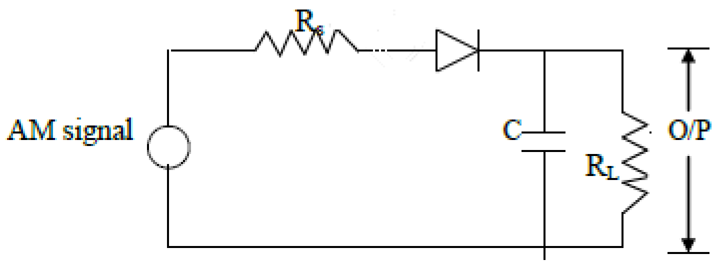
In this expression, the first and the fourth terms are unwanted terms whereas the second and third terms together represents the AM wave .

Clubing the second and third terms together , we obtain

$$v_2(t) = \frac{E_c}{2}\left[1 + \frac{4}{\pi E_c}x(t)\right]\cos(2\pi f_c t) + unwanted\ terms$$

This is the required expression for the AM wave with m=[4/πE_c] . The unwanted terms can be eliminated using a band-pass filter (BPF) .

2. Explain the detection of AM signals using Envelope Detector. [CO1-L2-Nov/Dec2011]



Envelope detector is used to detect high level modulated levels, whereas square-law detector is used to detect low level modulated signals (i.e., below 1v). It is also based on the switching action or switching characteristics of a diode. It consists of a diode and a resistor-capacitor filter. The operation of the envelope detector is as follows. On a positive half cycle of the input signal, the diode is forward biased and the capacitor C charges up rapidly to the peak value of the input signal. When the input signal falls below this value, the diode becomes reverse biased and the capacitor C discharges slowly through the load resistor Rl . The discharging process continues until the next positive half cycle. When the input signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated.

The charging time constant RsC is very small when compared to the carrier period 1/fc i.e., RsC << 1/fc

Where Rs = internal resistance of the voltage source.

C = capacitor fc = carrier frequency i.e., the capacitor C charges rapidly to the peak value of the signal.

The discharging time constant RlC is very large when compared to the charging time constant i.e., 1/fc << RlC << 1/W

Where Rl = load resistance value

W = message signal bandwidth i.e., the capacitor discharges slowly through the load resistor. Advantages:

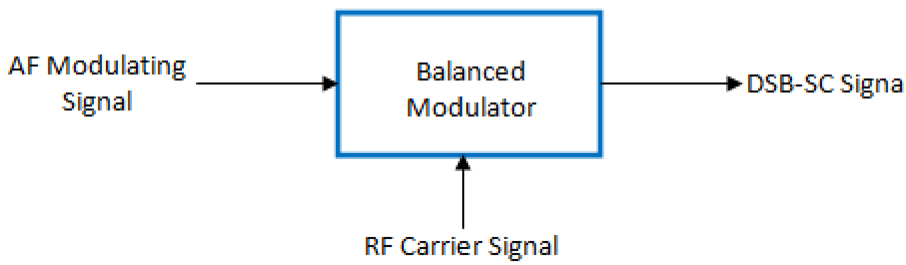
It is very simple to design ¬ It is inexpensive ¬ Efficiency is very high when compared to Square Law ¬ detector

Disadvantage: Due to large time constant, some distortion occurs which is ¬ known as diagonal clipping i.e., selection of time constant is somewhat difficult

Application: It is most commonly used in almost all commercial AM Radio ¬ receivers.

3. Explain about balanced modulator to generate DSB-SC signal. [CO1-L2-Nov/Dec2013]

The balanced modulators are used to suppress the unwanted carrier in AM wave .



Block Diagram of Balanced Modulator

The carrier and modulating signals are applied to the inputs of the balanced modulator and we get the DSB signal with supressed carrier at the output of the balanced modulator . Hence, the output consists of the upper and lower sidebands only .

Principle of Operation

The principle of operation of a balanced modulator states that if two signals at different frequencies are passed through a non-linear resistance then at the output, we get an AM signal with suppressed carrier . The device having a non-linear resistance can be a diode or a JFET or even a bipolar transistor .

Types of Balanced Modulator

The suppression of carrier can be done using the following two balanced modulators :

1. Using the diode ring modulator or lattice modulator
2. Using the FET balanced modulator

Balanced Modulator using AM Modulator

The block diagram of a balanced modulator is shown in fig.5 .

It consists of two standard amplitude modulators arranged in the balanced configuration so as to suppress the carrier completely .

Working Operation and Analysis

The carrier signal c(t) is connected to both AM modulators M_1 and M_2 .

The message signal x(t) is applied as it is to M_1 and its inverted version -x(t) is applied to M_2.

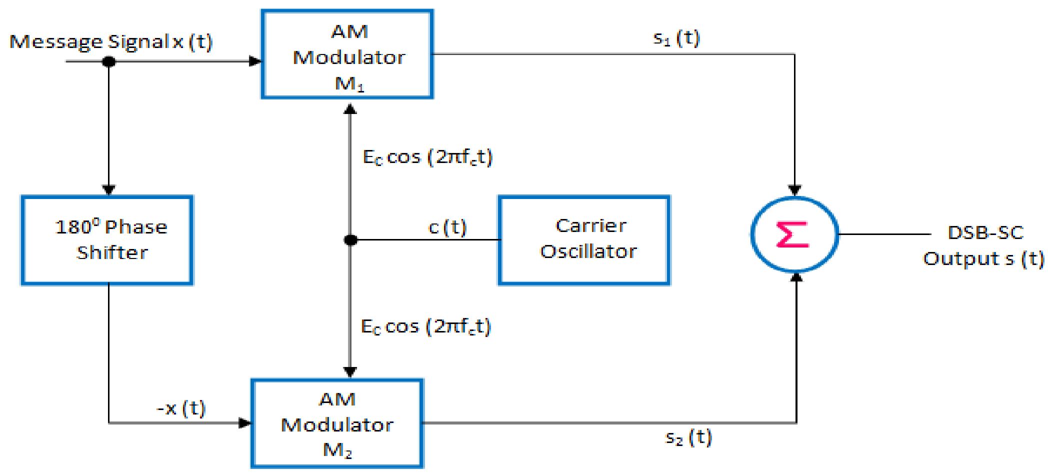


Fig : Balance Modulator for DSB-SC genaration

At the outputs of modulators M_1 and M_2 , we get standard AM signals $s_1(t)$ and $s_2(t)$ as under :

output of M_1 : $s_1(t) = E_c[1 + m\,x(t)] \cos(2\pi f_c t)$

output of M_2 : $s_2(t) = E_c[1 - m\,x(t)] \cos(2\pi f_c t)$

These are then applied to a subtractor and the subtractor produces the desired DSB-SC signal as under :

Subtractor output = $s_1(t) - s_2(t) = E_c[1 + m\,x(t)] \cos(2\pi f_c t) - E_c[1 - m\,x(t)] \cos(2\pi f_c t)$

$= E_c \cos(2\pi f_c t)[\{1 + m\,x(t)\}-\{1 - m\,x(t)\}]$

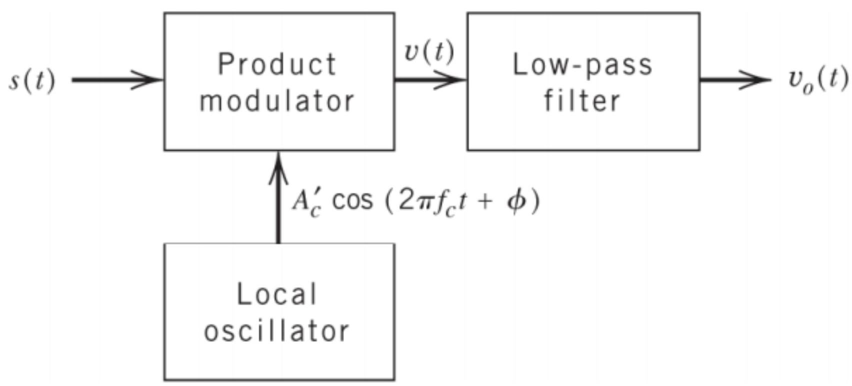
$= E_c \cos(2\pi f_c t)[1 + m\,x(t)- 1 + m\,x(t)]$

$= 2m\, E_c x(t) \cos(2\pi f_c t)$

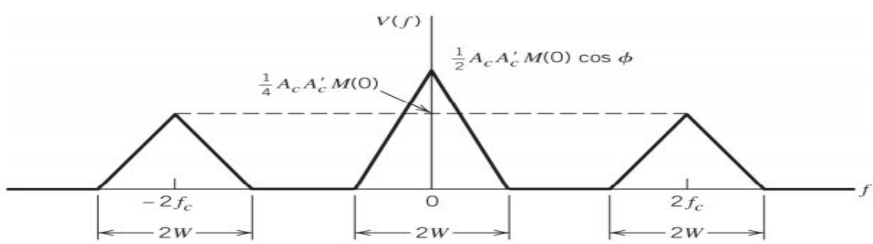
The R.H.S. of this expression consists of product of x(t) and c(t) = $E_c$cos (2π f_ct) . Hence, it represents a DSB-SC signal .

4. Discuss the coherent detection of DSB-SC modulated wave with a block diagram of detector and explain. **[CO1-L2-May/June 2015]**

Coherent Detection It is assumed that the local oscillator signal is exactly coherent or synchronized in both synchronized, in both frequency and phase frequency and phase, with carrier wave with carrier wave c(t) used in the product modulator to generate s(t). This method of demodulation is known as coherent detection or synchronous (or) demodulation.



For more general demodulation process, we assume it is a arbitrary phase difference



The first term in equn is removed by low pass filter.provided that the cut-ff frequency of this filter is greater than W but less than that 2fc –w. This is shifted by choosing fc>W. Therefore

$$v_0(t) = \frac{1}{2} A_c A_c{}^1 cos\emptyset m(t)$$

Vo(t) is proportional to m(t) when the phase error Φ is a constant attenuated by a factor equal to cosΦ

$$v_0(max) = \frac{1}{2} A_c A_c{}^1 m(t) \quad when\ \emptyset = 0$$

$$v_0(min) = 0 \quad when\ \emptyset = +or - \pi/2$$

When the phase error Φ is constant, the detector provides an undistorted version of the original baseband signal m(t)

In practice, we usually find that the phase error φ varies randomly with time, due to random variations in communication channel. The result is that at the detector outp , py g ut, the multiplying factor cosφ also varies randomly with time, which is obviously undesired.

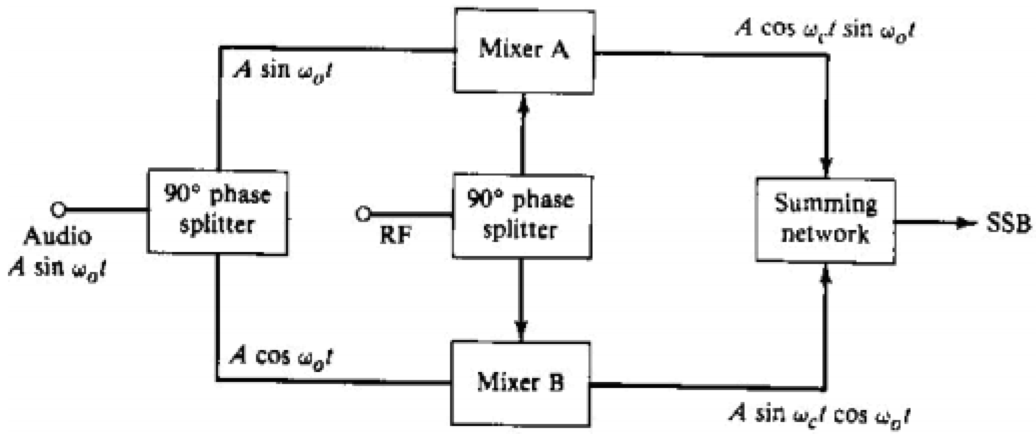
Provision must be made in the system to maintain the local oscillator in the receiver in perfect synchronism, in both frequency and phase with the carrier wave used to generate the frequency and phase, with the carrier wave used to generate the DSB-SC modulated signal in the transmitter.

The resulting system complexity is the price that must be paid for suppressing the carrier wave to save transmitter power.

5. Explain about the generation of SSB using Phase shift Method[CO1-L2-Nov/Dec2015]

Standard amplitude modulation and DSBSC are wasteful of bandwidth beause they both require a transmission bandwidth equal twice the message bandwidth

Thus the channel needs to provide only the same bandwidth as the message signal. When only one side band is transmitted the modulation is referred to as single side band modulation.



The phase discrimination method of generating an SSB modulated wave involves two separate simultaneous modulation process and subsequent combination of the resulting modulation products.

The system uses two product modulators I and Q supplied with carrier waves in phase quadrature to each other.

The incoming baseband signal m(t) is applied to product modulator I producing DSBSC wave that contains reference phase sidebands symmetrically spaced about the carrier frequency fc

The Hilbert transform m`(t) of m(t) is applied to the product modulator Q producing a modulated wave that contains sidebands having identical amplitude spectra to those of modulator I, but with spectra such that a vector addition or subtraction of two modulator outputs results in cancellation of one set of sidebands and reinforcement of the other set.

The use of a plus sign at the summing junction yields an SSB wave with only the lower side band and the use of minus sign yields SSB wave with only the upper side band.

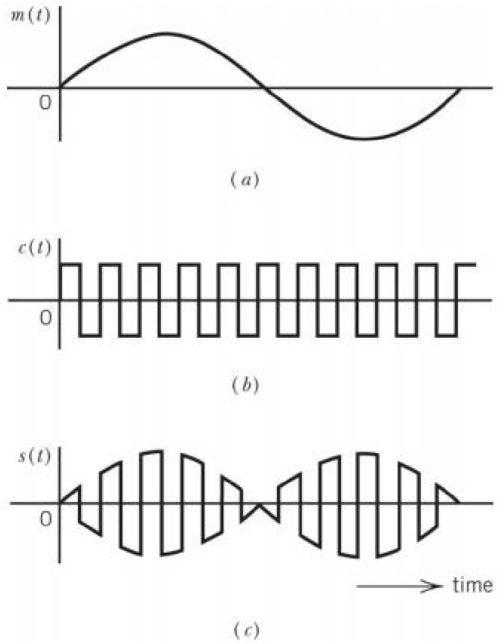
6. Draw the circuit diagram of Ring Modulator and explain with its operation? [CO1-L2-Nov/Dec2010]

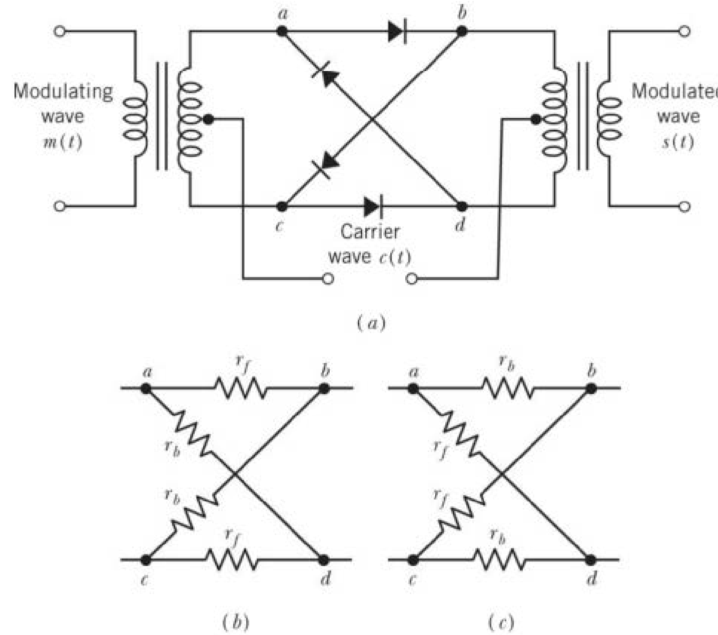
Ring modulator is one of the most useful product modulator, well suited for generating a DSB-SC wave. ◊ The diodes are controlled by a square The diodes are controlled by a square-wave carrier wave carrier c(t) of frequency) of frequency fc, which is applied longitudinally by means of two center-tapped transformers. ◊ If the transformers are perfectly balanced and the diodes are identical, there is no leakage of the modulation frequency into the modulation output .

The operation of the circuit.

Assuming that the diodes have a constant forward resistance rf when switched on and a constant backward resistance rb when switched off. And they switch as the carrier wave c(t) goes through zero.

On one half-cycle of the carrier wave, the outer diodes are switched to their forward resistance rf and the inner diodes are switched to their backward resistance rb. On the other half-cycle of the carrier wave, the diodes operate 27 resistance rb. On the other half cycle of the carrier wave, the diodes operate in the opposite condition.





The output voltage has the same magnitude as the output voltage, but they have opposite polarity.

In fact, the ring modulator acts as a commutator.

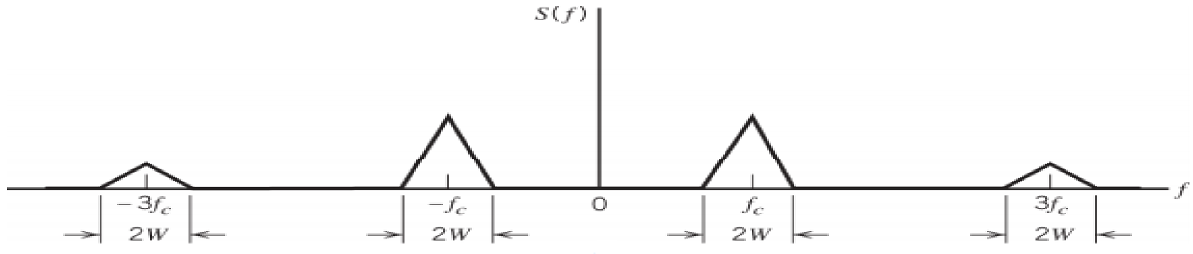
It is sometimes referred to as a double-balanced modulator, because it is balanced with respect to both the baseband signal and the square-wave carrier.

Assuming that Assuming that m(t) is limited to the frequency band) is limited to the frequency band $-W \le f \le W$, the spectrum of the modulator output consists of sidebands around each of the odd harmonics of the square-wave carrier m(t).

To prevent sideband overlap Î $f_c > W$.

We can use a band-pass filter of mid-band frequency f_c and bandwidth 2W to select the desired pair of sidebands around the carrier frequency f_c.

The circuitry needed for the generation of a DSB-SC modulated wave consists of a ring modulator followed by a band-pass filter.



7. Draw the block diagram for the generation and demodulation of a VSB signal and explain the Principle of operation. [CO1-L2-May/June2012]

A vestigial-sideband system is a compromise between DSB and SSB It inherits the advantages of DSB and SSB but avoids their SSB. It inherits the advantages of DSB and SSB but avoids their disadvantages.

VSB signals are relatively easy to generate and their bandwidth is VSB signals are relatively easy to generate and their bandwidth is only slightly (typically 25 percent) greater than that of SSB signals.

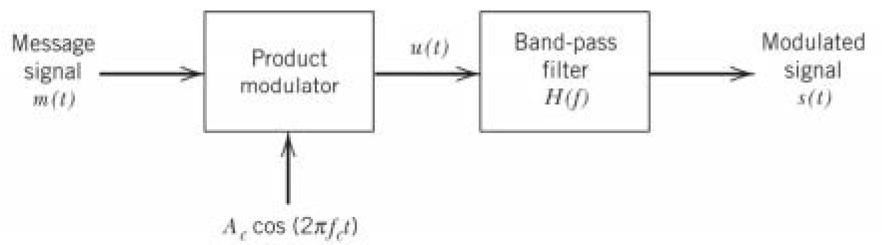
In VSB, instead of rejecting one sideband completely as in SSB, a gradual cutoff of one sideband is accepted. All of the one sideband is transmitted and a small amount (vestige) of the other sideband is transmitted as well. The filter is allowed to have a nonzero transition band.

The roll-off characteristic of the filter is such that the partial suppression of the transmitted sideband in the neighborhood of the carrier is exactly compensated for by the partial transmission of the 42 carrier is exactly compensated for by the partial transmission of the corresponding part of the suppressed sideband.

Our goal is to determine the particular H(f) required to produce a modulated signal modulated signal s (t) with desired spectral characteristics such) with desired spectral characteristics, such that the original baseband signal m (t) may be recovered from s (t) b y coherent detection.

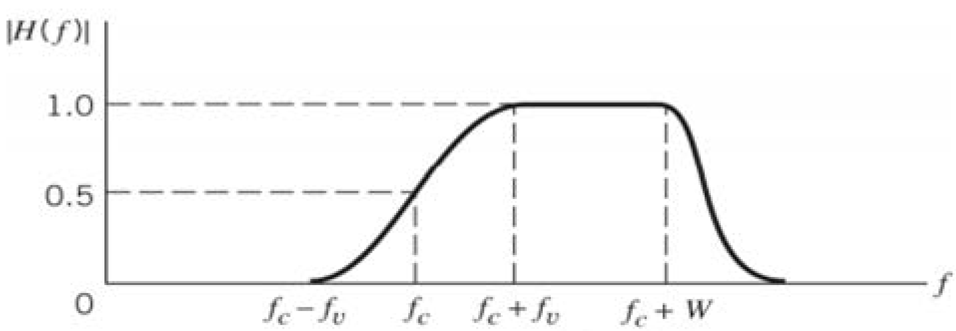
To generate VSB modulated wave, we pass a DSBSC modulated wave through a sideband shaping filter. The design of the filter depends on the desired spectrum of the VSB modulated wave.

The relation between transfer function H(f)of the filter and the spectrum S(f) of the VSB modulated wave is given by – S(f)= Ac/2[M(f-fc) + M(f+fc)]H(f),
where M(f) is message spectrum.
To determine the specifications of the filter transfer function H(f) so that S(f) defines the spectrum of the s(t), we pass s(t) through a coherent detector.
Thus, multiplying s(t) by a locally generated sine wave cos(2 π fc t), which is synchronous with the carrier wave Ac cos(2 π fc t),
we get v(t)= cos(2 π fc t)s(t).
The relation in frequency domain gives the Fourier transform of v(t) as V(f) = 0.5[S(f-fc) + S(f+fc)]
The final spectrum is given by : - Vo(f)=Ac/4 M(f) [H (f - fc) + H (f + fc)]



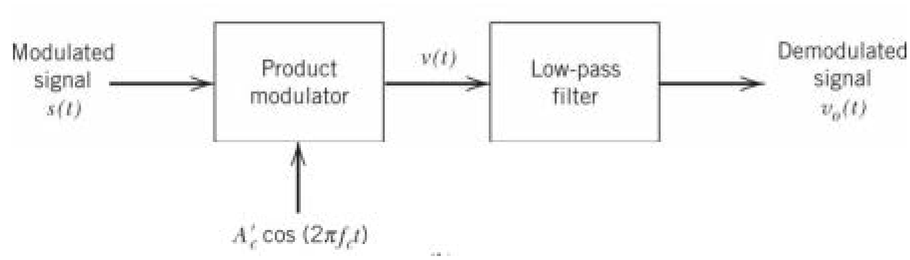
$$S(f) = U(f)H(f)$$

$$= \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]H(f)$$



Amplitude response of VSB filter

Detection of VSB wave plus Carrier:



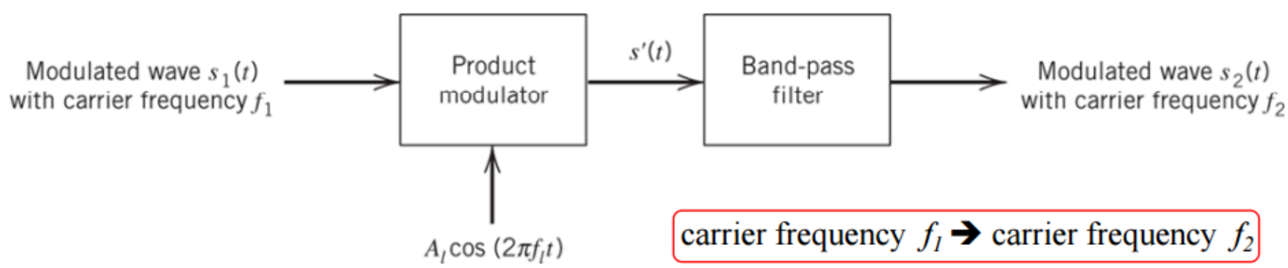
H (f - fc) + H (f + fc) =1,$-w \leq f \leq w$

8. Write short notes of frequency translation and FDM? [CO1-L2-Nov/Dec2013]

The basic operation involved in single-sideband modulation is in fact a form of a form of frequency translation frequency translation.

SSB modulation is sometimes referred to as frequency changing, mixing, or heterodyning. The mixer consists a product modulator followed by a band-pass filter.

Ban d - pass filter bandwidth: equal to that of the modulated signal s1(t) used as input.



Due to frequency translation performed by the mixer : We may set

Assume $f_2 > f_1$

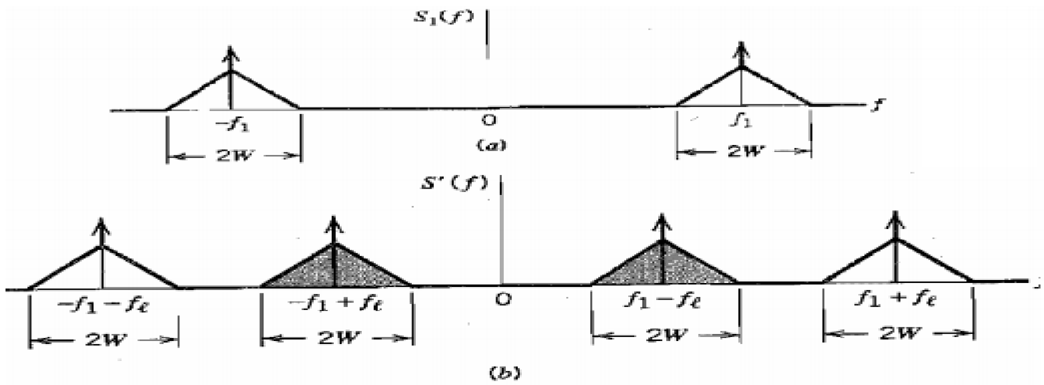
$f_2 = f_1 + f_1$

$f_1 = f_1 - f_1$

Assume $f_1 > f_2$

$f_2 = f_1 - f_1$

$f_1 = f_1 - f_2$

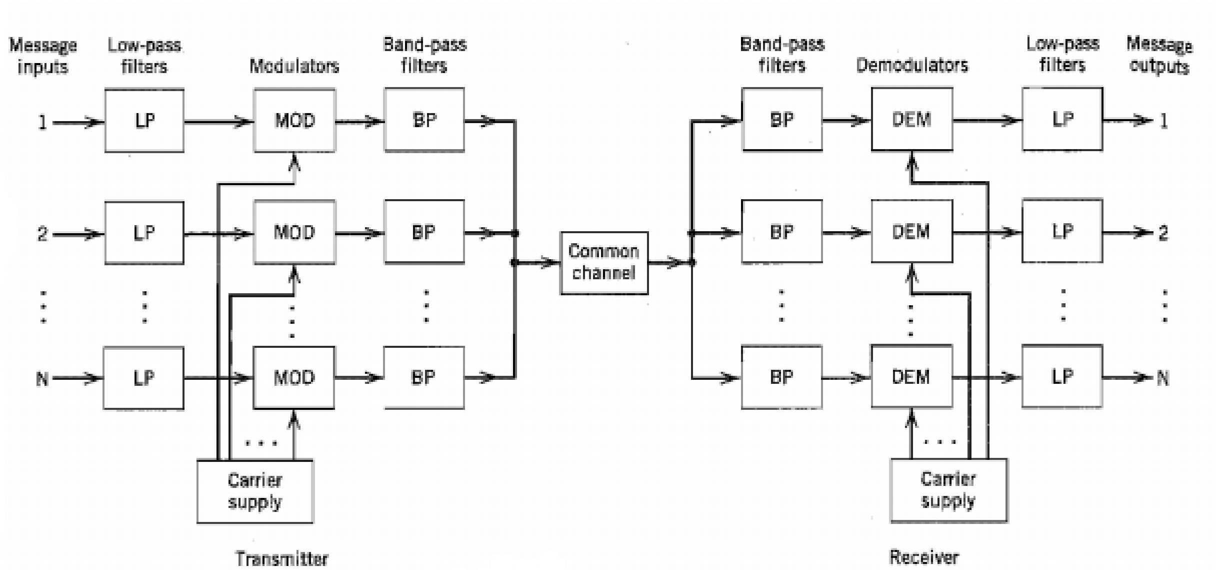


(*a*) Spectrum of modulated signal $s_1(t)$ at the mixer input.
(*b*) Spectrum of the corresponding signal $s'(t)$ at the output of the product modulator in the mixer.

The band-pass filter rejects the unwanted frequency and keeps the desired one. Mixing is a linear operation .

Frequency Division Multiplexing

It is a form of signal multiplexing which involves assigning non-overlapping frequency ranges to different signals or to each "user" of a medium. FDM can also be used to combine signals before final modulation onto a carrier wave. In this case the carrier signals are referred to as subcarriers: an example is stereo FM transmission, where a 38 kHz subcarrier is used to separate the left-right difference signal from the central left-right sum channel, prior to the frequency modulation of the composite signal. A television channel is divided into subcarrier frequencies for video, color, and audio. DSL uses different frequencies for voice and for upstream and downstream data transmission on the same conductors, which is also an example of frequency duplex. Where frequency-division multiplexing is used as to allow multiple users to share a physical communications channel, it is called frequency-division multiple access (FDMA).



Block diagram of FDM system.

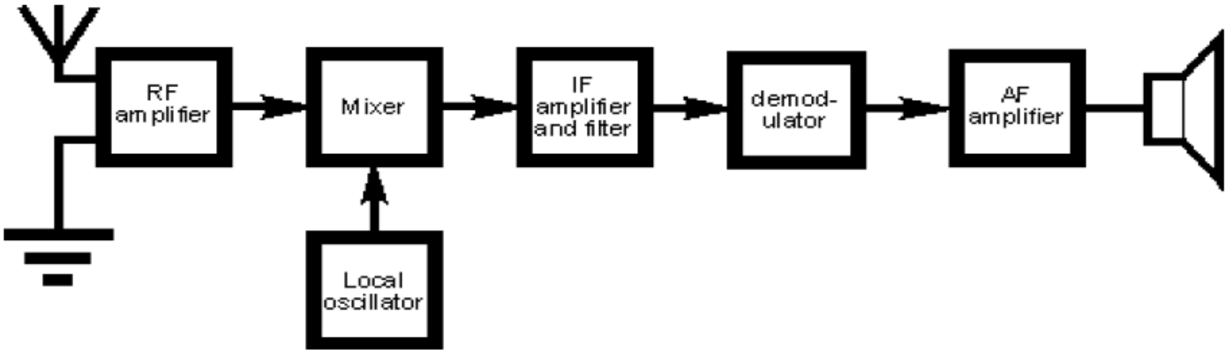
9. Explain the working of Super heterodyne receiver with its parameters. [CO1-L2-Nov/Dec2015]

Super heterodyne Radio Receiver:

A superheterodyne receiver(often shortened to superhet) uses frequency mixing to convert a received signal to a fixed intermediate frequency (IF) which can be more conveniently processed than the original radio carrier frequency.

Basic Superheterodyne Block Diagram and Functionality:

The basic block diagram of a basic superhet receiver is shown below. This details the most basic form of the receiver and serves to illustrate the basic blocks and their function.



Block Diagram of a Basic Superheterodyne Radio Receiver

The way in which the receiver works can be seen by following the signal as is passes through the receiver.Front end amplifier and tuning block: Signals enter the front end circuitry from the antenna. This circuit block performs two main functions:

Tuning: Broadband tuning is applied to the RF stage. The purpose of this is to reject the signals on the image frequency and accept those on the wanted frequency. It must also be able to track the local oscillator so that as the receiver is tuned, so the RF tuning remains on the required frequency. Typically the selectivity provided at this stage is not high. Its main purpose is to reject signals on the image frequency which is at a frequency equal to twice that of the IF away from the wanted frequency. As the tuning within this block provides all the rejection for the image response, it must be at a sufficiently sharp to reduce the image to an acceptable level. However the RF tuning may also help in preventing strong offchannel signals from entering the receiver and overloading elements of the receiver, in particular the mixer or possibly even the RF amplifier.

Amplification: In terms of amplification, the level is carefully chosen so that it does not overload the mixer when strong signals are present, but enables the signals to be amplified sufficiently to ensure a good signal to noise ratio is achieved. The amplifier must also be a low noise design. Any noise introduced in this block will be amplified later in the receiver.

Mixer / frequency translator block: The tuned and amplified signal then enters one port of the mixer. The local oscillator signal enters the other port. The performance of the mixer is crucial to many elements of the overall receiver performance. It should be as linear as possible. If not, then spurious signals will be generated and these may appear as 'phantom' received signals.

Local oscillator: The local oscillator may consist of a variable frequency oscillator that can be tuned by altering the setting on a variable capacitor. Alternatively it may be a frequency synthesizer that will enable greater levels of stability and setting accuracy.

Intermediate frequency amplifier, IF block : Once the signals leave the mixer they enter the IF stages. These stages contain most of the amplification in the receiver as well as the filtering that enables signals on one frequency to be separated from those on the next. Filters may consist simply of LC tuned transformers providing inter-stage coupling, or they may be much higher performance ceramic or even crystal filters, dependent upon what is required.

Detector / demodulator stage: Once the signals have passed through the IF stages of the superheterodyne receiver, they need to be demodulated. Different demodulators are required for different types of transmission, and as a result some receivers may have a variety of demodulators that can be switched in to accommodate the different types of transmission that are to be encountered. Different demodulators used may include:

AM diode detector: This is the most basic form of detector and this circuit block would simple consist of a diode and possibly a small capacitor to remove any remaining RF. The detector is cheap and its performance is adequate, requiring a sufficient voltage to overcome the diode forward drop. It is also not particularly linear, and finally it is subject to the effects of selective fading that can be apparent, especially on the HF bands.

Synchronous AM detector: This form of AM detector block is used in where improved performance is needed. It mixes the incoming AM signal with another on the same frequency as the carrier. This second signal can be developed by passing the whole signal through a squaring amplifier. The advantages of the synchronous is far less subject to the problems of selective fading.

SSB product detector: The SSB product detector block consists of a mixer and a local oscillator, often termed a beat frequency oscillator, BFO or carrier insertion oscillator, CIO. This form of detector is used for Morse code transmissions where the BFO is used to create an audible tone in line with the on-off keying of the transmitted carrier. Without this the carrier without modulation is difficult to detect. For SSB, the CIO re-inserts the carrier to make the modulation comprehensible.

Basic FM detector: As an FM signal carries no amplitude variations a demodulator block that senses frequency variations is required. It should also be insensitive to amplitude variations as these could add extra noise. Simple FM detectors such as the Foster Seeley or ratio detectors can be made from discrete components although they do require the use of transformers.

PLL FM detector: A phase locked loop can be used to make a very good FM demodulator. The incoming FM signal can be fed into the reference input, and the VCO drive voltage used to provide the detected audio output. o Quadrature FM detector: This form of FM detector block is widely used within ICs. IT is simple to implement and provides a good linear output.

Audio amplifier: The output from the demodulator is the recovered audio. This is passed into the audio stages where they are amplified and presented to the headphones or loudspeaker.

The parameters of the AM Receivers are Sensitivity, Selectivity, Fidelity, Image frequency rejection etc. some of which are explained below:

1. Selectively

The selectivity of an AM receiver is defined as its ability to accept or select the desired band of frequency and reject all other unwanted frequencies which can be interfering signals.

2. Fidelity

Fidelity of a receiver is its ability to reproduce the exact replica of the transmitted signals at the receiver output.

For better fidelity, the amplifier must pass high bandwidth signals to amplify the frequencies of the outermost sidebands, while for better selectivity the signal should have narrow bandwidth. Thus a trade off is made between selectivity and fidelity.

3. Sensitivity

Sensitivity of a receiver is its ability to identify and amplify weak signals at the receiver output.

It is often defined in terms of voltage that must be applied to the input terminals of the receiver to produce a standard output power which is measured at the output terminals.

10. State and prove Hilbert Transform. [CO1-H2-Nov/Dec2015]

Definition *The Hilbert transform $\hat{f}(t)$ of a function $f(t)$ is defined for all t by*

$$\hat{f}(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{f(\tau)}{t-\tau} d\tau,$$

when the integral exists.

It is normally not possible to calculate the Hilbert transform as an ordinary improper integral because of the pole at τ = t. However, the P in front of the integral denotes the Cauchy principal value which expanding the class of functions for which the integral in

If we apply the Fourier transform on a real function $f(t)$ then

$$\begin{aligned} f(t) &= \frac{1}{2\pi}\int_{-\infty}^{0} F(\omega)e^{i\omega t}d\omega + \frac{1}{2\pi}\int_{0}^{\infty} F(\omega)e^{i\omega t}d\omega \\ &= \frac{1}{2\pi}\int_{0}^{\infty} F(-\omega)e^{-i\omega t}d\omega + \frac{1}{2\pi}\int_{0}^{\infty} F(\omega)e^{i\omega t}d\omega \\ &= \frac{1}{2\pi}\int_{0}^{\infty} \left(F^*(\omega)e^{-i\omega t} + F(\omega)e^{i\omega t}\right) d\omega. \end{aligned}$$

This means that the positive frequency spectra is sufficient to represent a real signal.

Let us define a function $Z_f(\omega)$, that is zero for all negative frequencies and $2F(\omega)$ for all positive frequencies

$$Z_f(\omega) = F(\omega) + \operatorname{sgn}(\omega)F(\omega),$$

where

$$\operatorname{sgn}(\omega) = \begin{cases} 1 & \text{for} \quad \omega > 0 \\ 0 & \text{for} \quad \omega = 0 \\ -1 & \text{for} \quad \omega < 0 \end{cases}$$

The definition of the Fourier transform tells us that $f(t) \overset{F}{\Leftrightarrow} F(\omega)$ and therefore we know from that $F(\omega)\operatorname{sgn}(\omega)$ is the Fourier transform of $ig(t)$, thus

$$g(t) \overset{F}{\Leftrightarrow} F(\omega)\left(-i\operatorname{sgn}(\omega)\right).$$

It is a standard result that the inverse transform of $-i\operatorname{sgn}(\omega)$ equals $1/(\pi t)$, that is

$$g(t) = f(t) * \frac{1}{\pi t} = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{f(\tau)}{t-\tau} d\tau = Hf(t) = \widehat{f}(t),$$

and we see that $g(t)$ can be written as $\widehat{f}(t)$ which is known as the Hilbert transform of $f(t)$. Further more $g(t)$ is real.

The $\pm\pi/2$ phaseshift

The $\pm\pi/2$ phaseshift is interpreted in the frequency domain as a multiplication with the imaginary value $\pm i$, thus

$$H(\omega) = \begin{cases} -i = e^{-i\frac{\pi}{2}} & \text{for} \quad \omega > 0 \\ i = e^{i\frac{\pi}{2}} & \text{for} \quad \omega < 0 \end{cases}$$

$H(\omega)$ is unfortunately not a property of Fourier transform but the problem can be solved by expressing $H(\omega)$ as a limit of a bounded function $G(\omega)$, that is

$$G(\omega) = \begin{cases} -ie^{-\sigma\omega} & \text{for} \quad \omega > 0 \\ ie^{\sigma\omega} & \text{for} \quad \omega < 0 \end{cases}$$

where

$$\lim_{\sigma \to 0} G(\omega) = H(\omega).$$

It is now possible to use the inverse Fourier transform on $G(\omega)$, thus

$$g(t) \;=\; F^{-1}G(\omega)$$

where $g(t) \to h(t)$ when $\sigma \to 0$ and the inverse Fourier transform of the impulse response of $H(\omega)$ is

$$h(t) = \lim_{\sigma \to 0} g(t) = \lim_{\sigma \to 0} \frac{t}{\pi(\sigma^2 + t^2)} = \frac{1}{\pi t}.$$

A convolution between $f(t)$ and the impulse response $h(t)$ gives us

$$\hat{f}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(\tau)}{t - \tau} d\tau,$$

where $\hat{f}(t)$ is known as the Hilbert transform.

Unit –II

Angle Modulation

Part – A

1. What do you understand by narrowband FM? [CO2-L2-Nov/Dec2012]

When the modulation index is less than 1, the angle modulated systems are called low index. The bandwidth requirement of low index systems is approximately twice of the modulating.

2. Define frequency modulation. [CO2-L1-May/June2014]

Frequency modulation is defined as the process by which the frequency of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating or message signal.

3. Define modulation index of frequency modulation. [CO2-L1]

It is defined as the ratio of maximum frequency deviation to the modulating

$\beta = \delta f f m$

4. What do you meant by multitone modulation? [CO2-L2-May/June2010]

Modulation done for the message signal with more than one frequency component is called multitone modulation.

5. Define phase modulation. [CO2-L1]

Phase modulation is defined as the process of changing the phase of the carrier signal in accordance with the instantaneous amplitude of the message signal.

6. What are the types of Frequency Modulation?

Based on the modulation index FM can be divided into types. They are Narrow band FM and Wide band FM. If the modulation index is greater than one then it is wide band FM and if the modulation index is less than one then it is Narrow band FM

7. What is the basic difference between an AM signal and a narrowband FM signal? [CO2-L2-May/June2011]

In the case of sinusoidal modulation, the basic difference between an AM signal and a narrowband FM signal is that the algebraic sign of the lower side frequency in the narrow band FM is reversed.

8. What are the two methods of producing an FM wave? [CO2-L2-May/June2014]

Basically there are two methods of producing an FM wave. They are, i) Direct method: In this method the transmitter originates a wave whose frequency varies as function of the modulating source. It is used for the generation of NBFM

ii) Indirect method: In this method the transmitter originates a wave whose phase is a function of the modulation. Normally it is used for the generation of WBFM where WBFM is generated from NBFM.

9. Compare WBFM and NBFM[CO2-L2-Nov/Dec2013]

S.NO	WBFM	NBFM
1	Modulation index is greater than 1	Modulation index less than 1
2	Frequency deviation 75 KHz	Frequency deviation 5 KHz
3	Bandwidth 15 times NBFM	Bandwidth 2fm
4	Noise is more suppressed	Less suppressing of noise

10.Give the average power of an FM signal. [CO2-L1]

The amplitude of the frequency modulated signal is constant .The power of the FM signal is same as that of the carrier power.

P =1 2 E 2c

11. Define phase deviation. [CO2-L1]

The maximum phase deviation of the total angle from the carrier angle is called phase deviation.

12. Define frequency Deviation. [CO2-L1]

The maximum departure of the instantaneous frequency from the carrier frequency is called frequency deviation.

13. State the Carson's rule. [CO2-L2-May/June2015]

An approximate rule for the transmission bandwidth of an FM Signal generat ed by a single tone-modulating signal of frequency f m (max) is defined as

∴ BW=2[+ fm(max)]

14. Define the deviation ratio D for non-sinusoidal modulation. [CO2-L1]

The deviation ratio D is defined as the ratio of the frequency deviation f, which Corresponds to the maximum possible amplitude of the modulation signal m (t), to the highest modulation frequency.

D = Δf f m

15. What is the use of crystal controlled oscillator? [CO2-L2]

The crystal-controlled oscillator always produces a constant carrier frequency there by enhancing frequency stability.

16 What are the disadvantages of FM system? [CO2-L1]

1. A much wider channel is required by FM.
2.FM transmitting and receiving equipments tend to be more complex and hence it is expensive.

17. How will you generate message from frequency-modulated signals? [CO2-L2-May/June2011]

First the frequency-modulated signals are converted into corresponding amplitude modulated signal using frequency dependent circuits. Then the original signal is recovered from this AM signal.

18. What are the types of FM detectors? [CO2-L1-May/June2013]

The types of FM detectors are

(i) Slope detector and
(ii) Phase discriminator.

19. What are the types of phase discriminator? [CO2-L1]

The types of phase discriminator are (i) Foster seeley discriminator and (ii) Ratio detector.

20. What are the disadvantages of balanced slope detector? [CO2-L1-May/June2011]

1. Amplitude limiting cannot be provided
2. Linearity is not sufficient
3. It is difficult to align because of three different frequency to which various tuned circuits
to be tuned.
4. The tuned circuit is not purely band limited.

21. Write the advantages and disadvantages of foster-seely discrimination method? [CO2-L1]

Advantages:

a) It is much easier to design

b) Only two tuned circuits are necessary and they are tuned to same frequency c) Linearity

is better

Disadvantages:

a) It requires Amplitude limiting circuit.

22. What are the applications of phase locked loop? [CO2-L3-May/June2014]

Phase locked loops are used for various purposes in AM and FM communication.

(i)Automatic frequency correction in FM transmitter uses PLL to keep carrier frequency constant.

(ii)PLL is used direct FM Tramitter uses PLL to keep carrier frequency constant.

(iii) PLL is also used in FM demodulators.

23. Differentiate phase and frequency modulation. [CO2-L2-Nov/Dec2011]

S.No	Phase Modulation	Frequency Modulation
1	Phase of the carrier varies as per amplitude variations of modulating signal.	Frequency of the carrier varies as per amplitude variations of modulating signals.
2	Instantaneous phase deviation, $\theta\ (t) = k\ em\ \ (t)$	Instantaneous frequency deviation, $\Delta f\ (t) = k\ em\ \ (t)$
3	Modulation index = k Em	Modulation index = k Em

24. A 80 MHz carrier is frequency modulated by a sinusoidal signal of 1V amplitude and the frequency sensitivity is 100 Hz/V. Find the approximate bandwidth of the FM waveform if the modulating signal has a frequency of 10 kHz. [CO2-H2-May/June2013]

Ans: Frequency Sensitivity = 100 Hz/ volt.

Amplitude of modulating signal = 1V

Hence maximum frequency deviation, δ =100 Hz / volt ×1V= 100 kHz

Frequency of modulating signal, fm = 10kHz

∴ BW = 2 [δ + fm (max)]= 2 [100 +10×103]

BW = 20.2 kHz

25. What is the use of diversity reception? [CO2-L3-May/June2014]

Diversity reception is used when the signal fades into noise level. There are two types of diversity reception:

a) Space diversity

b) Frequency diversity.

a) Space diversity: It uses two or more receiving antennas separated by nine or more wavelengths. These are separate receivers for each antenna. The receiver with strongest signal is selected.

b) Frequency diversity: It uses single receiving antenna which works for two or more frequencies. The frequency which has strong signal is selected.

26. State the disadvantages of FM. [CO2-L1]

i) Bandwidth requirement of FM is much higher.

ii) FM transmitting and receiving equipment is more complex and costly. iii) Distance of reception is limited only to line of sight.

27. What do you understand by FM stereo multiplexing? [CO2-L2-Nov/Dec2012]

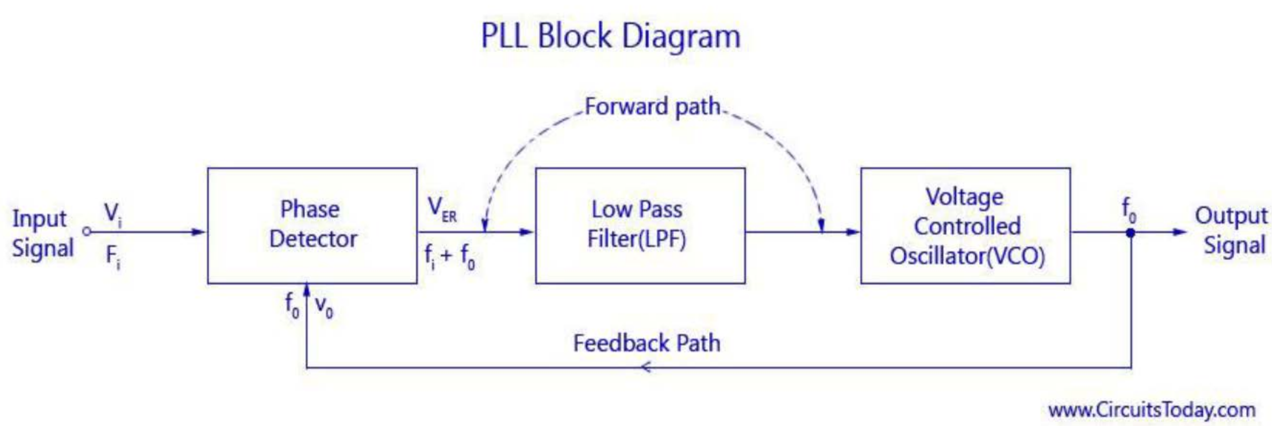
FM stereo multiplexing is used for stereo transmission. It is basically frequency division multiplexing. It is used for FM radio broadcasting. The left and right channel signals are used to generate sum and difference signals. The difference signal frequency modulates the carrier. The difference signal, FM difference signal, FM difference signal and carrier are combined together and

sent. Such FM multiplexed signal can be coherently received by stereo as well as mono receiver.

PART-B

1. Explain how FM can be used as a PLL [CO2-L2-May/June2013] [8]

A device called a phase-locked loop (PLL) can be used to demodulate an FM signal with better performance in a noisy environment than a frequency discriminator. The block diagram of a discrete-time version of a PLL as shown in figure,



The block diagram of a basic PLL is shown in the figure below. It is basically a flip flop consisting of a phase detector, a low pass filter (LPF),and a Voltage Controlled Oscillator (VCO) The input signal Vi with an input frequency fi is passed through a phase detector. A phase detector basically a comparator which compares the input frequency fiwith the feedback frequency fo .The phase detector provides an output error voltage Ver (=fi+fo),which is a DC voltage. This DC voltage is then passed on to an

LPF. The LPF removes the high frequency noise and produces a steady DC level, Vf (=Fi-Fo). Vf also represents the dynamic characteristics of the PLL.

The DC level is then passed on to a VCO. The output frequency of the VCO (fo) is directly proportional to the input signal. Both the input frequency and output frequency are compared and
adjusted through feedback loops until the output frequency equals the input frequency. Thus the PLL works in these stages – free-running, capture and phase lock.

As the name suggests, the free running stage refer to the stage when there is no input voltage
applied. As soon as the input frequency is applied the VCO starts to change and begin producing
an output frequency for comparison this stage is called the capture stage. The frequency comparison stops as soon as the output frequency is adjusted to become equal to the input frequency. This stage is called the phase locked state.

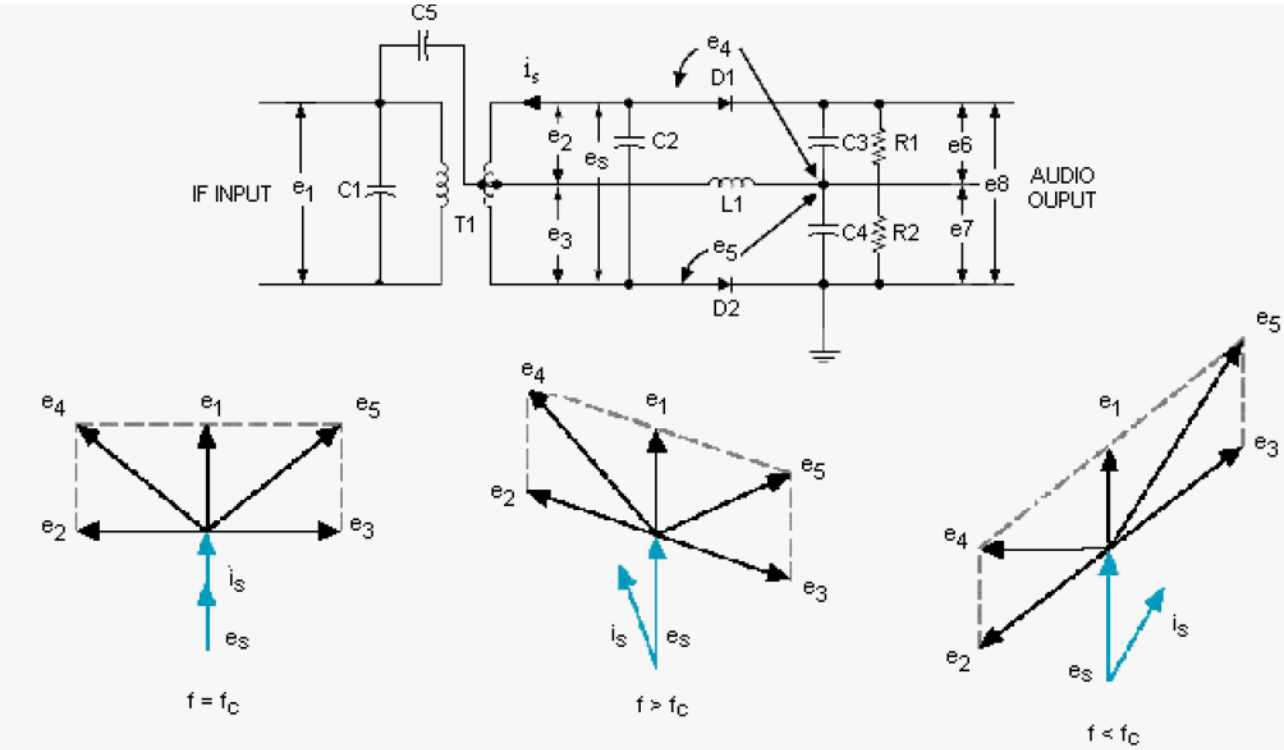
Comments on PLL Performance:

• The frequency response of the linearized loop characteristics of a band-limited differentiator. • The loop parameters must be chosen to provide a loop bandwidth that passes the desired baseband message signal but is as small as possible to suppress out-of-band noise.

• The PLL performs better than a frequency discriminator when the FM signal is corrupted by additive noise. The reason is that the bandwidth of the frequency discriminator must be large enough to pass the modulated FM signal while the PLL bandwidth only has to be large enough to pass the baseband message. With wideband FM, the bandwidth of the modulated signal can be significantly larger than that of the baseband message.

2. Draw the circuit diagram of Foster-seeley discriminator and explain its working. [CO2-L2-Nov/Dec2012] [10]

The Foster-Seely Discriminator is a widely used FM detector. The detector consists of a special center-tapped IF transformer feeding two diodes. The schematic looks very much like a full wave DC rectifier circuit. Because the input transformer is tuned to the IF frequency, the output of the discriminator is zero when there is no deviation of the carrier; both halves of the center tapped transformer are balanced. As the FM signal swings in frequency above and below the carrier frequency, the balance between the two halves of the center-tapped secondary are destroyed and there is an output voltage proportional to the frequency deviation.



The discriminator has excellent linearity and is a good detector for WFM and NBFM signals. Its major drawback is that it also responds to AM signals. A good limiter must precede a discriminator to prevent AM noise from appearing in the output.

3. Derive an expression for single tone FM wave and Narrow band FM wave? [CO2-H2-Nov/Dec2012] [16]

Single Tone Frequency Modulation

For the single tone frequency modulation,i.e the modulating signal x(t) be a sinusoidal signal of amplitude E_m and frequency f_m.

Therefore, $x(t) = E_m \cos(2\pi f_m t)$

The unmodulated carrier is represented by the expression :

$e_c = E_c \sin(\omega_c t + \varphi)$

Instantaneous frequency of an FM wave

In FM, the frequency f of the FM wave varies in accordance with the modulating voltage .

Thus,

$$f_i(t) = f_c + k_f x(t) = f_c + k_f.E_m \cos(2\pi f_m t)$$

$$or \qquad f_i(t) = f_c + \Delta f \cos(2\pi f_m t)$$

Where $\Delta f = k_f E_m$ and it is called as frequency deviation

Mathematical Expression for FM

FM wave is a sine wave having a constant amplitude and a variable instantaneous frequency .

As the instantaneous frequency is changing continuously, the angular velocity ω of an FM wave is the function of ω_c and ω_m.

Therefore, the FM wave is represented by,

$$s(t) = E_c \sin[F(\omega_c, \omega_m)]$$

$$s(t) = E_c \sin\theta(t)$$

$$Where \qquad \theta(t) = F(\omega_c, \omega_m)$$

$E_c \sin \Theta(t)$ is a rotating vector . If E_c is rotating at a constant velocity 'ω', then we could have writtenthat Θ(t)=ωt . in FM , this velocity is not constant. In fact, it is changing continuously.

The angular velocity of FM wave is given as,

$$\omega = [\omega_c + kE_m \cos\omega_m t]$$

Hence to find $\omega(t)$, we must integrate ω with respect to time.

$$Therefore, \qquad \theta(t) = \int \omega \, dt = \int [\omega_c + kE_m cos\omega_m t] dt$$

$$or \qquad \theta(t) = \omega_c \int [1 + \frac{kE_m}{\omega_c} cos\omega_m t] \, dt$$

$$= \omega_c [t + \frac{kE_m sin\omega_m t}{\omega_c \omega_m}]$$

$$= \omega_c t + \frac{kE_m \omega_c sin\omega_m t}{\omega_c \omega_m}$$

$$or \qquad \theta(t) = \omega_c t + \frac{kE_m sin\omega_m t}{f_m}$$

$$As\ per\ the\ definition, \quad \Delta f = kE_m$$

$$Thus, \qquad \theta(t) = \omega_c t + \frac{\Delta f \ sin\omega_m t}{f_m}$$

Substituting this value of $\theta(t)$ in the equation of $s(t)$, we get the equation for the FM wave as under ;

$$s(t) = E_c \sin[\omega_c t + \frac{\Delta f}{f_m} \sin\omega_m t]$$

$$But \ \frac{\Delta f}{f_m} = m_f$$

i. e. the modulation index of FM wave. Hence, the equation for FM wave is given as :

$$s(t) = E_c \sin[\omega_c t + m_f \sin\omega_m t]$$

This is the expression for FM wave, where, m_f represents the modulation index.

Narrow Band Fm Modulation

The case where $|\theta m(t)| \ll 1$ for all t is called narrow band FM. Using the approximations $\cos x \simeq 1$ and $\sin x \simeq x$ for $|x| \ll 1$, the FM signal can be approximated as:

$s(t) = Ac \cos[\omega ct + \theta m(t)]$

$= Ac \cos \omega ct \cos \theta m(t) - Ac \sin \omega ct \sin \theta m(t)$

$\simeq Ac \cos \omega ct - Ac\theta m(t) \sin \omega ct$

or in complex notation

s t = ACRE{ejwct (1 + jθm t }

This is similar to the AM signal except that the discrete carrier component Ac coswc(t) is 90° out

of phase with the sinusoid Ac sinwc(t) multiplying the phase angle θm(t). The spectrum of narrow band FM is similar to that of AM.

The Bandwidth of an FM Signal:

The following formula, known as Carson‘s rule is often used as an estimate of the FM signal bandwidth: $BT = 2(\Delta f + fm)$ Hz where Δf is the peak frequency deviation and fm is the maximum baseband message frequency component.

4. Discuss the working FM using Armstrong method. (or) Explain the indirect method of generation of FM wave [CO2-L2-Nov/Dec2013]

The direct methods cannot be used for the broadcast applications. Thus the alternative method i.e. indirect method called as the Armstrong method of FM generation is used.

In this method the FM is obtained through phase modulation. A crystal oscillator can be used hence the frequency stability is very high.

Operation:

- The crystal oscillator generates the carrier at low frequency typically at 1MHz. This is applied to the combining network and a 90° phase shifter.
- The modulating signal is passed through an audio equalizer to boost the low modulating frequencies .The modulating signal is then applied to a balanced modulator.
- The balanced modulator produced two side bands such that their resultant is 90° phase shifted with respect to the unmodulated carrier.

- The unmodulated carrier and 90° phase shifted sidebands are added in the combining network.
- At the output of the combining network we get FmFm wave. This wave has a low carrier frequency fcfc and low value of the modulation index mfmf .
- The carrier frequency and the modulation index are then raised by passing the FM wave through the first group of multipliers. The carrier frequency is then raised by using a mixer and then the fcfc and mfmf both are raised to required high values using the second group of multipliers.
- The FM signal with high fcfc and high mfmf is then passed through a class C power amplifier to raise the power level of the FM signal.
- The Armstrong method uses the phase modulation to generate frequency modulation. This method can be understood by dividing it into four parts as follows:

The block diagram of the Armstrong method is shown below:

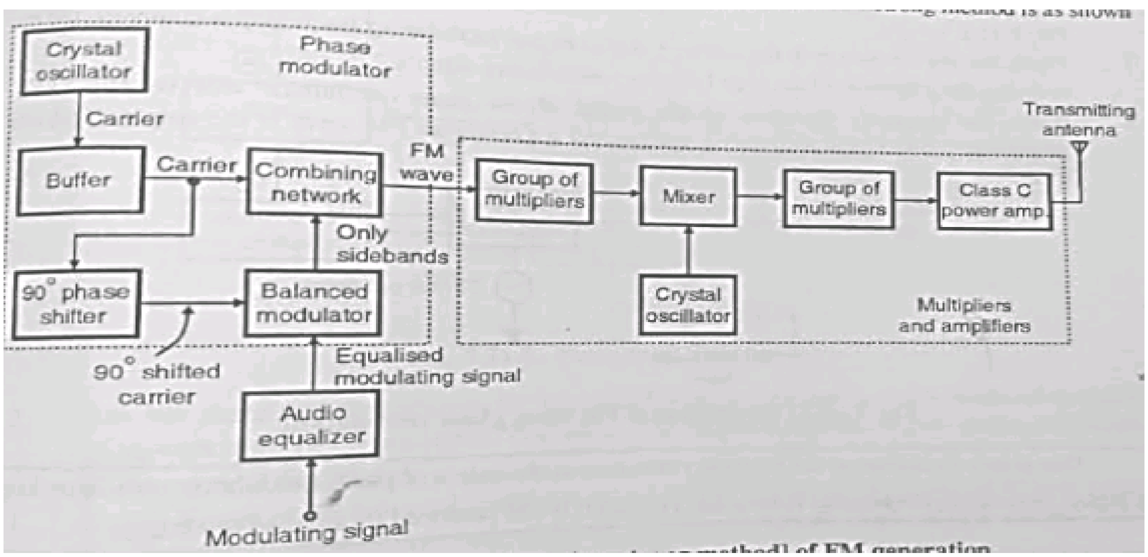


Fig: Indirect method [Armstrong method] of FM generation

1.Generation of FM from phase modulator:

The modulating signal is passed through a low pass RC filter. The filter output is then applied to a phase modulator along with carrier. Hence the extra deviation in the carrier fc due to higher modulating frequency is compensated by reducing the amplitude of the high frequency modulating signals.

Hence the frequency deviation at the output of the phase modulator will be effectively proportional only to the modulating voltage and we obtain an FM wave at the output of phase modulator.

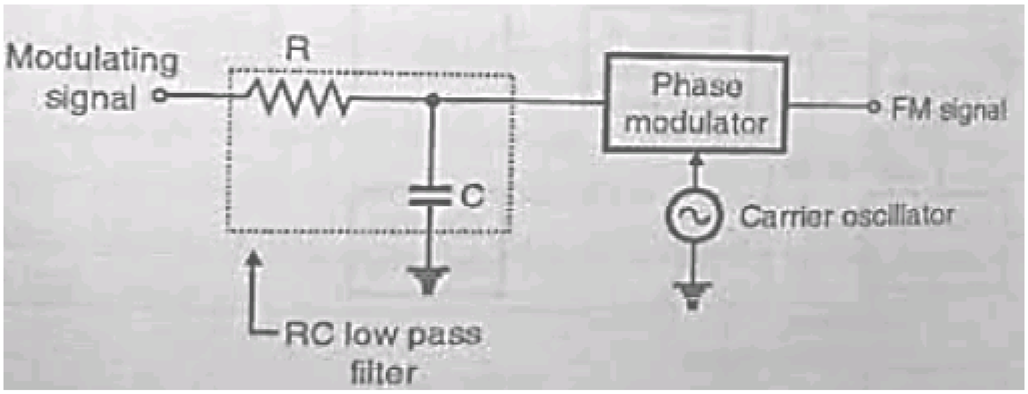


Fig: Generation of FM using phase modulation

2. Implementation of phase modulator

The crystal oscillator produces a stable un modulated carrier which is applied to the “90° phase shifter” as well as the “combining network” through a buffer.

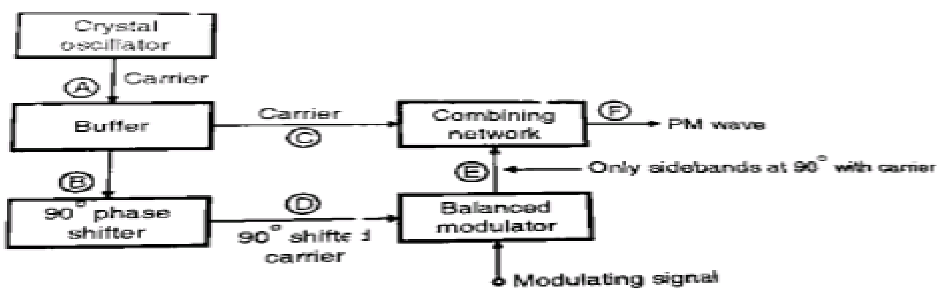


Fig: Phase modulator circuit

- The 90° phase shifter produces a 90° phase shifted carrier. It is then applied to the balanced modulator along with the modulation signal.
- At the output of the balanced modulator we get DSBSC signal i.e. AM signal without carrier. This signal consists of only two sidebands with their resultant in phase with their resultant in phase with the 90° phase shifted carrier.

3. Combining parts 1 and 2 to obtain The FM:

o Combining the parts 1 and 2 we get the block diagram of the Armstrong method of FM generation

4. Use of frequency multipliers and amplifiers:

o The FM signal produced at the output of phase modulator has a low carrier frequency and low modulation index. They are increased to an adequately high value with the help of frequency multipliers and mixer. The power level is raised to the desired level by the amplifier.

5. Explain about FM stereo multiplexing? [CO2-L2-Nov/Dec2012]

FM Stereo Multiplexing:

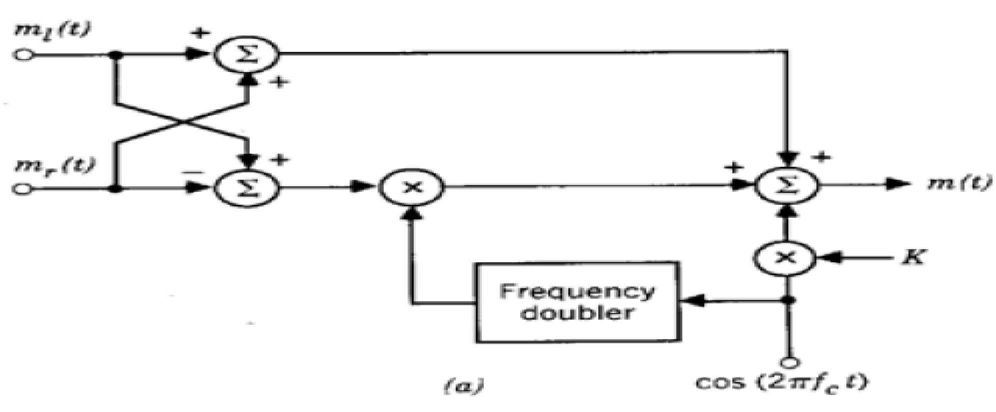
Stereo multiplexing is a form of frequency division multiplexing designed to transmit two separate signals via the same carrier. It is widely used in the FM radio broadcasting to send two different elements of a program. For example the different elements can be sections of orchestra, a vocalist and an accompanist. This gives a spatial dimension to its perception for the listener at the receiving end.

The two important factors that influence the FM stereo transmission are:

1. The transmission has to operate within the allocated FM broadcast channels.
2. It has to compatible with the monophonic receivers.

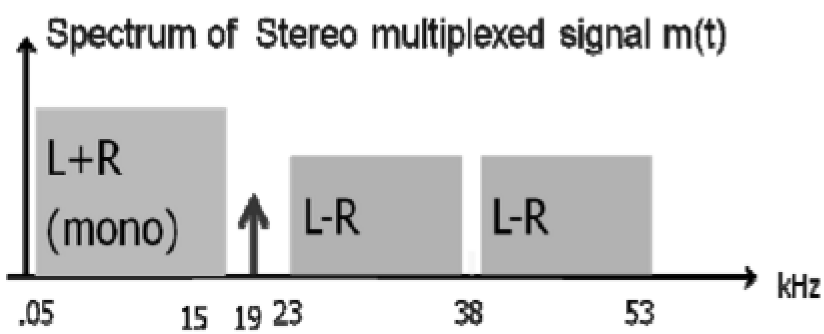
The FM stereo transmitter consists of a multiplexing system.

The block diagram of the multiplexer is shown in



fig

Let ml(t) and mr(t) denote the two signals from the two different microphones at the transmitter end of the system. They are applied to a matrixer that generates the sum signal
and the difference signal. The sum signal [ml(t)+mr(t)] is used in the base band form only.
The difference signal [ml(t) - mr(t)] along with a 38 kHz sub-carrier are applied to a product
modulator to generate a DSBSC modulated wave. The sub- carrier is generated from a frequency doubler using 19 kHz oscillator. The three signals: sum signal, difference signal
and a pilot carrier signal of frequency 19 kHz are combined/added to obtain the multiplexed
signal. The multiplexed signal can be defined as:



The multiplexed signal is used as a modulating signal for the FM modulator to produce an
FM signal for transmission.

6. Draw the frequency spectrum of FM and explain. Explain how Varactor diode can be used for frequency modulation. [CO2-L2-May/June2014] [8]

Frequency modulation: FM is that of angle modulation in which the instantaneous frequency fi(t) is varied linearly with the message signal m(t), as shown by

fi(t) =fc+kfm(t)

Where fc represents the frequency of the unmodulated carrier

kf represents the frequency sensitivity of the modulator(Hz/volt)

The frequency modulated wave s(t)=Accos[2πfct+2πkf otm(t)dt]

FM wave can be generated by first integrating m(t) and then using the result as the input to a phase modulator

PM wave can be generated by first differentiating m(t) and then using the result as the input to a frequency modulator. Frequency modulation is a Non-linear modulation process.

Single tone FM:

Consider $m(t)=A_m\cos(2\pi f_m t)$

The instantaneous frequency of the resulting FM wave

$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)$

$= f_c + \Delta f \cos(2\pi f_m t)$

where $\Delta f = k_f A_m$ is called as frequency deviation

$\theta(t) = 2\pi \int f_i(t)dt$

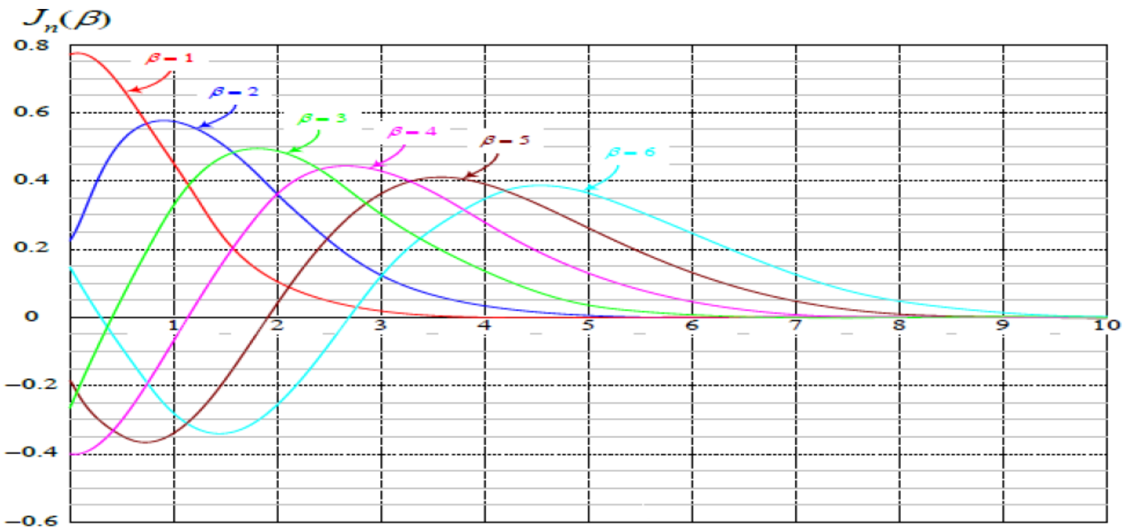
$= 2\pi f_c t + \Delta f / f_m \sin(2\pi f_m t)$

$= 2\pi f_c t + \beta \sin(2\pi f_m t)$

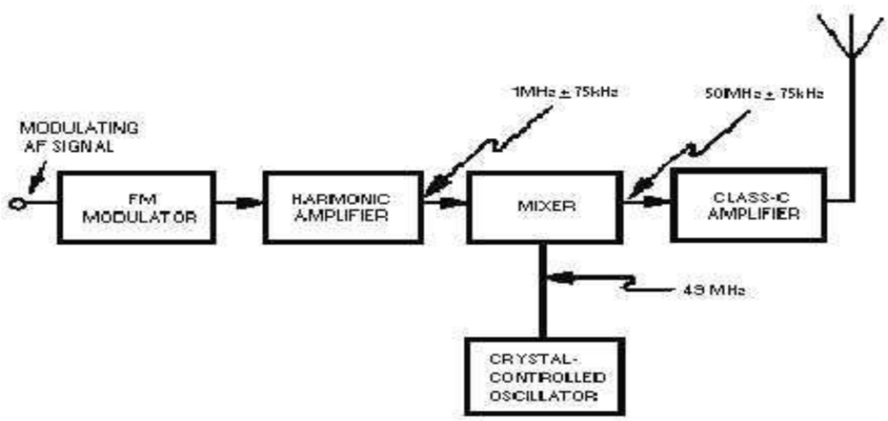
Where $\beta = \Delta f / f_m$ = modulation index of the FM wave

- When $\beta < 1$ radian then it is called as narrowband FM consisting essentially of a carrier, an upper side-frequency component, and a lower side-frequency component.
- When $\beta > 1$ radian then it is called as wideband FM which contains a carrier and an infinite number of side-frequency components located symmetrically around the carrier.

The envelope of an FM wave is constant, so that the average power of such a wave dissipated in a 1-ohm resistor is also constant.



Varactor Fm Modulator:



Another fm modulator which is widely used in transistorized circuitry uses a voltage-variable capacitor (VARACTOR). The varactor is simply a diode, or pn junction, that is designed to have a certain amount of capacitance between junctions. View (A) of figure 2 shows the varactor schematic symbol. A diagram of a varactor in a simple oscillator circuit is shown in view (B).

This is not a working circuit, but merely a simplified illustration. The capacitance of a varactor, as with regular capacitors, is determined by the area of the capacitor plates and the distance between the plates. The depletion region in the varactor is the dielectric and is located between the p and n elements, which serve as the plates. Capacitance is varied in the varactor by varying the reverse bias which controls the thickness of the depletion region. The varactor is so designed that the change in capacitance is linear with the change in the applied voltage. This is a special design characteristic of the varactor diode. The varactor must not be forward biased because it cannot tolerate much current flow. Proper circuit design prevents the application of forward bias.

7. Explain any two techniques of demodulation of FM. [CO2-L2-Nov/Dec2014]

FM demodulators

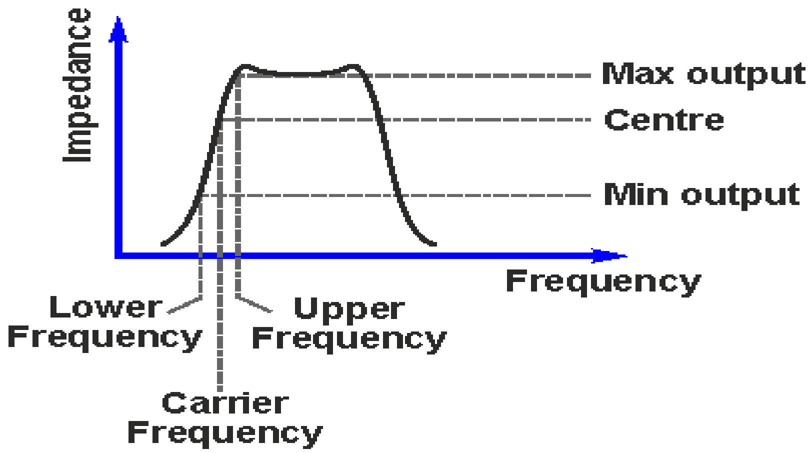
There are a number of circuits that can be used to demodulate FM. Each type has its own advantages and disadvantages, some being used when receivers used discrete components and others now that ICs are widely used.

Below is a list of some of the main types of FM demodulator or FM detector. In view of the widespread use of FM, even with the competition from digital modes that are widely used today, FM demodulators are needed in many new designs of electronics equipment.

FM Slope Detection Basics

The very simplest form of FM demodulation is known as slope detection or demodulation. It consists of a tuned circuit that is tuned to a frequency slightly offset from the carrier of the signal.

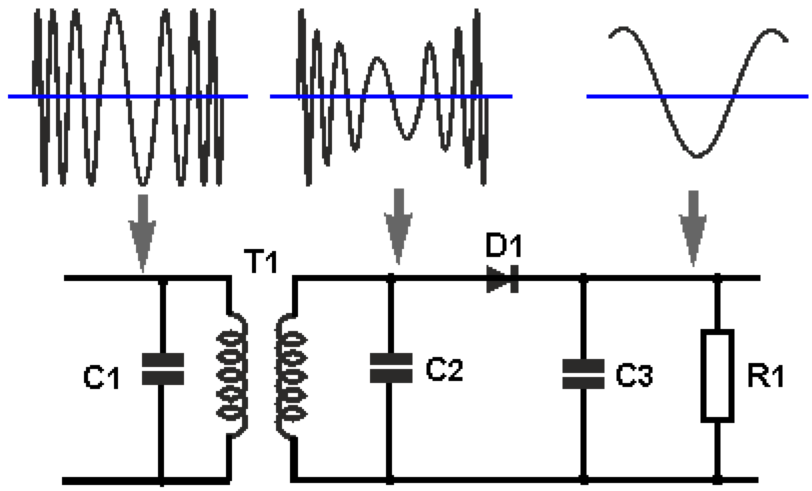
As the frequency of the signals varies up and down in frequency according to its modulation, so the signal moves up and down the slope of the tuned circuit. This causes the amplitude of the signal to vary in line with the frequency variations. In fact at this point the signal has both frequency and amplitude variations.



It can be seen from the diagram that changes in the slope of the filter, reflect into the linearity of the demodulation process. The linearity is very dependent not only on the filter slope as it falls away, but also the tuning of the receiver - it is necessary to tune the receiver off frequency and to a pint where the filter characteristic is relatively linear.

The final stage in the process is to demodulate the amplitude modulation and this can be achieved using a simple diode circuit. One of the most obvious disadvantages of this simple approach is the fact that both amplitude and frequency variations in the incoming signal appear at the output. However the amplitude variations can be removed by placing a limiter before the detector.

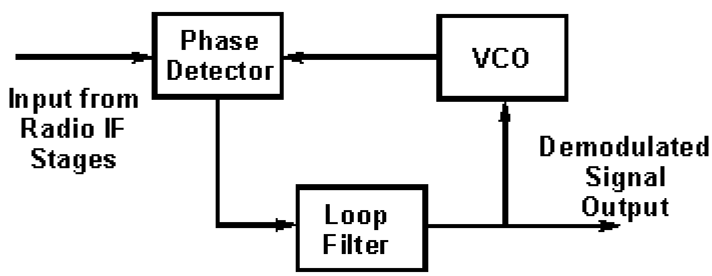
A variety of FM slope detector circuits may be used, but the one below shows one possible circuit with the applicable waveforms. The input signal is a frequency modulated signal. It is applied to the tuned transformer (T1, C1, C2 combination) which is offset from the centre carrier frequency. This converts the incoming signal from just FM to one that has amplitude modulation superimposed upon the signal.



This amplitude signal is applied to a simple diode detector circuit, D1. Here the diode provides the rectification, while C3 removes any unwanted high frequency components, and R1 provides a load.

PLL FM demodulation basics

The way in which a phase locked loop, PLL FM demodulator works is relatively straightforward. It requires no changes to the basic phase locked loop, itself, utilising the basic operation of the loop to provide the required output.



When used as an FM demodulator, the basic phase locked loop can be used without any changes. With no modulation applied and the carrier in the centre position of the pass-band the voltage on the tune line to the VCO is set to the mid position. However if the carrier deviates in frequency, the loop will try to keep the loop in lock. For this to happen the VCO frequency must follow the incoming signal, and in turn for this to occur the tune line voltage must vary. Monitoring the tune line shows that the variations in voltage correspond to the modulation applied to the signal. By amplifying the variations in voltage on the tune line it is possible to generate the demodulated signal.

PLL FM demodulator performance

The PLL FM demodulator is normally considered a relatively high performance form of FM demodulator or detector. Accordingly they are used in many FM receiver applications.

The PLL FM demodulator has a number of key advantages:

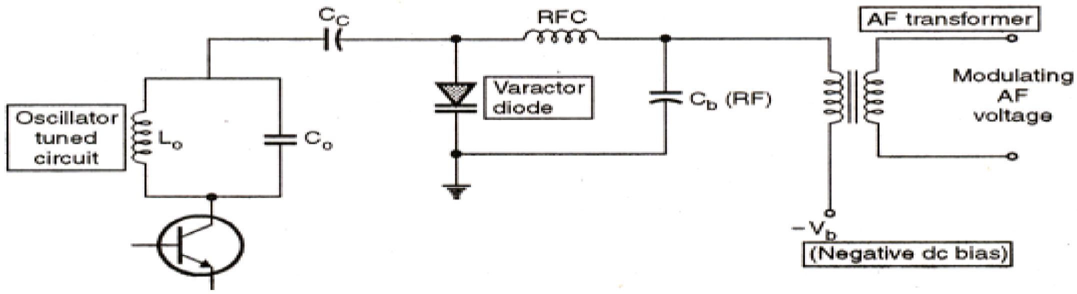
Linearity: The linearity of the PLL FM demodulator is governed by the voltage to frequency characteristic of the VCO within the PLL. As the frequency deviation of the incoming signal normally only swings over a small portion of the PLL bandwidth, and the characteristic of the VCO can be made relatively linear, the distortion levels from phase locked loop demodulators are normally very low. Distortion levels are typically a tenth of a percent.

Manufacturing costs: The PLL FM demodulator lends itself to integrated circuit technology. Only a few external components are required, and in some instances it may not be necessary to use an inductor as part of the resonant circuit for the VCO. These facts make the PLL FM demodulator particularly attractive for modern applications.

8. Discuss the direct method of generating a FM signal[CO2-L2-Nov/Dec2015]

Varactor Diode Modulator:

Varactor diode modulator is the direct method of FM generation wherein the carrier frequency is directly varied by the modulating signal. A varactor diode is a semiconductor diode whose junction capacitance varies linearly with applied voltage when the diode is reverse biased. Varactor diodes are used along with reactance modulator to provide automatic frequency correction for an FM transmitter. The varactor diode modulator circuit is shown in Fig5. for generation of FM wave.

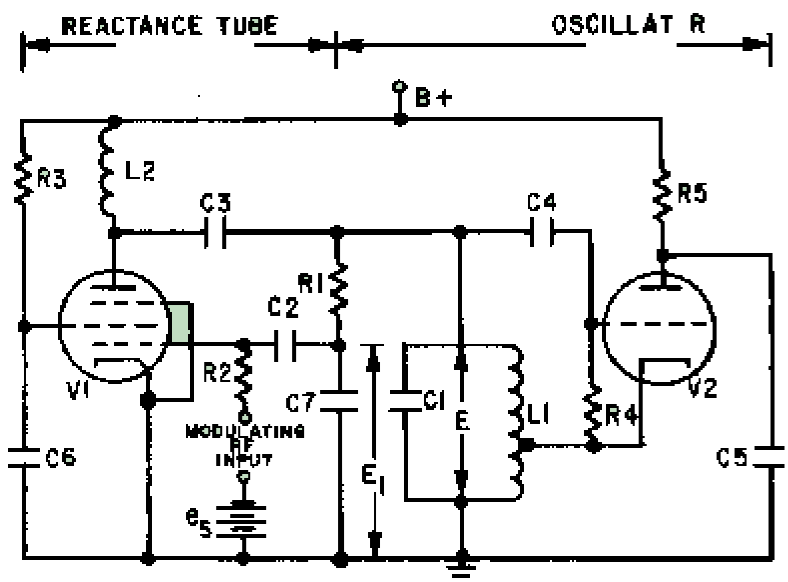


Varactor diode is arranged in reverse bias to offer junction capacitance effect. The modulating voltage which is in series with the varactor diode will vary the bias and hence the junction capacitance, resulting the oscillator frequency to change accordingly. The external modulating AF voltage adds to and subtracts from the dc bias, which changes the capacitance of the diode and thus the frequency of oscillation.Positive alternations of the modulating signal increase the reverse bias on the varactor diode, which decreases its capacitance and increases the frequency of oscillation. Conversely, negative alternations of the modulating signal decrease the frequency of oscillation.

The RFC and capacitor C b act as a filter which transmits only the AF variations to the varactor diode and blocks high frequency RF voltage from reaching the AF stage. The varactor diode FM modulators are widely accepted because they are simple to use, reliable and have the stability of a crystal oscillator. This method of FM generation is direct because the oscillator frequency is varied directly by the modulating signal, and the magnitude of frequency change is proportional to the amplitude of the modulating signal voltage. Varactor diode modulator is used for automatic frequency control and remote tuning. The drawback of varactor diode modulator is that since it uses a crystal, the peak frequency deviation is limited to relatively small values. Thus they are used mostly for low index applications such as two way mobile radio. Also since they are a two terminal device, the applications are quite limited.

Reactance Tube Modulator:

Reactance-Tube Modulation. - In direct modulation, an oscillator is frequency modulated by a REACTANCE TUBE that is in parallel (SHUNT) with the oscillator tank circuit. (The terms "shunt" or "shunting" will be used in this module to mean the same as "parallel" or "to place in parallel with" components.) This is illustrated in figure 2-11. The oscillator is a conventional Hartley circuit with the reactance-tube circuit in parallel with the tank circuit of the oscillator tube. The reactance tube is an ordinary pentode. It is made to act either capacitively or inductively; that is, its grid is excited with a voltage which either leads or lags the oscillator voltage by 90 degrees.



When the reactance tube is connected across the tank circuit with no modulating voltage applied, it will affect the frequency of the oscillator. The voltage across the oscillator tank circuit (L1 and C1) is also in parallel with the series network of R1 and C7. This voltage causes a current flow through R1 and C7. If R1 is at least five times larger than the capacitive reactance of C7, this branch of the circuit will be essentially resistive. Voltage E_1, which is across C7, will lag current by 90 degrees. E_1 is applied to the control grid of reactance tube V1. This changes plate current (I_p), which essentially flows only through the LC tank circuit. This is because the value of R1 is high compared to the impedance of the tank circuit. Since current is inversely proportional to impedance, most of the plate current coupled through C3 flows through the tank circuit.

Unit – III

Random Process

Part – A

1. Define random variables. **[CO3-L1-May/June2013]**

A random variable, usually written X, is a variable whose possible values are numerical outcomes of a random phenomenon. Random variable consists of two types they are discrete and continuous type variables.

2. What is meant by probability distribution?

The probability distribution of a discrete random variable is a list of probabilities associated with each of its possible values. It is also sometimes called the probability function or the probability mass function.

3. What are the conditions applied in the central limit theorem? [CO3-L2-May/June2012]

[1] The mean of the population of means is always equal to the mean of the parent population from which the population samples were drawn.

[2] The standard deviation of the population of means is always equal to the standard deviation of the parent population divided by the square root of the sample size (N).

[3] The distribution of means will increasingly approximate a normal distribution as the size N of samples increases.

4. Define stationary process. [CO3-L1-Nov/Dec2013]

Stationary process is a stochastic process whose joint probability distribution does not change when shifted in time. Consequently, parameters such as the mean and variance, if they are present, also do not change over time and do not follow any trends.

5.Write the equation for correlation? [CO3-L2-May/June2014]

The population correlation coefficient ρX,Y between two random variables X and Y with expected values μX and μY and standard deviations ςX and ςY is defined as:

$$\rho(x,y) = Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

6. What is meant by covariance? [CO3-L1]

Covariance is a measure of how much two variables change together, and the covariance function, or kernel, describes the spatial covariance of a random variable process or field.

$$C(x,y) = C0v(Z(x),Z(y))$$

7. Define random process. [CO3-L1-May/June2015]

A random process X(t) is a Gaussian process if for all n and all (t1 ,t2 ,…,tn), the random variables have a jointly Gaussian density function.

8. Write the equation of Autocorrelation? [CO3-L1-May/June2015]

The autocorrelation function of the output random process Y (t). By definition, we have RY (t, u) = E[Y (t)Y (u)]

where t and u denote the time instants at which the process is observe

9. Write the applications of random process? [CO3-L3-May/June2014]

The available noise power is directly proportional to temperature and it is independent of value of resistance. This power specified in terms of temperature is called as noise temperature. It is denoted by Te . It is given as,

 Te =(F −1)T A Gauss

functions in Bayesian inference.

 Wiener process (aka Brownian motion) is the integral of a white noise Gaussian process. It is not stationary, but it has stationary increments.

PART-B

1 Discuss about Central limit theorem in detail. [CO3-L2-May/June2012] [8]

Central Limit Theorem:

In probability theory, the central limit theorem (CLT) states that, given certain conditions, the arithmetic mean of a sufficiently large number of iterates of independent random variables, each with a well-defined expected value and well-defined variance, will be approximately normally distributed.

The Central Limit Theorem describes the characteristics of the "population of the means" which has been created from the means of an infinite number of random population samples of size (N), all of them drawn from a given "parent population". The Central Limit Theorem predicts that regardless of the distribution of the parent population:

[1] The mean of the population of means is always equal to the mean of the parent population from which the population samples were drawn.

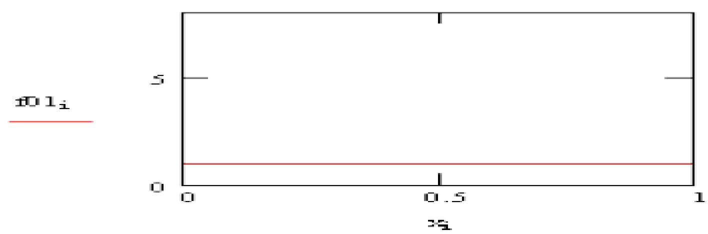
[2] The standard deviation of the population of means is always equal to the standard deviation of the parent population divided by the square root of the sample size (N).

[3] The distribution of means will increasingly approximate a normal distribution as the size N of samples increases.

A consequence of Central Limit Theorem is that if we average measurements of a particular quantity, the distribution of our average tends toward a normal one. In addition, if a measured variable is actually a combination of several other uncorrelated variables, all of them "contaminated" with a random error of any distribution, our measurements tend to be contaminated with a random error that is normally distributed as the number of these variables increases.Thus, the Central Limit Theorem explains the ubiquity of the famous bell-shaped "Normal distribution" (or "Gaussian distribution") in the measurements domain.

- Examples:

- Uniform distribution
- Triangular distribution
- 1/X distribution
- Parabolic distribution
- CLT Summary
- more statistical fine-print
- The uniform distribution on the left is obviously non-Normal. Call that the parent distribution

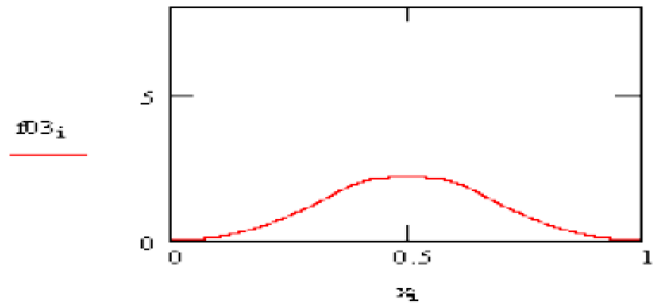


NonNormal Distribution of X

Non uniform distributions

To compute an average, Xbar, two samples are drawn, at random, from the parent distribution and averaged. Then another sample of two is drawn and another value of Xbar computed. This process is repeated, over and over, and averages of two are computed. The distribution of averages of two is shown on the left.

Repeatedly taking three from the parent distribution, and computing the averages, produce the probability density on the left.



Distribution of Xbar when sample size is 3

Distributions of Xbar

2. Explain in detail about Ergodic process. [CO3-L2-May/June2015] [8]

Ergodic process:

In the event that the distributions and statistics are not available we can avail ourselves of the time averages from the particular sample function. The mean of the sample function is referred to as the sample mean of the process X(t) and is defined as

$$\mu(X)T = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} X\lambda_0(t)dt$$

This quantity is actually a random-variable by itself because its value depends on the parameter sample function over it was calculated, the sample variance of the random process is defined as

$$\sigma^2(X)T = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} X\lambda_0(t)dt - \mu(X)Tdt$$

The time-averaged sample ACF is obtained via the relation is

$$(RXX)T = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} X(t) * X(t - Tdt(t)$$

$$\lim_{T--\infty} Var\{[\mu X]T\} = 0$$

It is similar sense a random process X(t) is said to be ergodic in ACF

$$\lim_{T--\infty} E\{[RXX(\tau)]\} = RXX(\tau)$$

$$\lim_{T--\infty} Var\{[RXX(\tau)]\} = 0$$

The concept of ergodicity is also significant from a measurement perspective because in Practical situations we do not have access to all the sample realizations of a random process. therefore have to be content in these situations with the time-averages that we obtain from a single realization. Ergodic processes are signals for which measurements based on a single sample function are sufficient to determine the ensemble statistics. Random signal for which this property does not hold are referred to as non-ergodic processes. As before the Gaussian random signal is an exception where strict sense eigodicity implies wide sense eigodicity.

3. Explain in detail about Random process and its Random variables. [CO3-L2-Nov/Dec2015] [8]

Random Variables:

Mathematically a random variable is neither random nor a variable

• It is a mapping from sample space into the real-line (“real-valued” random variable) or the complex plane (“complex-valued ” random variable) .

Suppose we have a probability space{S,$\Im$,P} . Let X : S → $\Re$be a function mapping the sample space S into the real line such that For each s ∈ S, there exists a unique X (s)∈$\Re$. Then X is called a random variable. Thus a random variable associates the points in the sample space with real numbers.

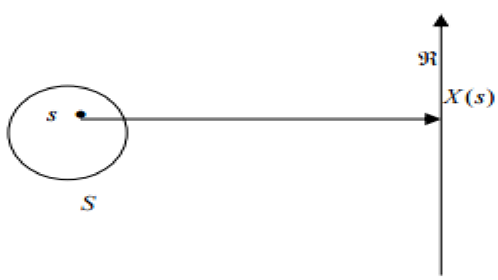


Figure Random Variable

Notations:

- Random variables are represented by upper-case letters.
- Values of a random variable are denoted by lower case letters
- $Y = y$ means that y is the value of a random variable X.

Discrete and Continuous Random Variables:

- A random variable X is called discrete if there exists a countable sequence of distinct real number x_i such that $\sum_i P_m(x_i) = 1$. $P_m(x_i)$ is called the ***probability mass function***. The random variable defined in Example 1 is a discrete random variable.
- A continuous random variable X can take any value from a continuous interval
- A random variable may also de mixed type. In this case the RV takes continuous values, but at each finite number of points there is a finite probability.

Probability Distribution Function:

We can determine the probability of any event involving values of the random variable A'.

$F_x(x)$ is a non-decreasing function of X.

$F_x(x)$ is right continuous

$F_x(x)$ approaches to its value from right

$F_x(-\infty) = 0$

$F_x(\infty) = 1$

Random Process:

- A random process can be defined as an indexed family of random variables $\{X(t),\ t \in T\}$ where T is an index set which may be discrete or continuous usually denoting time.
- The random process is defined on a common probability space $\{S, \Im, P\}$.
- A random process is a function of the sample point ξ and index variable t and may be written as $X(t, \xi)$.

- For a fixed $t(=t_0)$, $X(t_0,\xi)$ is a random variable.
- For a fixed $\xi(=\xi_0)$, $X(t,\xi_0)$ is a single realization of the random process and is a deterministic function.
- When both t and ξ are varying we have the random process $X(t,\xi)$.

The random process $X(t,\xi)$ is normally denoted by $X(t)$.

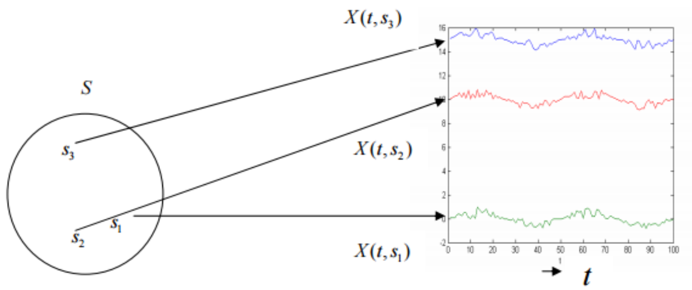


Figure Random Process

To describe $X(t)$ we have to use joint density function of the random variables at different t.

For any positive integer n, $X(t_1), X(t_2),.....X(t_n)$ represents n jointly distributed random variables. Thus a random process can be described by the joint distribution function $F_{X(t_1),X(t_2),....X(t_n)}(x_1,x_2.....x_n) = F(x_1,x_2.....x_n,t_1,t_2.....t_n), \forall n \in N$ and $\forall t_n \in T$

Otherwise we can determine all the possible moments of the process.

$E(X(t)) = \mu_x(t)$ = mean of the random process at t.

$R_X(t_1,t_2) = E(X(t_1)X(t_2))$ = autocorrelation function at t_1,t_2

$R_X(t_1,t_2,t_3) = E(X(t_1),X(t_2),X(t_3))$ = Triple correlation function at t_1,t_2,t_3, etc.

We can also define the auto-covariance function $C_X(t_1,t_2)$ of $X(t)$ given by

$$\begin{aligned} C_X(t_1,t_2) &= E(X(t_1)-\mu_X(t_1))(X(t_2)-\mu_X(t_2)) \\ &= R_X(t_1,t_2)-\mu_X(t_1)\mu_X(t_2) \end{aligned}$$

4. Write short notes on covariance function. [CO3-L2-May/June2012] [8]

In probability theory and statistics, covariance is a measure of how much two variables change together, and the covariance function, or kernel, describes the spatial covariance of a random variable process or field. For a random field or stochastic process Z(x) on a domain D, a covariance function C(x, y) gives the covariance of the values of the random field at the two locations x and y:

$$C(x,y) := \operatorname{cov}(Z(x), Z(y)).$$

The same C(x, y) is called the auto covariance function in two instances: in time series (to denote exactly the same concept except that x and y refer to locations in time rather than in space), and in multivariate random fields (to refer to the covariance of a variable with itself, as opposed to the cross covariance between two different variables at different locations, $Cov(Z(x_1), Y(x_2))$).

- ✓ **Mean & Variance of covariance functions:**

For locations $x_1, x_2, \ldots, x_N \in D$ the variance of every linear combination

$$X = \sum_{i=1}^{N} w_i Z(x_i)$$

can be computed as

$$\operatorname{var}(X) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i C(x_i, x_j) w_j.$$

A function is a valid covariance function if and only if this variance is non-negative for all possible choices of N and weights $w_1, \ldots, w_N$. A function with this property is called positive definite.

5. Write short notes on Auto correlation function. [CO3-L2-May/June2014] [8]

In statistics, the autocorrelation of a random process is the correlation between values of the process at different times, as a function of the two times or of the time lag. Let *X* be a stochastic process, and *t* be any point in time. (*t* may be an integer for a discrete-time process or a real number for a continuous process.) Then X_t is the value (or realization) produced by a given run of the process at time *t*. suppose that the process has mean μ_t and variance σ_t^2 at time *t*, for each *t*. Then the definition of the autocorrelation between times *s* and *t* is

$$R_i^t(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T [x_i(t) - M^t\{x_i(t)\}][x_i(t+\tau) - M^t\{x_i(t+\tau)\}]dt$$

- τ is the correlation variable (time shift).
- $|R_i^t|$ is between 0 and 1.
- If R_i^t is large (i.e. $R_i^t(\tau) \to 1$) then $x_i(t)$ and $x_i(t+\tau)$ are "similar". For example, a sinusoidal function is similar to itself delayed by one or more periods.
- If R_i^t is small then $x_i(t)$ and $x_i(t+\tau)$ are not similar – for example white noise would result in $R_i^t(\tau) = 0$.

6. With neat diagram Explain Linear filtering of Random process? [CO3-L2-May/June2015][8]

• A random process X(t) is applied as input to a linear time-invariant filter of impulse response h(t),

• It produces a random process Y (t) at the filter output as

X(t) →→→→→h(t)→→→ Y(t)

• Difficult to describe the probability distribution of the output random process Y (t), even when the probability distribution of the input random process X(t) is completely specified for $-\infty \le t \le +\infty$.

• Estimate characteristics like mean and autocorrelation of the output and try to analyse its behaviour.

• Mean The input to the above system X(t) is assumed stationary. The mean of the output random process Y (t) can be calculated

$$mY(t) = E[Y(t)] = E[\int_{-\infty}^{\infty} h(\tau)X(t - \tau)\, d\tau]$$

$$= \int_{-\infty}^{\infty} h(\tau)E[X(t - \tau)]\, d\tau$$

$$= mX \int_{-\infty}^{\infty} h(\tau)\, d\tau$$

$$= mXH(0)$$

where H(0) is the zero frequency response of the system.

✓ **Autocorrelation:**

The autocorrelation function of the output random process Y (t). By definition, we have

$$RY(t, u) = E[Y(t)Y(u)]$$

where t and u denote the time instants at which the process is observed. We may therefore use the convolution integral to write

$$RY(t, u) = E\left[\int_{-\infty}^{\infty} h(\tau 1)X(t - \tau 1)\, d\tau 1\right] \int_{\infty}^{\infty} h(\tau 2)X(t - \tau 2)\, d\tau 2\ ]$$

$$= \int_{-\infty}^{\infty} h(\tau 1)\, d\tau 1 \int_{-\infty}^{\infty} h(\tau 2)E\left[X(t - \tau 1)X(t - \tau 2)\right] d\tau 2$$

When the input X(t) is a wide-stationary random process, autocorrelation function of X(t) is only a function of the difference between the observation times $t - \tau 1$ and $u - \tau 2$.

Putting $\tau = t - u$, we get

$$RY(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\tau 1)h(\tau 2)RX(\tau - \tau 1 + \tau 2)\, d\tau 1\, d\tau 2$$

$$RY(0) = E[Y^2(t)]$$

The mean square value of the output random process Y (t) is obtained by putting $\tau = 0$ in the above equation.

$$E[Y^2(t)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\tau 1)h(\tau 2)RX(\tau 2 - \tau 1)\, d\tau 1\, d\tau 2$$

The mean square value of the output of a stable linear time-invariant filter in response to a wide-sense stationary random process is equal to the integral over all frequencies.

Unit – IV

Noise Characterization

Part – A

1. Define noise. [CO4-L1-May/June2013]

Noise is defined as any unwanted form of energy, which tends to interfere with proper reception and reproduction of wanted signal.

2. Give the classification of noise. [CO4-L1]

Noise is broadly classified into two types. They are

(i)External noise

(ii)Internal noise.

3. What are the types of External noise? [CO4-L1]

External noise can be classified into

1. Atmospheric noise
2. Extraterrestrial noises
3. Man –made noises or industrial noises

4. What are types of internal noise? [CO4-L1]

Internal noise can be classified into

1. Thermal noise
2. Shot noise
3. Transit time noise
4. Miscellaneous internal noise

5. What are the types of extraterrestrial noise and write their origin? [CO4-L2-Nov/Dec2014]

The two type of extraterrestrial noise are solar noise and cosmic noise Solar noise is the electrical noise emanating from the sun. Cosmic noise is the noise received from the center part of our galaxy, other distant galaxies and other virtual point sources.

6. Define transit time of a transistor.

Transit time is defined as the time taken by the electron to travel from emitter to the collector.

7. Define flicker noise. [CO4-L2-May/June2011]

Flicker noise is the one appearing in transistors operating at low audio frequencies. Flicker noise is proportional to the emitter current and junction temperature and inversely proportional to the frequency.

8. State the reasons for higher noise in mixers. [CO4-L2-May/June2013]

1. Conversion transconductance of mixers is much lower than the transconductance of amplifiers.

2. If image frequency rejection is inadequate, the noise associated with the image frequency also gets accepted.

9. Define signal to noise ratio. [CO4-L1]

Signal to noise ratio is the ratio of signal power to the noise power at the same point in a system.

10. Define thermal noise. Give the expression for the thermal noise voltage across a resistor. [CO4-L2-May/June2013]

The electrons in a conductor possess varying amounts of energy. A small fluctuation in this energy produces small noise voltages in the conductor. These random fluctuations produced by thermal agitation of the electrons are called thermal noise.

11. Define noise temperature. (In terms of hypothetical temperature) [CO4-L1-May/June2012]

The available noise power is directly proportional to temperature and it is independent of value of resistance. This power specified in terms of temperature is called as noise temperature. It is denoted by Te . It is given as,

$T_e = (F - 1)T$

12. What is shot noise? [CO4-L1-May/June2011]

When current flows in electronic device, the fluctuations number of electrons or holes generates the noise. It is called shot noise. Shot noise also depends upon operating conditions of the device.

13. Give the expression for noise voltage in a resistor. [CO4-L2-May/June2013]

The Mean –Square value of thermal noise voltage is given by,

K – Boltz man constant, R – Resistance

T – Obsolute temperature, B Bandwidth

$V_n^2 = 4\,kTBR$

14. What is White Noise? [CO4-L1]

Many types of noise sources are Gaussian and have flat spectral density over a wide frequency range. Such spectrum has all frequency components in equal portion, and is therefore called white noise. The power spectral density of white noise is independent of the operating frequency. The Power spectral density of White Noise is given as, S (f) = N o/2

15. What is narrowband noise? [CO4-L1-May/June2013]

The receiver of a communication system usually includes some provision for preprocessing the received signal. The preprocessing may take the form of a narrowband filter whose bandwidth is large enough to pass modulated component of the received signal essentially undistorted but not so large as to admit excessive noise through the receiver. The noise process appearing at the output
of such filter is called narrow band noise.

16. Define noise equivalent bandwidth. [CO4-L1-May/June2013]

The noise equivalent bandwidth of the filter is defined as the bandwidth of an ideal filter at which the noise power passed by real filter and ideal filter is same.

17. Define noise factor. [CO4-L1]

Noise factor (F) is defined as the ratio of signal to noise power ratio at the input to signal to noise power ratio at the output

18. Give the characteristics of shot noise. [CO4-L1]

(i) Shot noise is generated due to fluctuations in the number of electrons or holes. (ii) Shot noise has uniform spectral density.
(iii) Mean square noise current depends upon direct component of current. (iv) Shot noise depends upon operating conditions of the device.

19. What is FM threshold effect? [CO4-L1-May/June2014]

As the carrier to noise ratio is reduced, clicks are heard in the receiver output. As the carrier to noise ratio reduces further, crackling, or sputtering sound appears at the receiver output. Near the breaking point, the theoretically calculated output signal to noise ratio becomes large, but its actual value is very small. This phenomenon is called threshold effect.

20. What is capture effect in FM? [CO4-L1-May/June2015]

When the noise interference is stronger than FM signal, then FM receiver locks to interference. This suppresses FM signal. When the noise interference as well as FM signal are of equal strength, then the FM receiver locking fluctuates between them. This phenomenon is called capture effect.

21. What is meant by figure of merit of a receiver?

The ratio of output signals to noise ratio to channel signal to noise ratio is called figure of merit.

22. What is the Purpose of re-emphasis and de-emphasis in FM? [CO4-L2-May/June2012]

The PSD of noise at the output of FM receiver sally increases rapidly at high frequencies but the PSD of message signal falls off at higher frequencies. This means the message signal doesn‘t utilize the frequency band in efficient manner. Such more efficient use of frequency band and improved noise performance can be obtained with the help of re-emphasis and deemphasis.

23. What are extended threshold demodulators? [CO4-L1-Nov/Dec2013]

Threshold extension s also called threshold reduction. It is achieved with the help of FMFB demodulator. In the local oscillator is replaced by voltage controlled oscillator (VCO).The VC frequency changes as per low frequency variations of demodulated signal. Thus the receiver responds only to narrow band of noise centered around instantaneous carrier frequency. This reduces the threshold of FMFB receiver.

24. What is threshold effect with respect to noise? [CO4-L1-May/June2015]

When the carrier to noise ratio reduces below certain value, the message information is lost. The performance of the envelope detector deteriorates rapidly and it has no proportion with carrier to noise ratio. This is called threshold effect.

25. Define pre-emphasis and de-emphasis. [CO4-L1-May/June2012]

Pre-emphasis: It artificially emphasizes the high frequency components before modulation.This equalizes the low frequency and high frequency portions of the PSD and complete band is occupied.

De-emphasis: This circuit attenuates the high frequency components. The attenuation characteristic is exactly opposite to that of pre-emphasis circuit. De-emphasis restores the power distribution of the original signal. The signal to noise ratio is improved because of pre-emphasis and de-emphasis circuits.

26. Define signal to noise ratio. **[CO4-L1]**

Signal to noise ratio is the ratio of signal power to the noise power at the same point in a system.

27. What is threshold effect in an envelope detector? Explain. [CO4-L2-May/June2011]

When a noise is large compared to the signal at the input of the envelope detector, the detected output has a message signal completely mingled with noise. It means that if the input SNR is below a certain level, called threshold level, the noise dominates over the message signal, threshold is defined as value of the input signal to noise ratio (So/No) below which the output signal to noise ratio (Si/Ni) deteriorates much more rapidly than the input signal to noise ratio. The threshold effect in an envelope detector whenever the carrier power-to-noise power ratio approaches unity or less.

PART-B

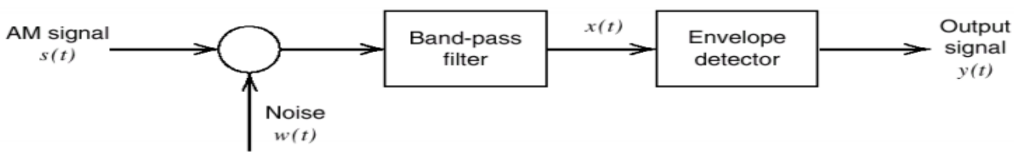
1. Discuss the noise performance of AM system using envelope detection. [CO4-L2-May/June2013] [8]

A full AM signal is given by

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

where $A_c\cos(2\pi f_c t)$ is the carrier wave, $m(t)$ is the message signal and bandwidth is W, k_a is a constant that determines the percentage modulation.

We would like to perform noise analysis for an AM system using an envelope detector.



We perform the noise analysis of the AM receiver by first determining the channel signal-to-noise ratio, and then the output signal-to-noise ratio.

We can easily obtain average power of the AM signal

$$s(t) = A_c\cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t)$$

$$P_s = \frac{1}{2}A_c^2 + \frac{1}{2}A_c^2 k_a^2 P$$

The average power of noise in the message bandwdith is WN_0 (the same as the DSB-SC system)

The channel signal-to-noise ratio for AM is therefore:

$$(SNR)_{C,AM} = \frac{A_c^{\,2}(1 + K_a^{\,2})}{2wN_0}$$

The dc term or constant term A_c may be removed simply by means of a blocking capacitor.

If we ignore the dc term A_c, we find that the remainder has a form similar to the output of a DSB-SC receiver using coherent detection.

The output signal-to-noise ratio of an AM using an envelope detector is approximately

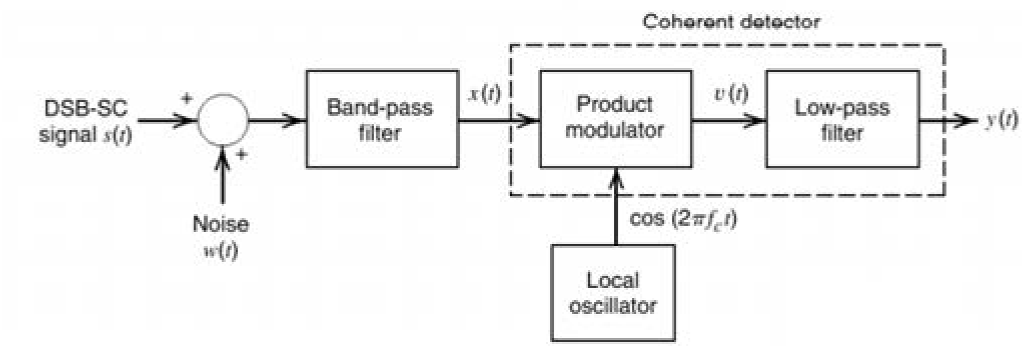
$$(SNR)_{0,AM} = \frac{A_C^{\,2} K_a^{\,2} P}{2wN_0}$$

The average noise power is small compared to the average carrier power at the envelope detector input.

The amplitude sensitive k_a is adjusted for a percentage modulation less than or equal to 100 percent. ($|k_a m(t)| <= 1$)

2. Discuss the noise performance of DSB-SC Receiver [CO4-L2-May/June2015] [16]

The model of a DSB-SC receiver using a coherent detector



The amplitude of the locally generated sinusoidal wave is assumed to be unity.

For the demodulation scheme to operate satisfactorily, it is necessary that the local oscillator be synchronized both in phase ^and in frequency with the oscillator generating the carrier wave i the transmitter. We assume that this synchronization has been achieved.

The DSB-SC component ot the modulated signal *s{t)* is expressed

$$s(t) = CA_c\cos(2\pi f_c t)m(t)$$

where C is the system dependent scaling factor. The purpose of which is to ensure that the signal component $s(t)$ is measured in the same units as the additive noise component.

$m(t)$ is the sample function of a stationary process of zero mean, whose power spectral density is limited to a maximum frequency W, i.e. W is the message bandwidth. The average power P of the message signal is the total area under the curve of power spectral density

$$P = \int S_M(f)df$$

In Example Process : Mixing of a Random Process with a Sinusoidal

$$Y(f) = X(t)\cos(2\pi f_c t + \Theta)$$

$$R_Y(\tau) = \frac{1}{2} R_X(\tau)\cos(2\pi f_c \tau)$$

$$S_Y(f) = \frac{1}{4}\left[S_X(f - f_c) + S_X(f + f_c)\right]$$

Therefore, the average power of the DSB-SC modulated signal component $s(t)$ is given by: $\dfrac{C^2 A_c^2 P}{2}$

The average power of the noise in the message BW is WN_0

The channel signal-to-noise ratio of the DSB-SC modulation system is:

$$(\text{SNR})_{C,\text{DSB}} = \frac{C^2 A_c^2 P}{2WN_0}$$

Next, we wish to determine the output signal-to-noise ratio.

Using the narrowband representation of the filtered noise $n(t)$, the total signal at the coherent detector input may be expressed as:

$$\begin{aligned} x(t) &= s(t) + n(t) \\ &= CA_c \cos(2\pi f_c t) m(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \end{aligned}$$

The output of the product-modulator component of the coherent detector is:

$$\begin{aligned} \upsilon(t) &= x(t)\cos(2\pi f_c t) \\ &= \frac{1}{2} CA_c m(t) + \frac{1}{2} n_I(t) \end{aligned}$$

$\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos(\alpha)\cos(\beta)$
$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin(\alpha)\cos(\beta)$

$$+\frac{1}{2}\left[CA_c m(t)+n_I(t)\right]\cos(4\pi f_c t)-\frac{1}{2}A_c n_Q(t)\sin(4\pi f_c t)$$

$$\Big\Downarrow \quad \boxed{\text{low-pass filter, BW}=W}$$

$$y(t)=\frac{1}{2}CA_c m(t)+\frac{1}{2}n_I(t)$$

The receiver output signal : $y(t)=\frac{1}{2}CA_c m(t)+\frac{1}{2}n_I(t)$

The average power of message component may be expressed as

$$P_{avg} = \frac{C^2 A_c^2 P}{4}$$

The average power of the noise at the receiver output is

$$\left(\frac{1}{2}\right)^2 2WN_0 = \frac{1}{2}WN_0$$

The output signal to noise ratio at the receiver output is

$$\frac{1}{2}^2 2wN_0 = \frac{1}{2}wN_0$$

The output signal noise ratio for DSB-SC

$$(SNR)_{O,DSB\ SC} = \frac{C^2 P A_C{}^2/4}{wN_0/2}$$

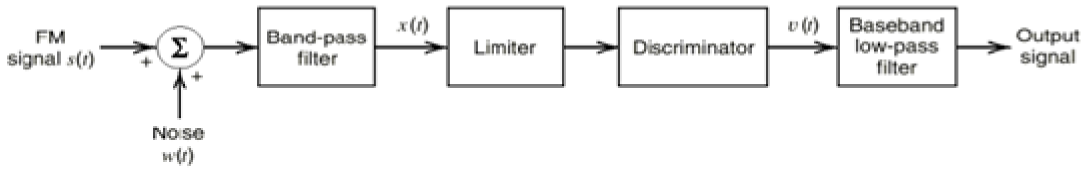
$$(SNR)_{O,DSB\ SC} = \frac{C^2 P A_C{}^2}{2wN_0}$$

We obtain the figure of merit

$$\frac{(SNR)_{O,}}{(SNR)_C} = 1$$

3. Explain the noise performance of FM Receiver [CO4-L2-May/June2014] [16]

The receiver model is given by:



The noise *w(t)* is modeled as white Gaussian noise of zero mean and power spectral density *N/2.*

The received FM signal *s{t)* has a carrier frequency X arid transmission handwidth $R_{7 \cdot t}$ such that only a negligihle amount of power lies outside the frequency band^ ± $R_7 I1$ for positive frequencies

The band-pass filter has a mid-band frequency^, and bandwidth B_T and therefore passes the FM signal essentially without distortion.

The message signal and In-phase noise component NI(t) of the filtered noise n(t) appear additively at the receiver output. The quadrature component NQ(t) of the noise n(t) is completely rejected by the coherent detector.

We note that these two results are independent of the input signal-to-noise ratio.

Thus, coherent detection distinguishes itself from other demodulation techniques in the important property: the output message component is unmutilated and the noise component always appears additively with the message, irrespective of the input signal-to-noise ratio.

In an FM system, the message information is transmitted by variations of"the instantaneous frequency of"a sinusoidal carrier wave, and its amplitude is maintained constant.

Any variations of* the carrier amplitude at the receiver input must result from noise or interference.

The Limiter is used to remove amplitude variations by clipping the modulated wave at the filter output almost to the zero axis. The resulting rectangular wave is rounded off by another bandpass filter that is an integral part of the limiter. thereby suppressing harmonics of the carrier frequency. <- The filler output is again sinusoidal, with an amplitude that is practically independent of the carrier amplitude at the receiver in put.

The *discriminator* consists of two components:

A *slope network* or *differentiator* with a purely imaginary transfer function that varies linearly with frequency. It produces a hybrid-modulated wave in which both amplitude and frequency vary in accordance with the message signal.

An *envelope detector* that recovers the amplitude variation and thus

reproduces the message signal, o The slope network and envelope detector are usually implemented as integral parts of a single physical unit.

The post-detection filter, labeled "baseband low-pass filter," has a bandwidth that is just large enough to accommodate the highest frequency component of the message signal, This filter removes the out-of-band components of the noise at the discriminator output and thereby keeps the effect of the utput noise to a minimum

- The filtered noise at the band-pass filter output is defined as:

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$n(t) = r(t)\cos\left[2\pi f_c t + \psi(t)\right]$$

$$r(t) = \left[n_I^2(t) + n_Q^2(t)\right]^{1/2} \sim \text{Rayleigh distribution}$$

$$\psi(t) = \tan^{-1}\left[\frac{n_Q(t)}{n_I(t)}\right] \qquad \sim \mathrm{U}(0,2\pi)$$

- The incoming FM signal $s(t)$ is given by

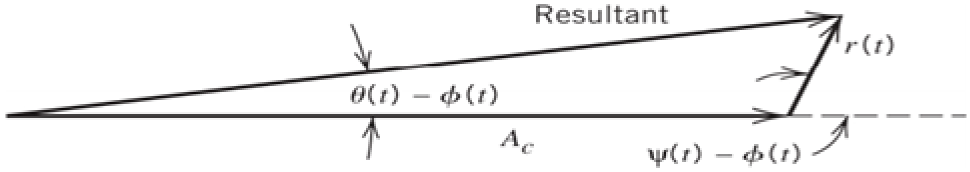
$$s(t) = A_c\cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right]$$

$$\phi(t) = 2\pi k_f \int_0^t m(\tau)\,d\tau$$

$$s(t) = A_c\cos\left[2\pi f_c t + \phi(t)\right]$$

The noisy signal at the band-pass filter output is:

$$x(t) = s(t) + n(t) = A_c\cos\left[2\pi f_c t + \phi(t)\right] + r(t)\cos\left[2\pi f_c t + \psi(t)\right]$$



$$\theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin\left[\psi(t) - \phi(t)\right]}{A_c + r(t)\cos\left[\psi(t) - \phi(t)\right]}\right\}$$

The envelope of $x(t)$ is of no interest to us, because any envelope variations at the band-pass output are removed by the limiter.

Our motivation is to determine the error in the instantaneous frequency of the carrier wave caused by the presence of the filtered noise $n(f)$.

With the discriminator assumed ideal, its output is proportional to t/2 where is the derivative of with respect to time.

We need to make certain simplifying approximations so that our analysis may yield useful results.

$$\theta(t)=\phi(t)+\tan^{-1}\left\{\frac{r(t)\sin[\psi(t)-\phi(t)]}{A_c+r(t)\cos[\psi(t)-\phi(t)]}\right\}$$

$$\theta(t)\simeq\phi(t)+\frac{r(t)}{A_c}\sin[\psi(t)-\phi(t)]$$

$$\theta(t)\simeq 2\pi k_f\int_0^t m(t)dt+\frac{r(t)}{A_c}\sin[\psi(t)-\phi(t)]$$

$$\upsilon(t)=\frac{1}{2\pi}\frac{d\theta(t)}{dt}\simeq k_f m(t)+n_d(t) \qquad (6.31)$$

(1) when CNR $\gg 1$, $A_c \gg r(t)$

$$\frac{r(t)\sin[\psi(t)-\phi(t)]}{A_c+r(t)\cos[\psi(t)-\phi(t)]} \sim \frac{r(t)\sin[\psi(t)-\phi(t)]}{A_c}$$

(2) $x \ll 1$, $\tan^{-1}(x)\approx x$

This means that the additive noise appearing at the discriminator output is determined effectively by the carrier amplitude A_c and the quadrature component $n_Q(t)$ of the narrowband noise n(t).

- In FM system, increasing the carrier power has a noise-quieting effect.

  Average power of output noise $=\dfrac{N_0}{A_c^2}\displaystyle\int_{-W}^{W} f^2 df=\dfrac{2N_0W^3}{3A_c^2}\propto\dfrac{2}{A_c^2}$

- We obtain $(\mathrm{SNR})_{O,\mathrm{FM}}=\dfrac{k_f^2P}{2N_0W^3/3A_c^2}=\dfrac{3A_c^2k_f^2P}{2N_0W^3}$

- The average power in the modulated signal $s(t)$ is $A^2{}_c/2$, and the average noise power in the message bandwidth is WN_o. The channel signal to noise ratio $(\mathrm{SNR})_{C,FM}$ is

$$(\mathrm{SNR})_{C,\mathrm{FM}}=\frac{A_c^2}{2WN_0}$$

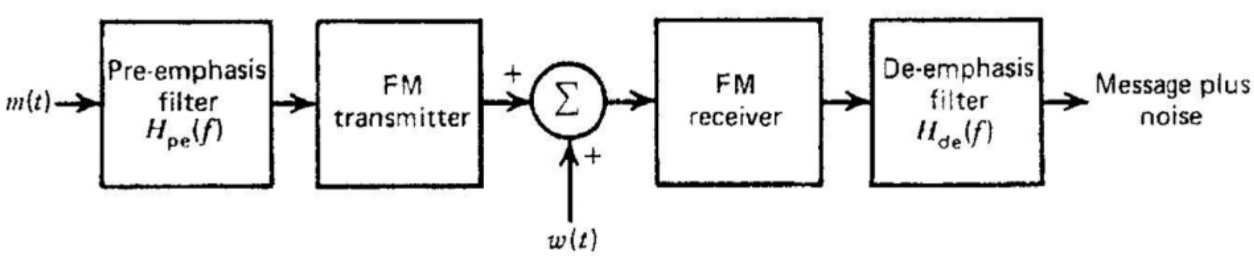
- Figure of merit for frequency modulation:

$$\left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\mathrm{FM}}=\frac{3k_f^2P}{W^2}$$

4. Compare the noise performance of AM and FM systems. [CO4-H1-ay/June2014]

Sr. No.	Parameter	AM envelope detection (Nonlinear)	AM DSBSC or SSB linear detection	FM
1.	(SWK)$_o$and (SNK)$_c$	(SNR)$_o$ ^(SWff)$_c$ *orm$_a$ = 1 modulation index	(S*K)$_o$-(S*R)$_c$	(SNr7)$_o$«|m2(SWR)$_e$ m, is modulation index
2.	Bandwidth B_7	2 W	2w (osesc) W *(SSBSC)*	8 *W* for m, - 2 *161V* form, ■ S
3.	Threshold effect	Present	Absent	Present
4.	Noise performance	Poor	Better	Good

5. Explain the significance of pre-emphasis and de-emphasis in FM systems.
[CO4-L2-May/June2015] [8]

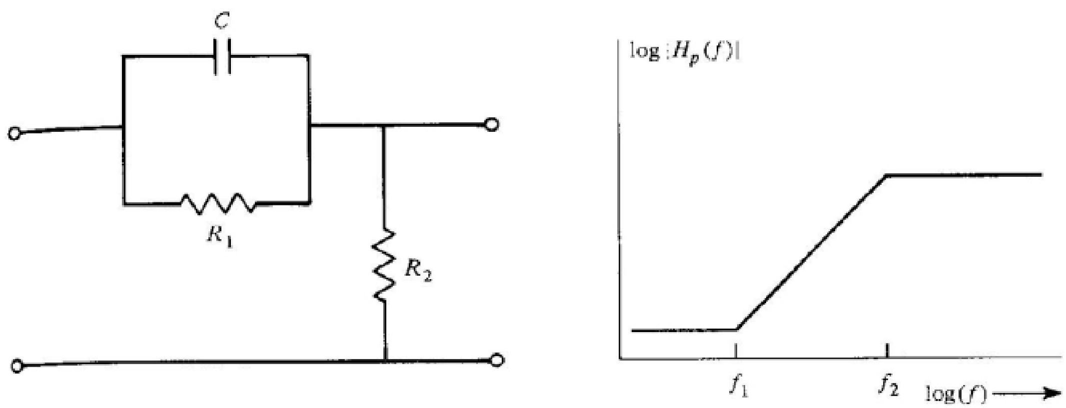


Hpe(f): used to artificially emphasize the high frequency components of the message prior to modulation, and hence, before noise is introduced. Hde(f): used to de-emphasize the high frequency components at the receiver, and restore the original PSD of the message signal. In theory, Hpe(f) f , Hde(f) 1/f . This can improve the output

SNR by around 13 dB.Dolby noise reduction uses an analogous pre-emphasis technique to reduce the effects of noise (hissing noise in audiotape recording is also concentrated on high frequency).

Pre-Emphasis:-

In an FM system the higher frequencies contribute more to the noise than the lower frequencies. Because of this all FM systems adopt a system of pre-emphasis where the higher frequencies are increased in amplitude before being used to modulate the carrier. The transfer function sketched above is used for a pre-emphasis circuit for FM signals in the FM band. The Time T = 75µs. For FM systems in the FM band m ~ 5 resulting in a S/N improvement of 19dB. With preemphasis this can be increased by 4dB for a total of 23dB.



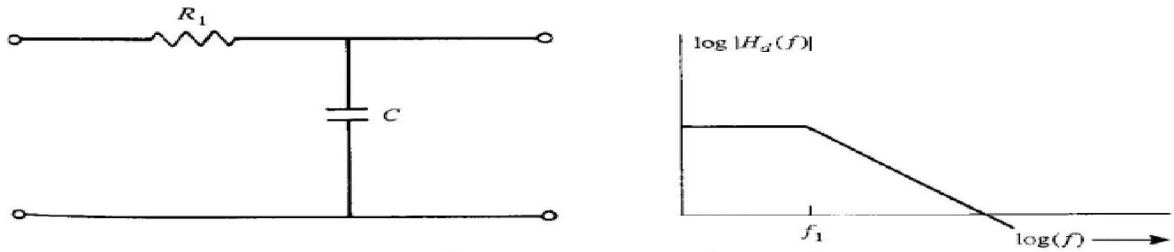
$$H_p(f) = K\,\frac{1+j(f/f_1)}{1+j(f/f_2)} \quad \text{where } f_1 = \frac{1}{2\pi\tau_1} = \frac{1}{2\pi R_1 C},\; f_2 = \frac{1}{2\pi\tau_2} = \frac{R_1+R_2}{2\pi R_1 R_2 C}$$

(a) Preemphasis Filter

(b) Bode Plot of Preemphasis Frequency Response

De-Emphasis:-

At the receiver the higher frequencies must be deemphasized in order to get back the original baseband signal. The transfer function of the de-emphasis circuit is shown above.

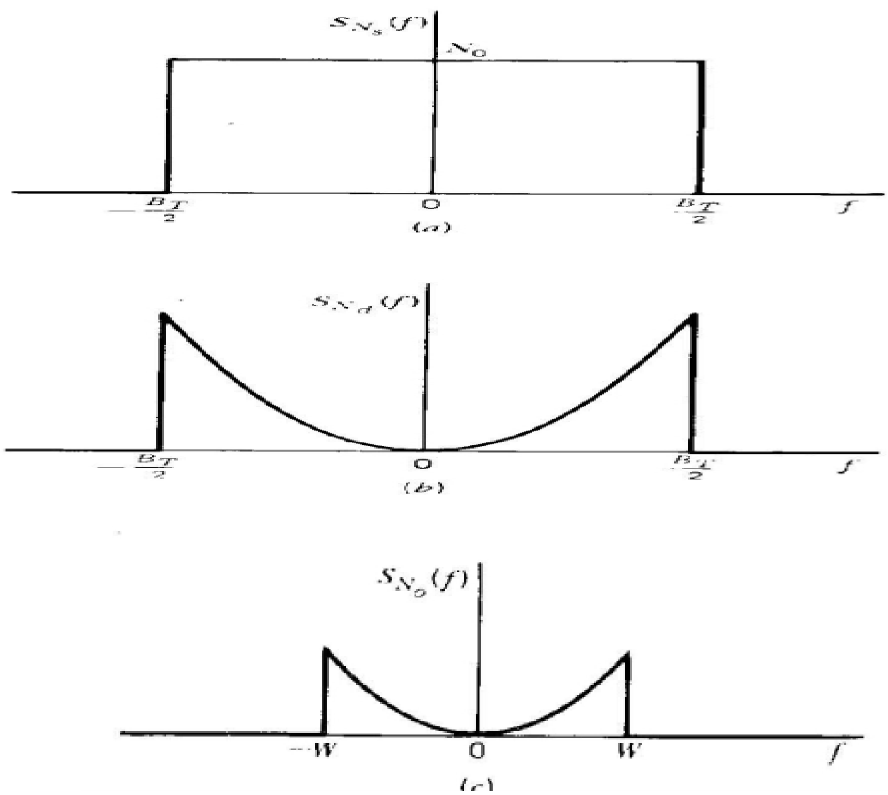


$H_d(f) = \frac{1}{1 + j(f/f_1)}$ where $f_1 = \frac{1}{2\pi\tau_1} = \frac{1}{2\pi R_1 C}$

(c) Deemphasis Filter (d) Bode Plot of Deemphasis Characteristic

6. Derive the noise power spectral density of the FM demodulation and explain its performance with diagram. [CO4-H1-Nov/Dec2014] [8]

Power spectral densities for FM noise analysis:



Average noise power at the receiver output:

$$P_N = \int_{-W}^{W} S_D(f)df$$

Thus,

$$P_N = \int_{-W}^{W} \left(\frac{1}{2\pi A}\right)^2 |j2\pi f|^2 N_0 df = \frac{2N_0W^3}{3A^2}$$

Average noise power at the output of a FM receiver $\propto \frac{1}{\text{carrier power } A^2}$

A Noise , called the *quieting effect*

$$SNR_O = \frac{3A^2k_f^2P}{2N_0W^3} \equiv SNR_{FM}$$

Transmitted power of an FM waveform:

$$P_T = \frac{A^2}{2}$$

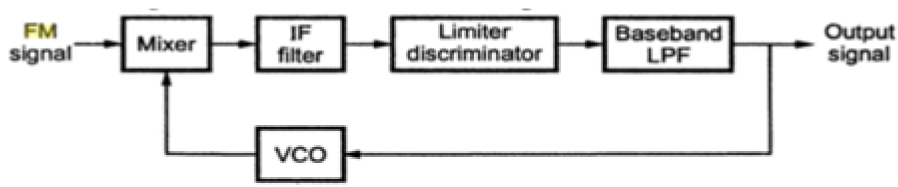
From $SNR_{baseband} = \frac{P_T}{N_0W}$:

$$SNR_{FM} = \frac{3k_f^2P}{W^2} SNR_{baseband} = 3\beta^2 \frac{P}{m_p^2} SNR_{baseband}$$

Where

7. Explain the FM threshold reduction and capture effect in FM? [CO4-L2-May/June2013]

FM Threshold Reduction or Extension (FMFB Demodulator) The threshold reduction or extension can be achieved in FM demodulator with negative feedback. Fig. shows the block diagram of FMFB demodulator.



FMFB demodulator

In the above figure observe that local oscillator is replaced by voltage controlled oscillator (VCO). The instantaneous output frequency of VCO is controlled by demodulated signal. Therefore output frequency of VCO changes as per low frequency variations of demodulated signal, in other words VCO frequency does not depends upon high frequency variations of narrowband noise. Thus FMFB demodulator acts as a tracking filter. It tracks only the slowly varying frequency of wideband FM waves. Therefore it responds only to narrowband of noise centered around instantaneous carrier frequency. This reduces the threshold of FMFB receiver. The threshold reduction of about ***5-7*** dB is possible.

Capture Effect

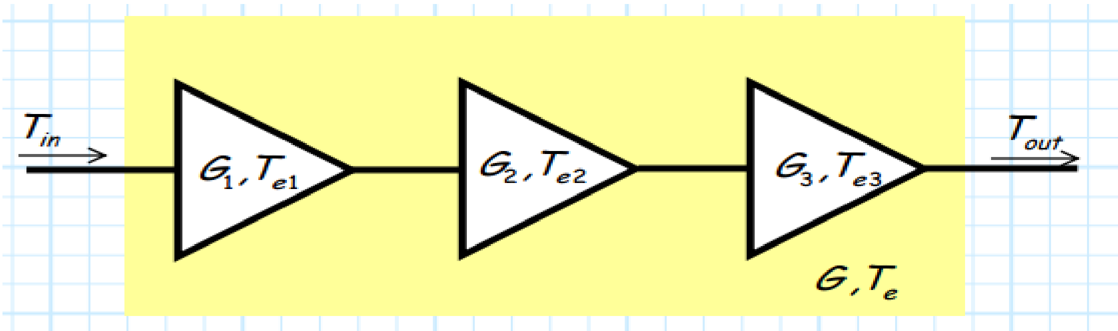
The FM system minimizes the effects of noise interference. This can be effective when interference is weak compared to FM signal. But if the interference is stronger than FM signal, then FM receiver locks to interference. This suppresses FM signal. When noise interference as well as FM signal are of equal strength, then the FM receiver locking fluctuates between them. This phenomenon is called capture effect.

8. What is noise temperature? Derive the expression for effective noise temperature for a cascaded system[CO4-H1-May/June2015] [10]

In electronics, noise temperature is a temperature (in Kelvin's) assigned to a component such that the noise power delivered by the noisy component to a noiseless matched resistor is given by

$$PRL = kBTsBn$$

Engineers often model noisy components as an ideal component in series with a noisy resistor. The source resistor is often assumed to be at room temperature, conventionally taken as 290 K (17 °C, 62 °F).Say we cascade three microwave devices, each with a different gain and equivalent noise temperature:



These three devices together can be thought of as one new microwave device.
First of all, we must define this temperature as the value Te such that:

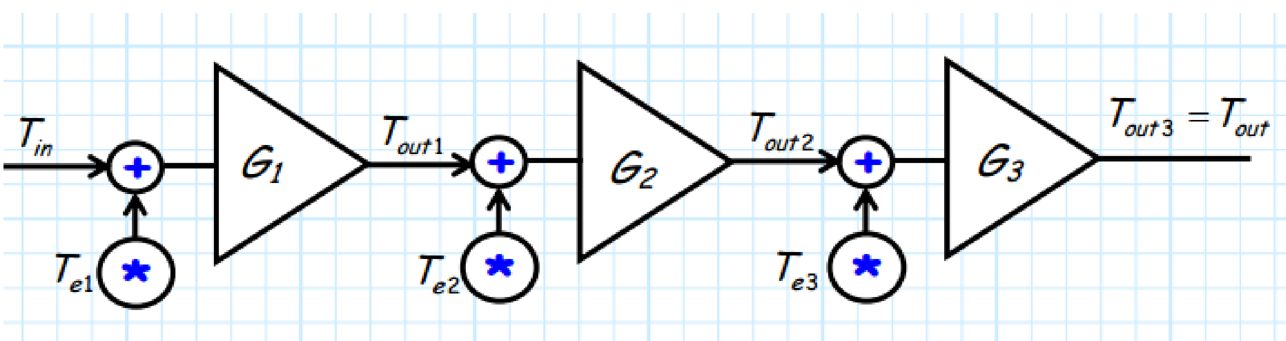
$$T_{out} = G(T_{in} + T_e)$$

The value G is the total system gain; in other words, the overall gain of the three cascaded devices. This gain is particularly easy to determine, as is it simply the product of the three gains:

$$G = G_1 G_2$$

Now for the hard part! To determine the value of Tout , we must use our equivalent noise model that we studied earlier:

Thus, we cascade three models, one for each amplifier:



We can observe our model and note three things:

$$T_{out1} = G1(T_{in} + T_{e1})$$

$$T_{out2} = G1(T_{out1} + T_{e1})$$

$$T_{out3} = G1(T_{out2} + T_{e1})$$

Combining these three equations, we find:

$$T_{out3} = G1G2G3(T_{in} + T_{e1}) + G2G3(T_{e2}) + G3(T_{e3})$$

a result that is likewise evident from the model. Now, since Tout out =T 3, we can determine the overall (i.e., system) equivalent noise temperature Te :

Moreover, we will find if we cascade an N number of devices, the overall noise equivalent temperature will be:

$$T_e = T_{e1} + \frac{T_{e2}}{G1} + \frac{T_{e3}}{G1G2} + \cdots .. + \frac{T_{eN}}{G1G2G3 \ldots .GN - 1}$$

9. What is narrowband noise discuss the properties of the Quadrature components of a narrowband noise. [CO4-L2-May/June2012] [8]

The signals of interest are usually passed through the filter and then given to the receiver. Such filter is narrow band and its mid-band frequency is large enough compared to bandwidth. The noise appearing at the output of such a filter is called narrowband noise. The spectral components of narrowband noise are concentrated about the midband frequency ±/o.

Quadrature Components of Narrowband Noise

We have represented noise as superposition of its spectral components over .1 wide range. The noise **n** (f) is represented as,

Let us represent the noise components in the neighborhood of $\mathbf{f_0}$**.** Here /o is any arbitrary frequency. Let /,> be defined as,

$$= \lim_{\Delta f \to 0} \left\{ \sum_{n=1}^{\infty} a_n \cos[2\pi f_0 t + 2\pi(n-m)\Delta f t] + \sum_{n=1}^{\infty} b_n \sin[2\pi f_0 t + 2\pi(n-m)\Delta f t] \right\}$$

Expanding sine and cosine terms in above equation and rearranging them we get,

$$n(t) = \lim_{\Delta f \to 0} \left\{ \left[\sum_{n=1}^{\infty} a_n \cos 2\pi(n-m)\Delta f t + \sum_{n=1}^{\infty} b_n \sin 2\pi(n-m)\Delta f t \right] \cos 2\pi f_0 t - \left[\sum_{n=1}^{\infty} a_n \sin 2\pi(n-m)\Delta f t - \sum_{n=1}^{\infty} b_n \cos 2\pi(n-m)\Delta f t \right] \sin 2\pi f_0 t \right\}$$

In the above equation let,

$$n_c(t) = \lim_{\Delta f \to 0} \left\{ \sum_{n=1}^{\infty} a_n \cos 2\pi(n-m)\Delta f t + \sum_{n=1}^{\infty} b_n \sin 2\pi(n-m)\Delta f t \right\}.$$

and,

$$n_s(t) = \lim_{\Delta f \to 0} \left\{ \sum_{n=1}^{\infty} a_n \sin 2\pi(n-m)\Delta f t - \sum_{n=1}^{\infty} b_n \cos 2\pi(n-m)\Delta f t \right\}$$

The above equation represents noise in terms of two components ; i.e. cosine and sine of same frequency fa. Such a representation is useful for narrowband noise representation. The noise representation given by equation above is called Quadrature components representation.) are the stationary random processes and their values depend upon a„ and/;,,. We have seen earlier that a„ andb,, are random variables of zero mean and they are uncorrelated. Hence it can be proved

It can be shown that the power spectral densities of and

n, (f) are same and they are given as follows :

Properties of the Components of Narrowband Noise

(i) The inphase component n_e(t) and quadrature component n,(r) of narrowband noise have same power spectral density as that of noise n(t).

ii)The n_r(f) and n„(f) are stationary random processes and they are uncorelated.

(iii)If the noise n(t) is Gaussian, then the quadrature omponents are also Gaussian.

(iv)The variance of quadrature components is same as that of noise n(t).

(v) The quadrature components are statistically independent if the narrowband

9. Define noise and explain the types of noise.[CO4-L2-Nov/Dec2013] [8]

Noise is ever present and limits the performance of virtually every system. The presence of noise degrades the performance of the Analog and digital communication systems. This chapter deals with how noise affects different Analog modulation techniques. After studying this chapter the

should be familiar with the following Various performance measures of communication systems

SNR calculations for DSB-SC, SSB-SC, Conventional AM, FM (threshold effect, threshold extension, pre-emphasis and deemphasis) and PM.

Figure of merit of All the above systems

Comparisons of all analog modulation systems – Bandwidth efficiency, power efficiency, ease of implementation.

Shot Noise:

Shot noise consists of random fluctuations of the electric current in an electrical conductor, which are caused by the fact that the current is carried by discrete charges (electrons). The strength of this noise increases for growing magnitude of the average current flowing through the conductor. Shot noise is to be distinguished from current fluctuations in equilibrium, which happen without any applied voltage and without any average current flowing. These equilibrium current fluctuations are known as Johnson-Nyquist noise.

Shot noise is important in electronics, telecommunication, and fundamental physics.

The strength of the current fluctuations can be expressed by giving the variance of the current *I*, where <*I*> is the average ("macroscopic") current. However, the value measured in this way depends on the frequency range of fluctuations which is measured ("bandwidth" of the measurement): The measured variance of the current grows linearly with bandwidth. Therefore, a more fundamental quantity is the noise power, which is essentially obtained by dividing through the bandwidth (and, therefore, has the dimension ampere squared divided by Hertz). It may be defined as the zero-frequency Fourier transform of the current-current correlation function.

Thermal Noise:

Thermal noise (Johnson–Nyquist noise, **Johnson noise**, or **Nyquist noise**) is the electronic noise generated by the thermal agitation of the charge carriers (usually the electrons) inside an electrical conductor at equilibrium, which happens regardless of any applied voltage.

Thermal noise is approximately white, meaning that the power spectral density is nearly equal throughout the frequency spectrum (however see the section below on extremely high frequencies). Additionally, the amplitude of the signal has very nearly a Gaussian probability density function.

This type of noise was first measured by John B. Johnson at Bell Labs in 1928. He described his findings to Harry Nyquist, also at Bell Labs, who was able to explain the results.

White Noise:

White noise is a random signal (or process) with a flat power spectral density. In other words, the signal contains equal power within a fixed bandwidth at any center frequency. White noise draws its name from white light in which the power spectral density of the light is distributed over the visible band in such a way that the eye's three color receptors (cones) are approximately equally stimulated. In statistical sense, a time series rt is called a white noise if {rt} is a sequence of independent and identically distributed (iid) random variables with finite mean and variance. In particular, if rt is normally distributed with mean zero and variance σ , the series is called a Gaussian white noise.

An infinite-bandwidth white noise signal is a purely theoretical construction. The bandwidth of white noise is limited in practice by the mechanism of noise generation, by the transmission medium and by finite observation capabilities. A random signal is considered "white noise" if it is observed to have a flat spectrum over a medium's widest possible bandwidth.

Unit – V

Information Theory

Part – A

1. What is entropy? [CO5-L1-May/June2014]

Entropy is also called average information per message. It is the ratio of total information to number of messages. i.e.,Entropy, H = Total information Number of messages

2. What is channel redundancy? [CO5-L1]

Redundancy = 1 – code efficiency

Redundancy should be as low as possible.

3. Name the two source coding techniques. [CO5-L1-May/June2013]

The source coding techniques are, a) prefix coding b)

Shannon-fano coding c) Huffman coding

4. Write the expression for code efficiency in terms of entropy.

Code efficiency = Entropy(H)

Average code word length(N)

5. What is memory less source? Give an example. [CO5-L1-May/June2012]

The alphabets emitted by memory less source do not depend upon previous alphabets. Every alphabet is independent. For example a character generated by keyboard represents memory less source.

6. Explain the significance of the entropy H(X/Y) of a communication system where X is the transmitter and Y is the receiver. [CO5-L2-May/June2014]

a) H(X/Y) is called conditional entropy. It represents uncertainty of X, on average, when Y is known.

b) In other words H(X/Y) is an average measure of uncertainty in X after Y is received.

c) H(X/Y) represents the information lost in the noisy channel.

7. What is prefix code? [CO5-L1]

In prefix code, no codeword is the prefix of any other codeword. It is variable length code.

The binary digits (codewords) are assigned to the messages as per their probabilities of occurrence.

8. Define information rate. [CO5-L1-Nov/Dec2012]

Information rate(R) is represented in average number of bits of information per second.

It is calculated as, R = r H Information bits / sec

9. Calculate the entropy of source with a symbol set containing 64 symbols each with a Probability pi = 1/ 64 . [CO5-H1-May/June2015]

Here, there are M = 64 equally likely symbols. Hence entropy of such source is given as,H = log 2

M = log 2 64 = 6 bits / symbol

10. State the channel coding theorem for a discrete memory less channel. [CO5-L2-Nov/Dec2014]

Statement of the theorem:

Given a source of ‗M' equally likely messages, with M >>1, which is generating information at a rate. Given channel with capacity C. Then if,R ≤ C There exits a coding technique such that the output of the source may be transmitted over the channel with a probability of error in the received message which may be made arbitrarily small.

Explanation: This theorem says that if R ≤ C ; it is possible to transmit information without any error even if noise is present. Coding techniques are used to detect and correct the errors.

11. What is information theory? [CO5-L1]

Information theory deals with the mathematical modeling and analysis of a communication system rather than with physical sources and physical channels

12. Explain Shannon-Fano coding. [CO5-L2-May/June2015]

An efficient code can be obtained by the following simple procedure, known as Shannon – Fano algorithm.

Step 1: List the source symbols in order of decreasing probability.

Step 2: Partition the set into two sets that are as close to equiprobable as possible, and sign 0 to the
upper set and 1 to the lower set.

Step: Continue this process, each time partitioning the sets with as nearly equal probabilities as possible until further partitioning is not possible.

13. Define bandwidth efficiency. [CO5-L1]

The ratio of channel capacity to bandwidth is called bandwidth efficiency.

i.e, Bandwidth efficiency = Channel Capacity Bandwidth (B)

14. Define channel capacity of the discrete memory less channel. [CO5-L1-May/June2012]

The channel capacity of the discrete memory less channel is given as maximum average mutual information. The maximization is taken with respect to input probabilities.

PART-B

1. Explain in detail about source coding theorem and data compaction. [CO5-L2-May/June2014] [12]

Source Coding Theorem (Shannon's first theorem):

The theorem can be stated as follows: Given a discrete memoryless source of entropy H(S) , the average code-word length L for any distortionless source coding is bounded as $L \geq H(s)$

This theorem provides the mathematical tool for assessing data compaction, i.e. lossless data compression, of data generated by a discrete memoryless source.

The entropy of a source is a function of the probabilities of the source symbols that constitute the alphabet of the source.

Entropy of Discrete Memoryless Source Assume that the source output is modeled as a discrete random variable, S , which takes on symbols from a fixed finite alphabet

$$S = \{S_0, S_1, \dots . S_{K-1}\}$$

With Probabilities

$$P(S = S_k) = P_k, k = 0,1,2, \dots k - 1 \; with \sum_{k=0}^{k-1} P_k = 1$$

Define the amount of information gain after observing the event k S = s as the logarithmic function

$$I(S_K) = log_2 \left(\frac{1}{P_k}\right) bits$$

the entropy of the source is defined as the mean of $I(S_k)$ over source alphabet S given by

$$H(s) = E[I(s_{k)}]$$

$$= \sum_{k=0}^{k-1} P_k I(S_K)$$

$$= \sum_{k=0}^{k-1} P_k \, log_2 (\frac{1}{P^k})$$

The entropy is a measure of the average information content per source symbol. ß The source coding theorem is also known as the "noiseless coding theorem" in the sense that it establishes the condition for error-free encoding to be possible.

Data Compaction:

Removal of redundant information from a file or data stream. The term *data compression* is commonly used to mean the same thing, although, strictly, while compression permits the loss of information in the quest for brevity, compaction is lossless. The effects of compaction are thus exactly reversible.

Generally, in the context of discrete and continuous systems, the output from discrete systems, if it is to be abbreviated, is losslessly compacted. Data compaction is appropriate, by way of example, for files containing text (including source programs) and machine code. In fax transmission, the position of black pixels is discretely encoded, and so again data compaction is employed.

Data compaction may be carried out in a probabilistic or statistical manner, and a particular algorithm may be suited to one or other of these. A data compaction algorithm may be more or less *effective* (in achieving a high ratio of compaction) and more or less *efficient* (in economy of time taken for encoding and decoding). To a large extent, these demands conflict. For example, Huffman coding is optimally effective when unconstrained, but may require a highextension of the source, and need the output stream to have a small alphabet (ultimately binary, which requires bit manipulation possibly on a large scale); Huffman can thus be very inefficient. On the other hand, Lempel–Ziv compaction is very efficient, and within given time constraints may be more effective than a similarly constrained Huffman code.

2. Explain in detail Huffman coding algorithm and compare this with the other types of coding [CO5-L3-May/June2012] [12]

Huffman coding is based on the frequency of occurrence of a data item (pixel in images). The principle is to use a lower number of bits to encode the data that occurs more frequently. Codes are stored in a ***Code Book*** which may be constructed for each image or a set of images. In all cases the code book plus encoded data must be transmitted to enable decoding.

The Huffman algorithm is now briefly summarized:

Huffman source encoding follows the steps

1. Arrange symbols in descending order of probabilities
2. Merge the two least probable symbols (or subgroups) into one subgroup
3. Assign ’0’ and ’1’ to the higher and less probable branches, respectively, in the subgroup
4. If there is more than one symbol (or subgroup) left, return to step 2

3. Extract the Huffman code words from the different branches (bottom-up) [CO5-L3-May/June2014] [12]

Example for 8 symbols

Symb.	Prob.	Coding Steps							Code word
m_i	p_i	1	2	3	4	5	6	7	
m_1	0.27						1	0	01
m_2	0.20					0		1	10
m_3	0.17				0		0	0	000
m_4	0.16				1		0	0	001
m_5	0.06		0	0		1		1	1100
m_6	0.06		1	0		1		1	1101
m_7	0.04	0		1		1		1	1110
m_8	0.04	1		1		1		1	1111

- Intermediate probabilities: $m_{7,8} = 0.08$; $m_{5,6} = 0.12$; $m_{5,6,7,8} = 0.2$; $m_{3,4} = 0.33$; $m_{2,5,6,7,8} = 0.4$; $S_{1,3,4} = 0.6$

step 2

m_1	0.27	
m_2	0.20	
m_3	0.17	
m_4	0.16	
m_{78}	0.08	
m_5	0.06	0
m_6	0.06	1

step 3

m_1	0.27	
m_2	0.20	
m_3	0.17	
m_4	0.16	
m_{56}	0.12	0
m_{78}	0.08	1

step 4

m_1	0.27	
m_2	0.20	
m_{5678}	0.20	
m_3	0.17	0
m_4	0.16	1

step 5

m_{34}	0.33	
m_1	0.27	
m_2	0.20	0
m_{5678}	0.20	1

step 6

m_{25678}	0.40	
m_{34}	0.33	0
m_1	0.27	1

step 7

m_{134}	0.60	0
m_{25678}	0.40	1

4. Define Entropy? Explain the properties of entropy with suitable example. [CO5-H1-May/June2013] **[12]**

The information due to message m1 will be

$$I_1 = log_2(\frac{1}{p_1})$$

Since there are P_1L number of messages of m_1, the total information due to all messages of m_1 will be

$$I_1(Total) = p_1 L log_2(\frac{1}{p_1})$$

similarly the total information due to all messages of m_2 will be

$$I_2(Total) = p_2 L log_2(\frac{1}{p_2})$$

Thus the total information carried due to the sequence of L messages will be

$$I_{Total} = I_1(Total) + I_2(Total) + \cdots + I_M(Total)$$

$$I_{Total} = p_1 Llog_2\left(\frac{1}{p_1}\right) + p_2 Llog_2\left(\frac{1}{p_2}\right) + \cdots + p_M Llog_2\left(\frac{1}{p_M}\right)$$

The average information per message will be

$$= p_1 Llog_2\left(\frac{1}{p_1}\right) + p_2 Llog_2\left(\frac{1}{p_2}\right) + \cdots + p_M Llog_2\left(\frac{1}{p_M}\right)$$

Putting for probabilities, we get

$$H = \frac{1}{M} log_2(M) + \frac{1}{M} log_2(M) + \cdots + \frac{1}{M} log_2(M)$$

In the above equation there are M numbers of terms in summation. Hence after adding these terms above equation becomes

$$H = log_2 M$$

Properties of entropy

1.Entropy is zero if the event is sure or it is impossible

H=0 if Pk= 0 or 1

2. when Pk= 1/M for all the M symbols, then the symbols are equally likely for such source entropy is given as H= $\log_2 M$

3. Upper bound on entropy is given as H_{max} = $\log_2 M$

5. Prove that the upper bound on entropy is given as $H_{max} = \leq log_2 M$,here M is the number of messages emitted by the source. [CO5-H2-May/June2014] [10]

To prove the above property we will use the following property of natural algorithm

$$ln\ x \leq x - 1\ for\ x \geq 0$$

Let us consider any two probability distributions{P_1,P_2,…P_M} and {q_1,q_2,…q_M} on the alphabet X={x_1,x_2,…x_M} of the discrete memory less source.

Then let us consider the term $\sum_{k=1}^{M} p_k \, log_2(\frac{q_k}{p_k})$.this term can be written as

$$\sum_{k=1}^{M} p_k \, log_2(\frac{q_k}{p_k}) = \sum_{k=1}^{M} p_k \frac{log_{10}(\frac{q_k}{p_k})}{log_{10}2}$$

Multiply RHS by $log_{10}e$ and rearrange terms as follows

$$\sum_{k=1}^{M} p_k \, log_2(\frac{q_k}{p_k}) = \sum_{k=1}^{M} p_k \frac{log_{10}e}{log_{10}2} . \frac{log_{10}(\frac{q_k}{p_k})}{log_{10}2}$$

$$\sum_{k=1}^{M} p_k log_2 \, e \; log_e(\frac{q_k}{p_k})$$

Here $log_e(\frac{q_k}{p_k}) = \ln(\frac{q_k}{p_k})$ Hence above equation becomes

$$\sum_{k=1}^{M} p_k \, log_2(\frac{q_k}{p_k}) = log_2 e \sum_{k=1}^{M} p_k \ln(\frac{q_k}{p_k})$$

From the above equation can write

$$\ln(\frac{q_k}{p_k}) \leq (\frac{q_k}{p_k} - 1).$$

Hence above equation becomes

$$\sum_{k=1}^{M} p_k \, log_2(\frac{q_k}{p_k}) \leq log_2 e \sum_{k=1}^{M} p_k (\frac{q_k}{p_k} - 1)$$

$$\leq log_2 e \sum_{k=1}^{M} (q_k - p_k)$$

$$\leq log_2 e [\sum_{k=1}^{M} q_k - \sum_{k=1}^{M} p_k]$$

Note that $\sum_{k=1}^{M} q_k = 1 \; as \; well \; as \; \sum_{k=1}^{M} p_k = 1$.hence the above equation becomes

$$\sum_{k=1}^{M} p_k \, log_2(\frac{q_k}{p_k}) \leq 0$$

Now let us consider that qk=1/k for all k that is all symbols in the alphabet are

Equally likely .then above equation becomes

$$\sum_{k=1}^{M} p_k \left[\log_2 q_k + \log_2 \frac{1}{p_k}\right] \le 0$$

$$\sum_{k=1}^{M} p_k \log_2 q_k + \sum_{k=1}^{M} p_k \log_2 \frac{1}{p_k} \le 0$$

$$\sum_{k=1}^{M} p_k \log_2 \frac{1}{p_k} \le -\sum_{k=1}^{M} p_k \log_2 q_k$$

$$\le \sum_{k=1}^{M} p_k \log_2 \frac{1}{p_k}$$

Putting q_k=1/M in above equation,

$$\sum_{k=1}^{M} p_k \log_2 \frac{1}{p_k} \le \sum_{k=1}^{M} p_k \log_2 M$$

$$\le \log_2 M \sum_{k=1}^{M} p_k$$

Since $\sum_{k=1}^{M} p_k = 1$,the above equation becomes

$$\sum_{k=1}^{M} p_k \log_2 \frac{1}{p_k} \le \log_2 M$$

Hence $H(X) \le \log_2 M$

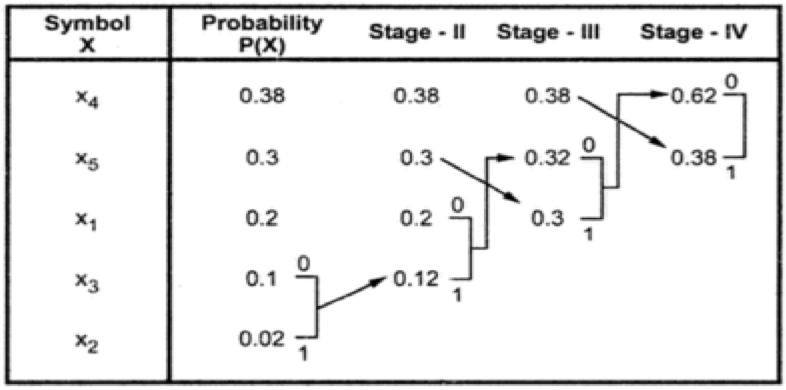
This is the proof of upper bound on entropy,the maximum value of entropy is

Hence $Hmax(X) \le \log_2 M$

7. **Encode the source symbols with following set of probabilities using Huffman coding. X={x_1, x2, x3, x4, x5}**

P(X)== {0.2, 0.02, 0.1, 0.38, 0.3} [CO5-H2- Nov/Dec 2014] [8]

Following table lists the stage wise combination of probabilities.



Following table lists the codes.

Symbol X	Probability P(X)	Digits obtained by tracing	Huffman code
x_4	0.38	1	1
x_5	0.3	1 0	0 1
x_1	0.2	0 0 0	0 0 0
x_3	0.1	0 1 0 0	0 0 1 0
x_2	0.02	1 1 0 0	0 0 1 1

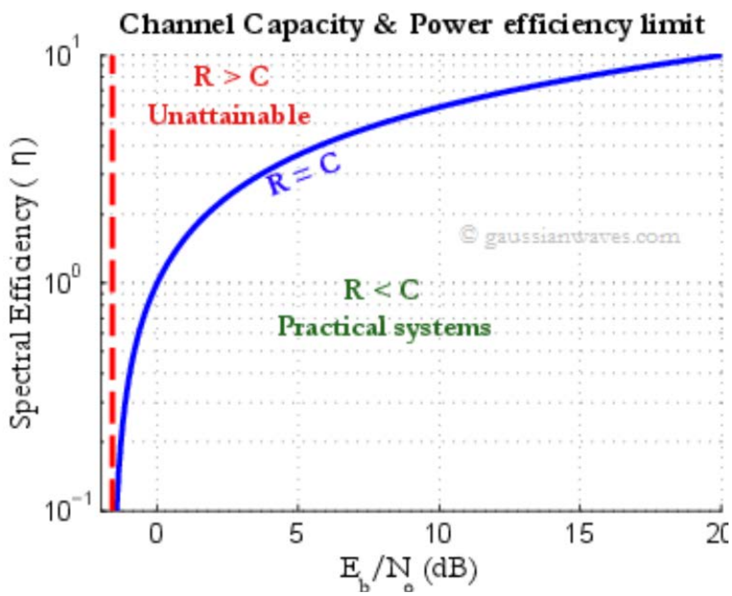
8. Derive the channel capacity (or) information Capacity theorem. [CO5-H1-Nov/Dec 2013] [10]

In information theory, the Shannon–Hartley theorem tells the maximum rate at which information can be transmitted over a communications channel of a specified bandwidth in the presence of noise. It is an application of the noisy channel coding theorem to the archetypal case of

a continuous-time analog communications channel subject to Gaussian noise. The theorem establishes Shannon's channel capacity for such a communication link, a bound on the maximum

amount of error-free digital data (that is, information) that can be transmitted with a specified bandwidth in the presence of the noise interference, assuming that the signal power is bounded, and that the Gaussian noise process is characterized by a known power or power spectral density. The law is named after Claude Shannon and Ralph Hartley.

$$C = Blog_2\left(1+\frac{S}{N}\right) = \frac{Blog_{10}(1+\frac{S}{N})}{log_{10}(2)}$$



Considering all possible multi-level and multi-phase encoding techniques, the Shannon–Hartley

theorem states the channel capacity *C*, meaning the theoretical tightest upper bound on the information rate (excluding error correcting codes) of clean (or arbitrarily low bit error rate) data that can be sent with a given average signal power *S* through an analog communication channel subject to additive white Gaussian noise of power *N*, is:

$$C = Blog_2\left(1+\frac{S}{N}\right)$$

Where *C* is the channel capacity in bits per second;

B is the bandwidth of the channel in hertz (passband bandwidth in case of a modulated signal);

S is the average received signal power over the bandwidth (in case of a modulated signal,
often denoted *C*, i.e. modulated carrier), measured in watts (or volts squared);

N is the average noise or interference power over the bandwidth, measured in watts (or volts squared); and

S/N is the signal-to-noise ratio (SNR) or the carrier-to-noise ratio (CNR) of the communication signal to the Gaussian noise interference expressed as a linear power ratio (not as logarithmic decibels).

9. Write a short note on rate distortion theory. [CO5-L2- Nov/Dec 2013] [8]

Rate–distortion theory is a major branch of information theory which provides the theoretical foundations for lossy data compression; it addresses the problem of determining the minimal number of bits per symbol, as measured by the rate *R*, that should be communicated over a channel, so that the source (input signal) can be approximately reconstructed at the receiver (output signal) without exceeding a given distortion *D*.

When working with stationary sources with memory, it is necessary to modify the definition of the rate distortion function and it must be understood in the sense of a limit taken over sequences of increasing lengths.

$$R(D) = \lim_{n-\infty} R_n(D)$$

where

$$R_n(D) = \frac{1}{n} I(Y^n, X^n)$$

and

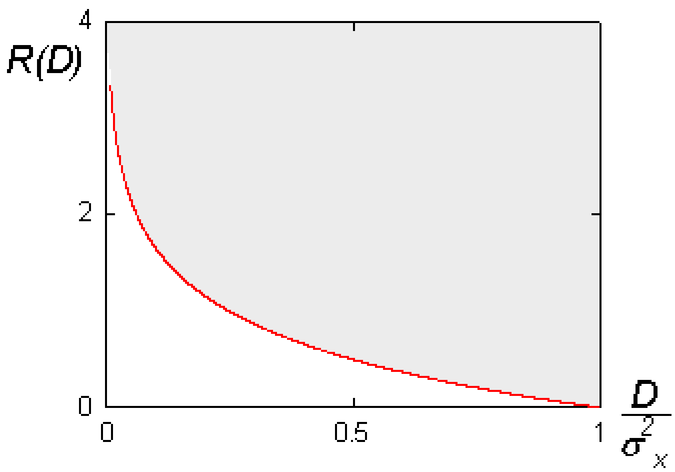
$$Q = \{Q(Y^n|X^n): E[d(X^n, Y^n)] \leq D\}$$

where superscripts denote a complete sequence up to that time and the subscript 0 indicates initial state.

If we assume that $P_X(x)$ is Gaussian with variance σ^2, and if we assume that successive samples of the signal X are stochastically independent (or equivalently, the source is memoryless, or the signal is uncorrelated), we find the following analytical expression for the rate–distortion function:

$$R(D) = \{\frac{1}{2} log_2 \left(\frac{\sigma_x{}^2}{D}\right), if\ 0 \leq D \leq \sigma_x{}^2$$

The following figure shows what this function looks like:



Rate–distortion theory tell us that 'no compression system exists that performs outside the gray area'. The closer a practical compression system is to the red (lower) bound, the better it performs. As a general rule, this bound can only be attained by increasing the coding block length parameter. Nevertheless, even at unit block lengths one can often find good (scalar) quantizes that operate at distances from the rate–distortion function that are practically relevant.

10. Consider that a binary source is transmitting equiprobable symbols 0's and 1's at the rate of 100 bits/sec, and the probability of error for each symbol in the

channel is 0.1. Calculate the rate of transmission over the channel. [CO5-H2-Nov/Dec 2012] [8]

This is the case of binary symmetric channel as shown below.

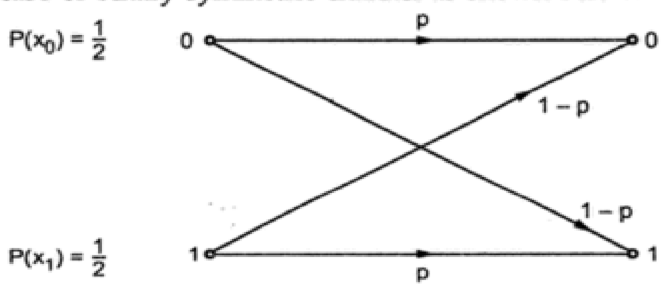


Fig. 4 Binary symmetric channel

Here probability of error is 0.1, hence 1 − p = 0.1.

Therefore p = 0.9.

Entropy of the source will be,

This is the case of binary symmetric channel as show$(1-P)log_2\frac{1}{(1-P)}$n below

Here probability of error is 0.1,hence 1-p=0.1,therefore=0.9

Entropy of the source will be

$$H(X) = P(X_0)log_2\frac{1}{P(X_0)} + P(X_1)log_2\frac{1}{P(X_1)}$$

$$= \frac{1}{2}log_2 2 + \frac{1}{2}log_2 2 = 1\ bits/symbol$$

For this channel,H(X/Y) is given as,

$$H\left(\frac{X}{Y}\right) = Plog_2\frac{1}{P} + (1-P)log_2\frac{1}{(1-P)}$$

$$0.9log_2\frac{1}{0.9} + (1-0.9)log_2\frac{1}{(1-0.9)} = 0.469$$

bits/symbol

Information transmission rate is given as

$$D_t = \left[H(X) - H\left(\frac{X}{Y}\right)\right] bits/sec$$

$$(1-0.469)100 = 53.1 bits/sec$$

11. Derive the channel capacity for Various Discrete Channel. [CO5-H1- Nov/Dec 2014] [16]

Chanel capacity of various discrete channels

Binary symmetric channel(BSC)

Let x = y = {0,1}

And the crossover probability be

$$P_{\frac{Y}{X}}\left(\frac{y}{x}\right) = \begin{Bmatrix} P, & y \neq x \\ 1-P, & y = x \end{Bmatrix}$$

$$\begin{array}{ccc} 0 & 0 & 1 \\ 0 & 1-p & p \\ 1 & p & 1-p \end{array}$$

To compute the capacity of BSC, we first examine mutual information

$$I(X,Y) = H(Y) - H\left(\frac{Y}{X}\right)$$

$$= H(Y) - H(Z)$$

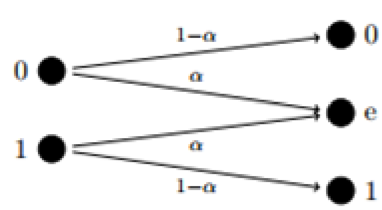
$$I(X,Y) = 1 - h_2(p)$$

$1 - h_2(p)$ bits of information can be communicated reliably

Binary Erasure channel

Let x = {0,1} y = {0,1,e}

And the crossover probability be



$$P_{\frac{Y}{X}}\left(\frac{y}{x}\right) = \begin{Bmatrix} \alpha, & y = e \\ 1-\alpha, & y = x \end{Bmatrix}$$

$$I(X,Y) = H(X) - H\left(\frac{X}{Y}\right)$$

$$H(X) - [H\left(\frac{X}{Y} = e\right)P(Y = e) + H\left(\frac{X}{Y} = 0\right)P(Y = 0) + H\left(\frac{X}{Y} = 1\right)P(Y = 1)]$$

$$H(X) - [H(X_\alpha + 0 + 0]$$

$$(1-\alpha)H(x)$$

To achieve equality,H(X)=1,X is Bernoulli

$$I(X:Y) = (1-\alpha)$$

$$C = (1-\alpha)$$

$(1-\alpha)$ bits of information can be communicated reliably.