SKP Engineering College

Tiruvannamalai – 606611

A Course Material

on

Electromagnetic Fields

By

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Quality Certificate

This is to Certify that the Electronic Study Material

Subject Code: EC6403

Subject Name: Electromagnetic Fields

Year/Sem: II/IV

Being prepared by me and it meets the knowledge requirement of the University curriculum.

Signature of the Authors

Name: G.Kumaran

Designation: Assistant Professor

This is to certify that the course material being prepared by Mr.G.Kumaran is of the adequate quality. He has referred more than five books and one among them is from abroad author.

Signature of HD   Signature of the Principal

Name:Mr.R.Saravanakumar   Name: Dr.V.Subramania Bharathi

Seal: Seal:

EC6403 ELECTROMAGNETIC FIELDS   **L T P C   3 1 0 4**

OBJECTIVES:

- To impart knowledge on the basics of static electric and magnetic field and the associated laws.
- To give insight into the propagation of EM waves and also to introduce the methods in computational electromagnetics.
- To make students have depth understanding of antennas, electronic devices, Waveguides is possible.

UNIT I STATIC ELECTRIC FIELD 9

Vector Algebra, Coordinate Systems, Vector differential operator, Gradient, Divergence, Curl, Divergence theorem, Stokes theorem, Coulombs law, Electric field intensity, Point, Line, Surface and Volume charge distributions, Electric flux density, Gauss law and its applications, Gauss divergence theorem, Absolute Electric potential, Potential difference, Calculation of potential differences for different configurations. Electric dipole, Electrostatic Energy and Energy density.

UNIT II CONDUCTORS AND DIELECTRICS 9

Conductors and dielectrics in Static Electric Field, Current and current density, Continuity equation, Polarization, Boundary conditions, Method of images, Resistance of a conductor, Capacitance, Parallel plate, Coaxial and Spherical capacitors, Boundary conditions for perfect dielectric materials, Poisson‟s equation, Laplace‟s equation, Solution of Laplace equation, Application of Poisson‟s and Laplace‟s equations.

UNIT III STATIC MAGNETIC FIELDS 9

Biot -Savart Law, Magnetic field Intensity, Estimation of Magnetic field Intensity for straight and circular conductors, Ampere‟s Circuital Law, Point form of Ampere‟s Circuital Law, Stokes theorem, Magnetic flux and magnetic flux density, The Scalar and Vector Magnetic potentials, Derivation of Steady magnetic field Laws.

UNIT IV MAGNETIC FORCES AND MATERIALS 9

Force on a moving charge, Force on a differential current element, Force between current elements, Force and torque on a closed circuit, The nature of magnetic materials, Magnetization and permeability, Magnetic boundary conditions involving magnetic fields, The magnetic circuit, Potential energy and forces on magnetic materials, Inductance, Basic expressions for self and mutual inductances, Inductance evaluation for solenoid, toroid, coaxial cables and transmission lines, Energy stored in Magnetic fields.

UNIT V TIME VARYING FIELDS AND MAXWELL'S EQUATIONS 9

Fundamental relations for Electrostatic and Magnetostatic fields, Faraday‟s law for Electromagnetic induction, Transformers, Motional Electromotive forces, Differential form of Maxwell‟s equations, Integral form of Maxwell‟s equations, Potential functions, Electromagnetic boundary conditions, Wave equations and their solutions, Poynting‟s theorem, Time harmonic fields, Electromagnetic Spectrum.

TOTAL (L:45+T:15): 60 PERIODS

OUTCOMES: Upon completion of the course, the students would be able to

- Analyze field potentials due to static changes and static magnetic fields.

- Explain how materials affect electric and magnetic fields.

- Analyze the relation between the fields under time varying situations.

- Discuss the principles of propagation of uniform plane waves.

TEXT BOOKS:

1. William H Hayt and Jr John A Buck, "Engineering Electromagnetics" , Tata Mc Graw-Hill Publishing Company Ltd, New Delhi, 2008

2. Sadiku MH, "Principles of Electromagnetics", Oxford University Press Inc, New Delhi, 2009

REFERENCES:

1. David K Cheng, "Field and Wave Electromagnetics", Pearson Education Inc, Delhi, 2004
2. John D Kraus and Daniel A Fleisch, "Electromagnetics with Applications", Mc Graw Hill Book Co, 2005
3. Karl E Longman and Sava V Savov, "Fundamentals of Electromagnetics", Prentice Hall of India, New Delhi, 2006
4. Ashutosh Pramanic, "Electromagnetism", Prentice Hall of India , New Delhi, 2006

CONTENTS

Unit – I

Static Electric Field

Part – A

1.Define volume charge density [CO1-L1-May/June 2012]

Volume charge density is defined as the ratio of total charge in coulomb to the total volume in cubic meters.

$$\rho_v = \frac{\text{total charge (c)}}{\text{total volume (m}^3)}$$

2.State stokes theorem. [CO1-L1]

The line integral of a vector around a closed path L is equal to the surface integral of curl F over open surface S enclosed by the closed path L.

$$\oint_L \vec{F} \cdot d\vec{L} = \iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

3.State Divergence Theorem. [CO1-L1]

The integral of the normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field throughout the volume enclosed that closed surface.

$$\oiint_S \vec{F} \cdot d\vec{S} = \iiint_V (\nabla \cdot \vec{F}) dV$$

4.Convert the point P(3,4,5) from Cartesian to spherical coordinates. [CO1-H2-Nov/Dec 2006]

Solution:

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{3^2 + 4^2 + 5^2} = 7.07$$
$$\cos \theta = \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{z}{7.07}$$
$$\tan \theta = \frac{y}{x} = 1.333$$

spherical coordinates (r ,θ, φ)
=(7.07 , $\cos^{-1}(\frac{z}{7.07})$ $\tan^{-1}(1.333)$)

5.State coulombs law. [CO1-L1]

Coulombs' law states that the force between two point charges Q1 and Q2 act along the line joining the two point charges is directly proportional to the product of two charges and inversely proportional to the square of the distances between the charges.

$$F = k\frac{Q_1 Q_2}{r^2} \text{ N}$$

where Q1, Q2 are point charges.

r – is the distance between two charges.

K – is a constant of proportionality and it depends on permittivity of the medium and is given by,

$$k = \frac{1}{4\pi\varepsilon_0}$$

ε0 = 8.854 x 10-12 F/m

k = 9 x 109 m/F

6. State Gauss law for electric fields[CO1-L1-April/May 2015]

The total electric flux passing through any closed surface is equal to the total charge enclosed by that surface.   ψ= Q

7. Define electric flux. [CO1-L1]

The lines of electric force are electric flux.

It is denoted by,

$$X = Q$$

8. Define electric flux density. [CO1-L1]

Electric flux density is defined as total flux passing through the unit surface area.

It is denoted by,

D= Q/A   C/m^3

9. Define electric field intensity. [CO1-L1]

Electric field intensity is defined as the electric force per unit positive charge.

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\vec{a_r}$$

10. Why gauss law cannot be applied to determine the electric field due to finite line charge? [CO1-H1]

Gauss law can be applied to any irregular closed surface that it should be large enough to enclose the entire charge. Hence, it is not applicable finite line charge.

11. What is a point charge? [CO1-L1]

Point charge is one whose maximum dimension is very small in comparison with any other length.

12. Define linear charge density. [CO1-L1-April/May 2015]
It is the charge per unit length

13. Define potential difference. [CO1-L1]
Potential difference is defined as the work done in moving a unit charge from one point to another point in an electric field
It is denoted by V,

$$V = -\int_{B}^{A} \vec{E} \cdot d\vec{L}$$

14. Define point charge. [CO1-L1]
A point charge means that electric charge which is separated on a surface or space whose geometrical dimensions are very small compared to other dimensions, in which the effect of electric field to be studied. Thus the point charge has a location but not the dimensions.

15. Define Absolute potential. [CO1-L1]
When the potential is measured with respect to specified reference position which is assumed to be at zero potential, is called absolute potential of that point such a reference position is generally assumed to be at infinity, which is at zero potential.

16. Distinguish between potential and potential difference. [CO1-L2]
Refer qn. No. 15
Such absolute potential are denoted as V_A, V_B etc
The potential difference is the difference between absolute potentials.
$V_{AB} = V_A - V_B$ v

17. Define charge density. [CO1-L1]
Charge density is defined as total charge per unit length or unit surface or unit volume.

18. Define surface charge density. [CO1-L1]
It is defined as,

$$\rho_s = \frac{\text{total charge in coulomb (c)}}{\text{total area in square meters (m}^3)}$$

19. What is a unit vector? What is its function while representing a vector? [CO1-L1]
unit vector has a function to indicate the direction. Its magnitude is always unity. Thus for any vector, to indicates its direction a unit vector is used.

20. When will you say that a vector is solenoid and ir-rotational $\vec{F}$. [CO1-H2-Nov/Dec 2013]

If curl of vector $\vec{F}$ is Zero is irrotational and if divergence of a vector is Zero, $\vec{F}$ is said to be solenoidal.

(i) $\nabla \times \vec{F} = 0$ means irrotational field and

(ii) $\nabla \cdot \vec{F} = 0$ means solenoidal field

21. State the expression for electric field intensity due to infinite line charge. [CO1-L1]

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon r}\ \vec{a}_y\ v/m$$

Where   r-perpendicular distance of point p from the line charge

$\vec{a}_y$-unit vector in the direction of the perpendicular distance of point p from the line charge.

22. State the expression for electric field intensity due to charged circular ring. [CO1-L1]

$$\vec{E} = \frac{\rho_L\,r\,z}{2\varepsilon_0 (r^2 + z^2)^{3/2}}\ \vec{a}_y$$

23. Write the relation between potential and electric field. [CO1-L1-Nov/Dec 2013]

$$\vec{E} = -\nabla V$$

The electric field strength at any point is the negative of the potential gradient at that point.

24. What is the use of gauss law? [CO1-L1]

It is the alternative statement of coulomb's law. This gauss law can be used to find $\vec{E}$ or $\vec{D}$

For symmetrical charge distribution, such as point charge , an infinite line charge , an infinite sheet of charge and a spherical distribution of charge.

The gauss law is also used to find the charge enclosed or the flux passing through the closed surface.

25. State the principle of superposition theorem. [CO1-L1]

According to the superposition theorem, the total force on Q is vector sum of all forces surrounding Q i.e.,Q1,Q2,Q3.

26. Define electric scalar potential. [CO1-L1]

Potential at any point is defined as the work done in moving a unit positive charge from infinity to that point in an electric field

$$V = \frac{Q}{4\pi\epsilon_0 r} \text{ volts}$$

27. A uniform surface charge of $\sigma = 2\mu c/m^2$ **situated at z =2- plane. What is the value of flux density at (1, 1, 1) m?. [CO1-H1]**

$$\vec{D} = \frac{\rho_s}{2}\vec{a_n}$$

$$\vec{a_n} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{r} = (0.1)\vec{a_x} + (0.1)\vec{a_y} + (z.1)\vec{a_z}$$

$$\vec{r} = (-\vec{a_x} \text{-}\vec{a_y} \text{-}\vec{a_z})$$

$$|\vec{r}| = \sqrt{1+1+1} = \sqrt{3}$$

$$\vec{a_n} = \frac{(-\vec{a_x} - \vec{a_y} - \vec{a_z})}{\sqrt{3}}$$

$$\vec{D} = \frac{\rho_s}{2}\frac{(-\vec{a_x} - \vec{a_y} - \vec{a_z})}{\sqrt{3}}$$

$$\vec{D} = \frac{2\mu}{2}\frac{(-\vec{a_x} - \vec{a_y} - \vec{a_z})}{\sqrt{3}}$$

$$\vec{D} = \frac{(-\vec{a_x} - \vec{a_y} - \vec{a_z})}{\sqrt{3}} \times 10^{-6} c/m^3$$

28. State the relation between D & E. [CO1-L1]

The electric field intensity of $\vec{E}$ is given as,

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a_r}$$

The electric flux density of $\vec{D}$ is given as,

$$\vec{D} = \frac{Q}{4\pi r^2}\vec{a_r}$$

By dividing $\vec{D}$ and $\vec{E}$ we get,

$$\frac{\vec{D}}{\vec{E}} = \frac{\frac{Q}{4\pi r^2}\vec{a_r}}{\frac{Q}{4\pi\varepsilon_0 r^2}\vec{a_r}} = \varepsilon_0$$

29. How gauss law is applied, when there is no symmetry. [CO1-H1]

When there is no symmetry, gauss law cannot be applied directly. In such a case difference closed Gaussian surface is considered. The closed surface is so small that $\vec{D}$ is almost

constant everywhere on the surface. Finally the results can be obtained by decreasing the volume enclosed by Gaussian surface to approach to zero.

30. What is electric dipole. Write down the potential due to an electric dipole. [CO1-L1]

The two point charges of equal magnitude but opposite sign separated by a very small distance give raise to an electric dipole.

The potential at P due to an electric dipole.

$$V = \frac{m \cos \theta}{4\pi\epsilon_0 r^2}$$

31. Write down the formula for conversion of a point from Cartesian to cylindrical system. [CO1-L1]

The Cartesian Co-ordinate (x, y, z) can be converted into cylindrical Co-ordinates (ρ, ϕ, z) as,

Given	Transform
x	$\rho = \sqrt{x^2 + y^2}$
y	$\phi = \tan^{-1}(y/x)$
z	$z = Z$

32. Write down the formula for conversion of a point from Cylindrical to Cartesian system. [CO1-L1]

The cylindrical Co-ordinates (ρ, ϕ, z) can be converted into Cartesian Co-ordinates (x, y, z) as,

Given	Transform
ρ	$x = \rho\cos\theta$
ϕ	$y = \rho\sin\phi$
z	$z = Z$

33. Write down the formula for conversion of a point from Cartesian to Spherical system. [CO1-L1]

The Cartesian Co-ordinates (x, y, z) can be transformed into spherical Co-ordinates (r, θ, ϕ) as,

Given	Transform
x	$r = \sqrt{x^2 + y^2 + z^2}$,
y	$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right) = \cos^{-1}\left(\frac{z}{r}\right)$
z	$\phi = \tan^{-1}(y/x)$

34. Write down the formula for conversion of a point from Spherical to Cartesian Co-ordinate system. [CO1-L1]

The spherical Co-ordinates (r, θ, φ.) can be transformed into Cartesian Co-ordinates (x, y, z) as,

Given	Transform
r	x = r sinθ cosφ
θ	y = r sinθ sinφ
φ	z = r cosθ

35. Transform the Cartesian Co-ordinates x = 2, y = 1, z = 3, into spherical Co-ordinates. [CO1-L3]

Given | Transform

X = 2　　　　　　$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{4+1+9} = 3.74$

Y = 1　　　　　　$\theta = \cos^{-1}\left(\dfrac{z}{r}\right) = \cos^{-1}\left(\dfrac{3}{\sqrt{14}}\right) = 36.7°$

Z = 3　　　　　　$\phi = \tan^{-1}(y/x) = \tan^{-1}(1/2) = 26.56°$

Ans: Spherical co-ordinates are (3.74, 36.7°, and 26.56°)

36. Give the Cartesian co-ordinate of a point whose cylindrical co-coordinates are ρ=1, φ = 45°, z = 2. [CO1-L3-April/May 2015]

Given: ρ=1, φ = 45°, z = 2.

The Cartesian co-ordinates are,

X = ρ cosφ =1 cos 45 = 0.707

Y = ρ sinφ =1 sin 45 = 0.707

Z = z

Ans: Cartesian co-ordinates are (0.707, 0.707, 2)

37. Express the divergence of a vector in the three system of orthogonal co-ordinates. [CO1-L3]

For rectangular co-ordinate system

$$\nabla.\vec{B} = \frac{\partial Bx}{\partial x} + \frac{\partial By}{\partial y} + \frac{\partial Bz}{\partial z}$$

For cylindrical co- ordinate system

$$\nabla.\vec{B} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\ B\rho) + \frac{1}{\rho}\frac{\partial B\phi}{\partial \phi} + \frac{\partial Bz}{\partial z}$$

For spherical co-ordinate system

$$\nabla.\vec{B} = \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r}(r^2 \sin\theta B_r) + \frac{\partial}{\partial\theta}(r\sin\theta B_\theta) + \frac{\partial}{\partial\phi}(rB_\phi)\right]$$

38. Show that the vector $\vec{H} = 3y^4 z^2 \vec{a}_x + 4x^3 z^2 \vec{a}_y + 3x^2 y^2 \vec{a}_z$ is solenoidal. [CO1-H1]

$$\nabla.\vec{H} = (\vec{a}_x \frac{\partial}{\partial x} + \vec{a}_y \frac{\partial}{\partial y} + \vec{a}_z \frac{\partial}{\partial z}).(3y^4 z^2 \vec{a}_x + 4x^3 z^2 \vec{a}_y + 3x^2 y^2 \vec{a}_z)$$

$$= \frac{\partial}{\partial x}(3y^4 z^2) + \frac{\partial}{\partial y}(4x^3 z^2) + \frac{\partial}{\partial z}(3x^2 y^2)$$

$$= 0 + 0 + 0 = 0$$

Thus the given vector is solenoidal.

39. What is vector algebra? [CO1-L1]

The various mathematical operations such as addition, subtraction and multiplication are performed by vector algebra.

The following operations are discussed.

Scaling 2. Addition 3. Subraction.

40. What is vector addition? [CO1-L1]

Consider 2 coplanar vectors as shown in the figure.

The vectors that lie in the same plane are called coplanar vectors.

Let us find the sum of two vectors A and B

The procedure is to move one of the two vectors parallel to itself at the tip of the other vector.

Thus move A vector to itself at the tip of vector B.

Then join tip of vector A moved to the origin.

This vector represents resultant, which is the addition of two vector A and B.

Let us denote this resultant as vector C.

$$\vec{C} = \vec{A} + \vec{B}$$

The direction of vector C is from the origin O, to the tip of vector moved.

If vector B is moved parallel to itself to the tip of vector A we get the same resultant C.

The addition of vectors obey the commutative law.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

41. What is vector subtraction? [CO1-L1]

The subtraction of vectors are obtained by the rule of addition.

    If the vector B is to be subtracted from A then based on addition it is represented as

$$\vec{C} = \vec{A} + (-\vec{B})$$

    Thus reverse the direction of vector B by multiplying it with -1 and then add it to vector a to obtain the subtraction. This is shown as below

42. Show that the vector $2yx\vec{a}_x + (x^2 + 2yz)\vec{a}_y + (y^2 + 1)\vec{a}_z$ is irrotational. [CO1-H1]

$$\text{Let } \vec{F} = 2yx\vec{a}_x + (x^2 + 2yz)\vec{a}_y + (y^2 + 1)\vec{a}_z$$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 2xy & x^2 + 2yz & y^2 + 1 \end{vmatrix}$$

$$= \vec{a}_x \left[\frac{\partial(y^2+1)}{\partial y} - \frac{\partial(x^2+2yz)}{\partial z} \right] - \vec{a}_y \left[\frac{\partial}{\partial x}(y^2+1) - \frac{\partial}{\partial z}(2xy) \right] + \vec{a}_z \left[\frac{\partial}{\partial x}(x^2+2yz) - \frac{\partial}{\partial y}(2xy) \right]$$

$$= 0$$

$$\nabla \times \vec{F} = 0$$

    Hence $\vec{F}$ is irrotational

Part - B

1. **Illustrate how to perform the mathematical operations such as addition and subtraction using the laws of vector algebra. [CO1-H1]**
   VECTOR ALGEBRA:
   - The various mathematical operations such as addition, subtraction and multiplication.
   - The following operations are discussed.
   - 1.Scaling 2.Addition 3.Subraction.
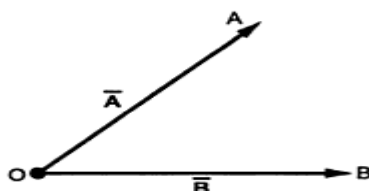
   SCALING OF A VECTOR:
   - Scaling is a multiplication by a scalar to a vector.
   - Such a multiplication changes the magnitude (length) of a vector but not its direction when the scalar is positive.
   - Let α-scalar with which vector is to be multiplied.
   - If α >1 then the magnitude of vector increases ,but direction remains same when multiplied.
   - If α <1 then the magnitude of vector decreases ,but direction remains same when multiplied.

- If α= -1 then the magnitude remains same, but the direction of vector reverses when multiplied.

(a) α > 1   (b) α < 1   (c) α = −1

ADDITION OF VECTORS:

- Consider 2 coplanar vectors as shown in the figure.
- The vectors that lie in the same plane are called coplanar vectors.
- Let us find the sum of two vectors A and B

- The procedure is to move one of the two vectors parallel to itself at the tip of the other vector.
- Thus move A vector to itself at the tip of vector B.
- Then join tip of vector A moved to the origin.
- This vector represents resultant, which is the addition of two vector A and B.
- Let us denote this resultant as vector C.

$$\vec{C} = \vec{A} + \vec{B}$$

- The direction of vector C is from the origin O, to the tip of vector moved.
- If vector B is moved parallel to itself to the tip of vector A we get the same resultant C.
- The addition of vectors obey the commutative law.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

- Another method of performing the addition of vectors is parallelogram rule.
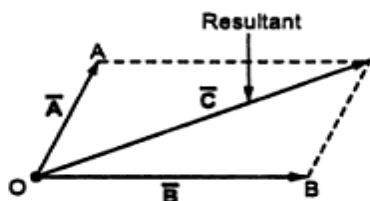
- The diagonal of the parallelogram represents the addition of two vector.
- More than two vectors can be added by using these vectors.
- For example consider 4 vectors as shown in the figure.

(a) Four vectors   (b) Sum of the four vectors

$$R = \overline{A} + \overline{B} + \overline{C} + \overline{D}$$
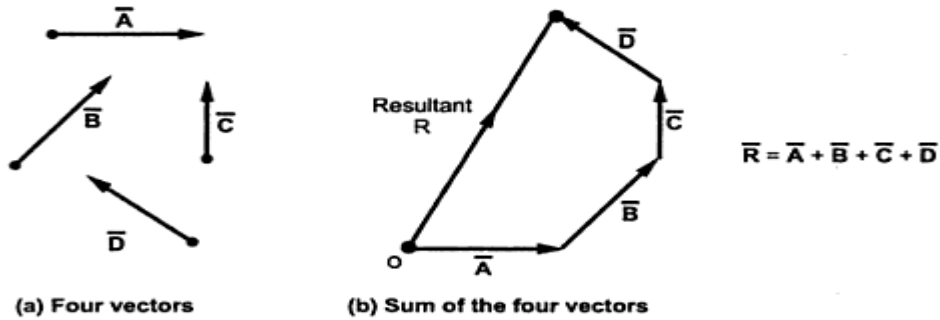
- These can be added by shifting these vectors one by one to the tip of other vectors to complete the polygon.
- The vector joining origin O to the tip of the last shifted vector represents the sum as shown in the figure.
- This method is called head to tail rule of addition of vectors.
- Once the coordinate systems are defined then the vectors can be expressed in terms of the components along the axes of the coordinate systems.
- Then by adding the components of the vectors the resultant vector which is the addition of the vectors can be obtained.
- The following basic laws of algebra are obeyed by the vectors $\vec{A}, \vec{B}$ & $\vec{C}$.

Law	Addition	Multiplication by scalar
Commutative	$\vec{A} + \vec{B} = \vec{B} + \vec{A}$	$\alpha\vec{A} = \vec{A}\alpha$
Associative	$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$	$\beta(\alpha\vec{A}) = (\beta\alpha)\vec{A}$
Distributive	$\alpha(\vec{A} + \vec{B}) = \alpha\vec{A} + \alpha\vec{B}$	$(\alpha + \beta)\vec{A} = \alpha\vec{A} + \beta\vec{A}$

SUBRACTION OF VECTORS:
- The subtraction of vectors are obtained by the rule of addition.
- If the vector B is to be subtracted from A then based on addition it is represented as

$$\vec{C} = \vec{A} + (-\vec{B})$$

Thus reverse the direction of vector B by multiplying it with -1 and then add it to vector a to obtain the subtraction. This is shown as below.

(a) Vectors   (b) Subtraction of vectors

$$\overline{C} = \overline{A} - \overline{B}$$
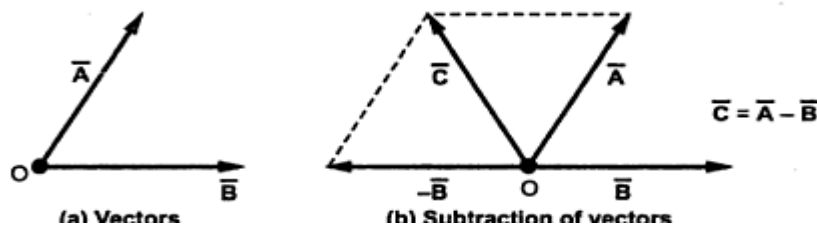
IDENTICAL VECTORS:

- Two vectors are said to be identical if their difference is zero

$$\vec{A} - \vec{B} = 0 \text{ i.e. } \vec{A} = \vec{B}$$

- Such two vectors are also called as equal vectors.

-

2. What is Div of a vector? Explain its properties. [CO1-L1]

DIVERGENCE OF A VECTOR:

It is defined as the net outflow of the flux per unit volume over a closed incremental surface.

The divergence of A at a given point P is the outward flux per unit volume as the volume shrinks about P.

$$\text{div}\,\vec{A} = \nabla \cdot \vec{A} = \lim_{\Delta v \to 0} \frac{\oiint_S \vec{A} \cdot d\vec{S}}{\Delta v}$$

- Physically the divergence of the vector quantity represents the rate of change of field strength in the direction of the field.
- If the div of a vector field is positive at a point P then something is diverging from the small volume surrounding that point which acts as a source.
- If negative then something is converging into a small volume surrounding that point which acts as a sink.
- If div is zero at a point P then the rate at which something entering into a small volume surrounding at that point is equal to the rate at which it is leaving that volume.

Positive Divergence   Negative Divergence   Zero Divergence
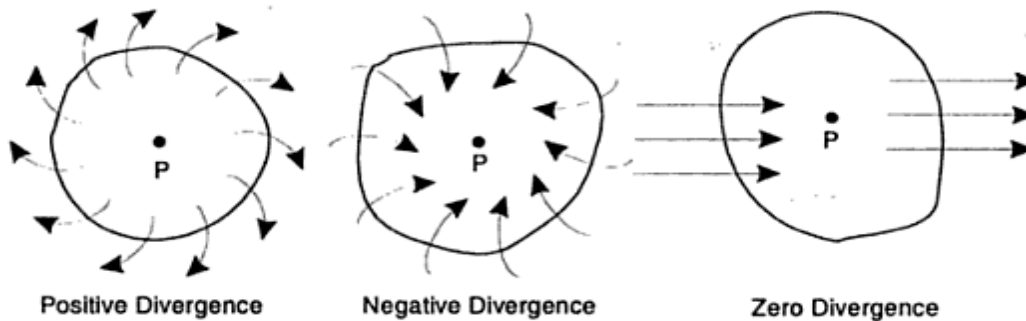
- The divergence of a vector A is represented as

$$\nabla \cdot \vec{A} \text{ (or) div}\,\vec{A}$$

- In Cartesian system

  $\nabla \cdot \vec{A}$ is given as

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

PROPERTIES OF A DIVERGENCE OF A VECTOR FIELD:

- It produces a scalar field.
- The div of a scalar V, div V makes no sense.

$$\nabla \cdot (\vec{A} + \vec{B}) = \nabla \cdot \vec{A} + \nabla \cdot \vec{B}$$

- The vector field whose divergence is zero is called the solenoidal field.

3.Explain briefly about Divergence theorem and prove it. [CO1-L1]
DIVERGENCE THEOREM:
Definition:

The theorem states that the integral of the normal component of a vector function over a closed surface is equal to the integral of the divergence of that vector throughout the volume 'V' enclosed by the surface 'S'.
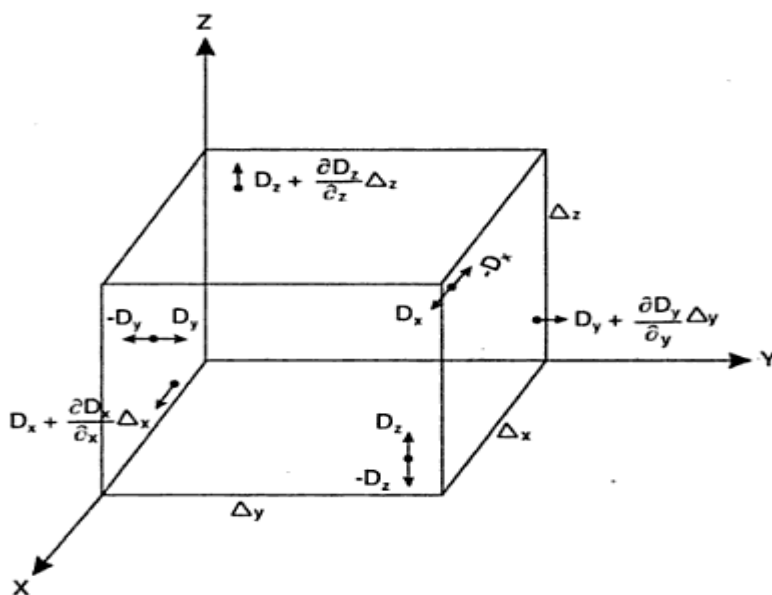
$$\oiint_S \vec{D} \cdot d\vec{S} = \oiiint_V (\nabla \cdot \vec{D}) dv$$

PROOF:

- Consider a small volume

$$\Delta x \, \Delta y \, \Delta z = \Delta v$$

Is placed in an electric field with flux density vector $\vec{D}$ having components D_X, D_Y, D_Z in the   three coordinate directions as shown in the below figure.

- The total flux density is given as

$$\vec{D} = D_x \vec{a}_x + D_y \vec{a}_y + D_z \vec{a}_z$$

- The normal outward component of vector $\vec{D}$ at the backface is $-D_x$ since the field is directed outwards.

- If the field changes between the back and front faces the normal component at the front face can be represented using Taylor's theorem as

$$D_x + \frac{\partial D_x}{\partial x} \Delta x$$

- Similarly the normal component of vector D at the left side face is-D_y and at the right side face is

$$D_y + \frac{\partial D_y}{\partial x} \Delta y$$

At the bottom face it is -D_z and at the top face is

$$D_z + \frac{\partial D_z}{\partial x} \Delta z$$

The outward flux of vector $\vec{D}$ over the back face is

$$-D_x \Delta y \Delta z$$

And over the front face is

$$(D_x + \frac{\partial D_x}{\partial x} \Delta x) \, \Delta y \Delta z$$

Adding up the outward flux of vector $\vec{D}$ over the entire volume, the total flux becomes

$$\Delta \psi = \left(-D_x + D_x + \frac{\partial D_x}{\partial x} \Delta x\right) \Delta y \Delta z + \left(-D_y + D_y + \frac{\partial D_y}{\partial y} \Delta y\right) \Delta x \Delta z$$

$$+ \left(-D_z + D_z + \frac{\partial D_z}{\partial z} \Delta z\right) \Delta y \Delta x$$

On simplifying

$$\Delta \psi = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \Delta x \Delta y \Delta z$$

The total electric flux over the surface of the volume is also equal to the normal component of vector $\vec{D}$ over the surface of the volume. That is

$$\Delta \psi = \oiint_S \vec{D} \cdot d\vec{S}$$

$$\Delta \psi = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \Delta x \Delta y \Delta z$$

Now RHS of the divergence theorem can be expanded in Cartesian coordinates as

$$\nabla \cdot \vec{D} = \left(\vec{a}_x \frac{\partial}{\partial x} + \vec{a}_y \frac{\partial}{\partial y} + \vec{a}_z \frac{\partial}{\partial z}\right) \cdot \left(D_x \vec{a}_x + D_y \vec{a}_y + D_z \vec{a}_z\right)$$

$$= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

So

$$\iiint_V (\nabla \cdot \vec{D}) \, dV = \iiint_V \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \right) \Delta x \Delta y \Delta z$$

Hence from the above equation,

$$\oiint_S \vec{D} \cdot d\vec{S} = \iiint_V (\nabla \cdot \vec{D}) \, dV$$

PROBLEMS:

1.Determine the divergence of these vector fields.

$$\vec{P} = x^2 yz\vec{a}_x + xz\vec{a}_z$$

$$\vec{Q} = \rho \sin \varphi \vec{a}_\rho + \rho^2 z\vec{a}_\varphi + z \cos \varphi \vec{a}_z$$

$$\vec{T} = \frac{1}{r^2} \cos \theta \vec{a}_r + r \sin \theta \cos \varphi \vec{a}_\theta + \cos \theta \vec{a}_\varphi$$

$$\text{(i) } \nabla \cdot \vec{P} = \frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z}$$

$$= 2xy + 0 + x$$

$$= 2xy + x$$

$$\text{(ii) } \nabla \cdot \vec{Q} = \frac{1}{\rho} \frac{\partial(\rho Q_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial(Q_\varphi)}{\partial \varphi} + \frac{\partial Q_z}{\partial z}$$

$$= \frac{1}{\rho} \frac{\partial(\rho^2 \sin\varphi)}{\partial x} + \frac{1}{\rho} \frac{\partial(\rho^2 z)}{\partial \varphi} + \frac{\partial(z \cos \varphi)}{\partial z}$$

$$= 2\sin\varphi + 0 + \cos \varphi)$$

$$= 2\sin\varphi + \cos \varphi)$$

$$\text{(iii) } \nabla \cdot \vec{T} = \frac{1}{r^2} \frac{\partial(r^2 T_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(T_\theta \, \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial(T_\varphi)}{\partial \varphi}$$

$$= \frac{1}{r^2} \frac{\partial(\cos\theta)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(r \sin^2 \theta \, \cos\varphi)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial(\cos\theta)}{\partial \varphi}$$

$$= 0 + 2\cos\theta\cos\varphi + 0$$

$$= 2\cos\theta\cos\varphi$$

4. What is curl of a vector and state its properties. [CO1-L1]

CURL OF A VECTOR:

- The circulation of a vector field around a closed path is given by curl of a vector. Mathematically it is defined as,

$$\text{Curl of } \vec{F} = \lim_{\Delta S_N \to 0} \frac{\oint_L \vec{F} \cdot d\vec{L}}{\Delta S_N}$$

ΔS_N –Area enclosed by the line integral in normal direction

- The maximum circulation of vector $\vec{F}$ per unit area as area tends to zero whose direction is normal to the surface is called curl of $\vec{F}$
- It is expressed as

$$\nabla \times \vec{F} = \text{curl of } \vec{F}$$

- In various coordinate systems the curl of $\vec{F}$ is given by

1.Cartesian co-ordinate system:

$$\nabla \times \vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right]\vec{a}_x + \left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right]\vec{a}_y + \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]\vec{a}_z$$

2.Cylindrical co-ordinate system

$$\nabla \times \vec{H} = \frac{1}{r}\begin{vmatrix} \vec{a}_r & r\vec{a}_\varphi & \vec{a}_z \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \varphi} & \dfrac{\partial}{\partial z} \\ H_r & rH_\varphi & H_z \end{vmatrix}$$

3.Spherical co-ordinate system

$$\nabla \times \vec{H} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \vec{a}_r & r\vec{a}_\theta & r\sin\theta\,\vec{a}_\varphi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \varphi} \\ H_r & rH_\theta & r\sin\theta H_\varphi \end{vmatrix}$$

PROPERTIES OF CURL:

- The curl of a vector is a vector quantity.
- $\nabla \times (\overline{A} + \overline{B}) = \nabla \times \overline{A} + \nabla \times \overline{B}$
- $\nabla \times \nabla \times \overline{A} = \nabla(\nabla \cdot \overline{A}) - \nabla^2 \overline{A}$
- The divergence of a curl is zero.

  $\nabla \cdot (\nabla \times \overline{A}) = 0$

- The curl of a scalar makes no sense

$\nabla \times \alpha$ = No sense if α is scalar.

- The curl of gradient of a vector is zero.

$$\nabla \times \nabla V = 0$$

- $\nabla \times \overline{A} \times \overline{B} = \overline{A}(\nabla \cdot \overline{B}) - \overline{B}(\nabla \cdot \overline{A}) + (\overline{B} \cdot \nabla)\overline{A} - (\overline{A} \cdot \nabla)\overline{B}$

PROBLEM:

Given the general vector, $\overline{A} = (\sin 2\phi)\overline{a}_\phi$ in cylindrical co-ordinates. Find curl of $\overline{A}$ at $(2, \pi/4, 0)$.

Solution : In cylindrical co-ordinates $\nabla \times \overline{A}$ is given by,

$$\nabla \times \overline{A} = \left[\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\overline{a}_r + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right]\overline{a}_\phi + \left[\frac{1}{r}\frac{\partial(r A_\phi)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \phi}\right]\overline{a}_z$$

Now $A_r = 0$, $A_\phi = \sin 2\phi$ and $A_z = 0$

$\therefore$

$$\nabla \times \overline{A} = \left[0 - \frac{\partial \sin 2\phi}{\partial z}\right]\overline{a}_r + [0-0]\overline{a}_\phi + \left[\frac{1}{r}\frac{\partial(r \sin 2\phi)}{\partial r} - 0\right]\overline{a}_z$$

$$= [0-0]\overline{a}_r + 0\,\overline{a}_\phi + \frac{\sin 2\phi}{r}\overline{a}_z = \frac{\sin 2\phi}{r}\overline{a}_z$$

At $\left(2, \frac{\pi}{4}, 0\right)$, $r = 2$, $\phi = \frac{\pi}{4}$, $z = 0$

$\therefore$

$$\nabla \times \overline{A} = \frac{\sin\left(2 \times \frac{\pi}{4}\right)}{2}\overline{a}_z$$

$$= \frac{\sin\left(\frac{\pi}{2}\right)}{2}\overline{a}_z$$

$$= 0.5\ \overline{a}_z$$

5.What is stokes theorem? Derive and prove it using a closed surface S. [CO1-L1]
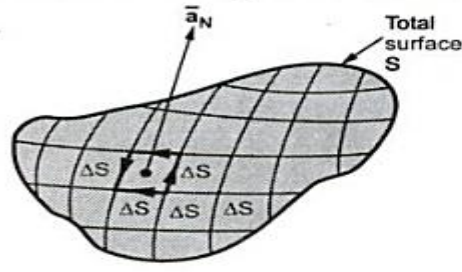**STOKE'S THEOREM:**
The stoke's theorem relates the line integral to a surface integral. It states that The line integral of vector H around a closed path L is equal to the integral of curl of vector H over the open surface S enclosed by the closed path L. Mathematically it is expressed as,

$$\oint_L \vec{H} \cdot d\vec{L} = \iint_S (\nabla \times \vec{H}) \cdot d\vec{S}$$
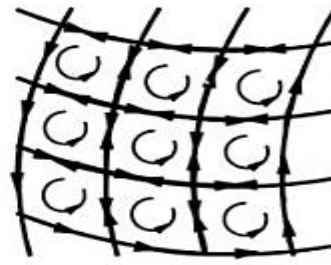
dL = Perimeter of the total surface S
PROOF OF STOKE'S THEOREM:
- Consider a surface S which is splitted into number of incremental surfaces
- Each incremental surface is having area Δs as shown in the figure.

(a)   (b)

- Applying definition of the curl to any of these incremental surfaces we can write,

$$(\nabla \times \overline{H})_N = \frac{\oint \overline{H} \cdot d\overline{L}_{\Delta S}}{\Delta S}$$

  N = Normal to ΔS according to right hand rule

  dLΔS = Perimeter of the incremental surface ΔS

- Now the curl of H in the normal direction is the dot product of curl of H with $\overline{a}_N$ normal to the surface Δs, according to right hand rule

$$\therefore \qquad (\nabla \times \overline{H})_N = (\nabla \times \overline{H}) \cdot \overline{a}_N$$

$$\therefore \qquad \oint \overline{H} \cdot dL_{\Delta S} = (\nabla \times \overline{H}) \cdot \overline{a}_N \ \Delta S$$

$$\therefore \qquad \oint \overline{H} \cdot dL_{\Delta S} = (\nabla \times \overline{H}) \cdot \Delta \overline{S}$$

- To obtain total curl of every incremental surface add the closed line integrals for each Δs.
- From fig B it can be seen that at a common boundary between the two incremental surfaces the line integral is getting cancelled as the boundary is getting traced in two opposite directions.
- The summation of all closed line integrals for each and every Δs ends up in a single closed line integral for the outer boundary of total surface S.

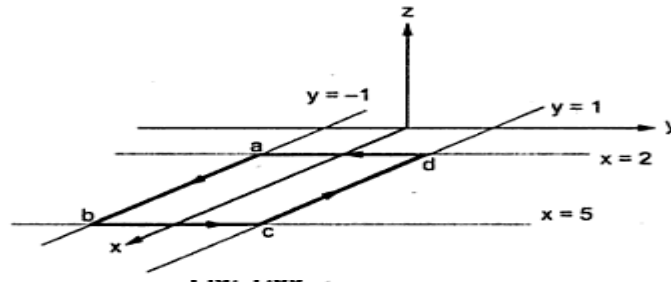$$\oint_L \vec{H} \cdot d\vec{L} = \iint_S (\nabla \times \vec{H}) \cdot d\vec{S}$$

dL = Perimeter of the total surface S

PROBLEMS:

Evaluate both sides of the Stoke's theorem for the field $\vec{H} = 6xy\vec{a}_x - 3y^2\vec{a}_y \ A/m$ and the rectangular path around the region, $2 \le x \le 5, -1 \le y \le 1, z = 0$. Let the positive direction of $d\vec{S}$ be $\vec{a}_z$.

Solution : According to Stoke's theorem,

$$\oint_L \vec{H} \cdot d\vec{L} = \iint_S (\nabla \times \vec{H}) \cdot d\vec{S}$$

$$\oint \overline{H} \cdot d\overline{L} = \int_{ab} + \int_{bc} + \int_{cd} + \int_{da} \overline{H} \cdot d\overline{L}$$

$$\int_{ab} \overline{H} \cdot d\overline{L} = \int_{x=2}^{5} \left(6xy\,\overline{a}_x - 3y^2\,\overline{a}_y\right) \cdot dx\,\overline{a}_x$$

$$= \int_{x=2}^{5} 6xy\;dx = 6y \left[\frac{x^2}{2}\right]_2^5$$

$$= \frac{6y}{2}\,[25-4] = 63\,y$$

Now $y = -1$ for path ab, $\int_{ab} \overline{H} \cdot d\overline{L} = 63(-1) = -63$

Similarly $\int_{bc} \overline{H} \cdot d\overline{L} = \int_{y=-1}^{1} -3y^2\;dy = \frac{-3y^3}{3} = -[y^3]_{-1}^{1} = -[1-(-1)] = -2$

$$\int_{cd} \overline{H} \cdot d\overline{L} = \int_{x=5}^{2} 6xy\;dx = 6\left[\frac{x^2}{2}\right]_5^2 (y) = \frac{6y}{2}[4-25] = -63y$$

But $y = 1$ for path cd hence $\int_{cd} \overline{H} \cdot d\overline{L} = -63$

$$\int_{da} \overline{H} \cdot d\overline{L} = \int_{y=1}^{-1} -3y^2\;dy = -[y^3]_{1}^{-1} = -[(-1)^3 - (1)^3] = -[-1-1] = +2$$

∴ $\oint \overline{H} \cdot d\overline{L} = -63 - 2 - 63 + 2 = -126$ A

Now evaluate right hand side.

$$\nabla \times \overline{H} = \begin{vmatrix} \overline{a}_x & \overline{a}_y & \overline{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 6xy & -3y^2 & 0 \end{vmatrix}$$

$$= \overline{a}_x\,[0-0] + \overline{a}_y\,[0-0] + \overline{a}_z\,[0-6x]\,\overline{a}_z = -6x\,\overline{a}_z$$

∴ $\int_S (\nabla \times \overline{H}) \cdot d\overline{S} = \int_S (-6x\,\overline{a}_z) \cdot (dx\,dy\,\overline{a}_z)$

$d\overline{S} = dx\,dy\,\overline{a}_z$ normal to direction $\overline{a}_z$

∴ $\int_S (\nabla \times \overline{H}) \cdot d\overline{S} = \int_{y=-1}^{1} \int_{x=2}^{5} -6x\;dx\;dy = -6\left[\frac{x^2}{2}\right]_2^5 [y]_{-1}^{1}$

$$= -\frac{6}{2}[25-4][1-(-1)] = -3 \times 21 \times 2 = -126 \text{ A}$$

Thus both the sides are same, hence Stoke's theorem is verified.

1. **Define Coulombs law. Derive its proportionality constant K in a free space.**
   [CO1-L1-APRIL/MAY 2015]

COLOUMBS LAW:

The coulombs law states that force between 2 point charges Q_1 and Q_2

1.Acts along the line joining the 2 point charges.

2.Is directly proportional to the product $(Q_1 \, Q_2)$

3.Is inversely proportional to the square of the distance between them.
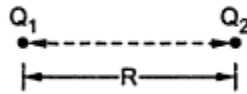
- Consider 2 pt charges Q_1 and Q_2 separated by a distance R
- The charge Q_1 exerts force on Q_2 while Q_2 exerts a force on Q_1
- The force acts along the line joining the Q_1 and Q_2
- If the charges have same polarity repulsive occurs.
- If the charges have different polarity attractive occurs.
- Mathematically it is expressed as

$$F \; \alpha \; \frac{Q_1 Q_2}{R^2}$$

- This force depends on the medium in which the point charges are located.
- The effect of medium is given as constant of proportionality (k)

$$F = k\frac{Q_1 Q_2}{R^2}$$

k = constant of proportionality

CONSTSNT OF PROPORTIONALITY (K):

- It is defined as

$$k = \frac{1}{4\pi\varepsilon}$$

$\varepsilon -$ permitivity of the medium where the charge located

In general ε is expressed as,

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

ε_0 = Permitivity of the space
ε_r = Relative permittivity of free space
ε = absolute permitivity

For free space or vacuum

$$\varepsilon_r = 1$$
$$\varepsilon = \varepsilon_0$$

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2}$$

Hence the coulombs law is expressed as,

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$$

7.Derive expression for differential vector operator. [CO1-H1]
THE VECTOR OPERATOR Δ (del):
- The divergence of an vector A is given by

$$\text{div } \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

- The div of an vector is a scalar quantity
- The divergence operation can be represented by the use of a mathematical operator called del operator (Δ) which is a vector operator.
- It is given by

$$\nabla = \frac{\partial}{\partial x}\vec{a}_x + \frac{\partial}{\partial y}\vec{a}_y + \frac{\partial}{\partial z}\vec{a}_z$$

- Now vector $\vec{A}$ is a vector field and ∇ is a vector differential operator.
- The resultant of a div is a scalar.
- To get scalar from two vectors it is necessary to take dot product of two

$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\nabla \cdot \vec{A} = \left(\vec{a}_x\frac{\partial}{\partial x} + \vec{a}_y\frac{\partial}{\partial y} + \vec{a}_z\frac{\partial}{\partial z}\right) \cdot \left(A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z\right)$$

$$\vec{a}_x \cdot \vec{a}_x = \vec{a}_y \cdot \vec{a}_y = \vec{a}_z \cdot \vec{a}_z = 1$$

$$\nabla \cdot \vec{A} = \left(\frac{\partial(A_x)}{\partial x} + \frac{\partial(A_y)}{\partial y} + \frac{\partial(A_z)}{\partial z}\right) = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \vec{A} = div \ \vec{A}$$

- ∇ is a mathematical operator and need not be involved always in the dot product.
- It may be operated on scalar field to obtain vector result, thus if m is a scalar then

$$\nabla m = \left(\vec{a}_x\frac{\partial}{\partial x} + \vec{a}_y\frac{\partial}{\partial y} + \vec{a}_z\frac{\partial}{\partial z}\right)m = \frac{\partial m}{\partial x} + \frac{\partial m}{\partial y} + \frac{\partial m}{\partial z}$$

- The ∇ operator does not have any other specific form in different coordinate systems

8.What is gradient of scalar? Explain its properties. [CO1-L1]
GRADIENT OF A SCALAR:
- Let W be the unique function of x,y,z coordinates in the Cartesian system.
- This is the scalar function and denoted as W (x,y,z)
- Consider the vector operator in Cartesian system denoted as Δ.
- It is defined as

$$\nabla\,(del) = \frac{\partial}{\partial x}\vec{a}_x + \frac{\partial}{\partial y}\vec{a}_y + \frac{\partial}{\partial z}\vec{a}_z$$

- The operation of the vector operator ∇ on a scalar function is called gradient of a scalar w.

$$\text{Grad W} = \nabla W = \left(\vec{a}_x\frac{\partial}{\partial x} + \vec{a}_y\frac{\partial}{\partial y} + \vec{a}_z\frac{\partial}{\partial z}\right)W$$

$$\text{Grad W} = \left(\vec{a}_x\frac{\partial W}{\partial x} + \vec{a}_y\frac{\partial W}{\partial y} + \vec{a}_z\frac{\partial W}{\partial z}\right)$$

- The gradient of scalar W in various coordinate systems are

S.No	Co-ordinate system	Grad W = ∇W
1	Cartesian	$\nabla W = \vec{a}_x\dfrac{\partial W}{\partial x} + \vec{a}_y\dfrac{\partial W}{\partial y} + \vec{a}_z\dfrac{\partial W}{\partial z}$
2	Cylindrical	$\nabla W = \vec{a}_r\dfrac{\partial W}{\partial r} + \dfrac{1}{r}\vec{a}_\varphi\dfrac{\partial W}{\partial \varphi} + \vec{a}_z\dfrac{\partial W}{\partial z}$
3	Spherical	$\nabla W = \vec{a}_r\dfrac{\partial W}{\partial r} + \dfrac{1}{r}\vec{a}_\theta\dfrac{\partial W}{\partial \theta} + \dfrac{1}{r\sin\theta}\vec{a}_\varphi\dfrac{\partial W}{\partial \varphi}$

PROPERTIES OF GRADIENT OF A SCALAR:
- The gradient ΔW gives the maximum rate of change of W per unit distance.
- The gradient ΔW always indicates the direction of the maximum rate of change of W
- The gradient ΔW at any point is perpendicular to the constant W surface,which passes through the point.
- The directional derivative of W along the unit vector a is

$$\nabla\ W\cdot\vec{a}$$

Which is projection of ΔW in the direction of unit vector a
- If U is the another scalar function then,

$$\dot{\nabla}(U+W) = \nabla U + \nabla W$$
$$\nabla(U W) = U\nabla W + W\nabla U$$

$$\nabla\left(\frac{U}{W}\right) = \frac{W\nabla U - U\nabla W}{W^2}$$

9.Give the types of charge distributions in various configurations. Find its charge density. [CO1-L3]

TYPES OF CHARGE DISTRIBUTIONS:

- There are 4 types of charge distributions.
- 1.Point charge 2.line charge 3.surface charge 4.volume charge.
  POINT CHARGE:
- The dimensions of a surface carrying charge are very very small compared to region surrounding it, then the surface can be treated to be a point. The corresponding charge is called point charge.
- The point charge has a position but not the dimensions.
- The point charge can be positive or negative.

Point charges
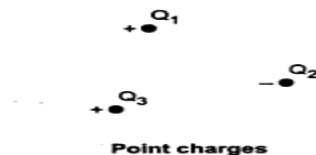
LINE CHARGE:

- The charge may be spreaded all along a line which may be finite or infinite.
- Such a charge uniformly distributed along a line is called a line charge.
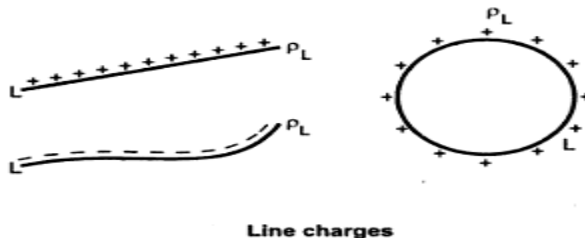
Line charges

The charge density of a line charge is denoted as ρ_L and defined as charge per unit length.

$$\rho_L = \frac{\text{Total charge in coulomb}}{\text{Total volume in square meters}}(C/m)$$

METHOD OF FINDING Q FROM ρ_L:

- The total charge is calculated by considering differential length dl of the line.
- Then by integrating the charge dQ on dl for the entire length the total charge Q can be calculated.
- Such an integral is called line integral.
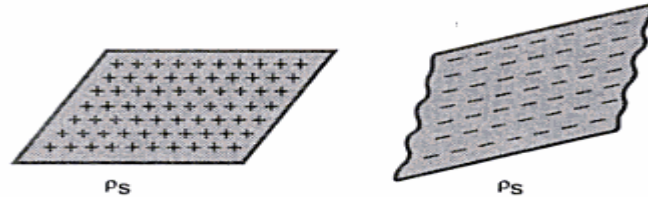- Mathematically dQ= ρ_L

$$Q = \int_L dQ = \int_L \rho_L dL$$

- If the line of length L is a closed path then integral is called closed contour integral.

$$Q = \oint_L \rho_L \, dL$$

SURFACE CHARGE:

- If is the charge is distributed uniformly over a two dimensional surface then it is called a surface charge or a sheet of charge

- The surface charge density is denoted as ρ_s and defined as charge per unit area.

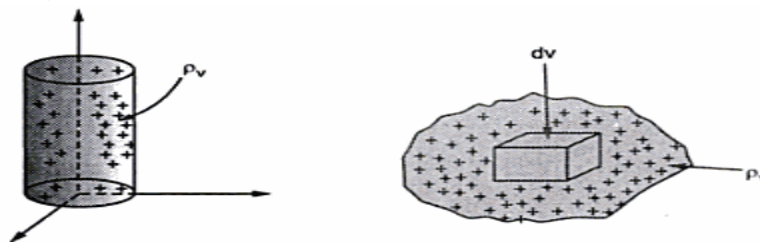$$\rho_s = \frac{\text{Total charge in coulomb}}{\text{Total volume in square meters}} (C/m^2)$$

METHOD OF FINDING Q FROM ρ_s:

- The total charge Q is calculated by considering elementary surface area dS.
- The elementary charge is given by ρ_s dS.
- By integrating Dq the total charge Q is obtained.
- Such a integral is called surfaceintegral.
- Mathematically it is given as

$$Q = \int_S dQ = \int_S \rho_S dS$$

VOLUME CHARGE:

- If the charge distributed uniformly in a volume then it is called volume charge.

- The volume charge density is denoted as ρ_v and defined as charge per unit volume.

$$\rho_v = \frac{\text{Total charge in coulomb}}{\text{Total volume in cubic meters}} (C/m^3)$$

METHOD OF FINDING Q FROM ρ_v:

- The charge Dq possessed by differential volume is ρ_v dv.
- The total charge is obtained by the integration Dq.
- Such a integral is called volume integral.

- Mathematically it is given as

$$Q = \iiint\limits_{vol} \rho_v \, dV$$

PROBLEM 1:

A charge is distributed on x axis of Cartesian system having a line charge density of $3x^2$ µC/m. Find the total charge over the lecnth of 10 m.

Solution: Given $\rho_L = 3x^2 \, \mu\frac{C}{m}$ and L = 10 m

      dl=dx in x direction

Charge $Q = \int_L \rho_L \, dL = \int_0^{10} 3x^2 \, dx = \left[\frac{3x^3}{3}\right] = 1000\mu C = 1 \text{ mC}$

PROBLEM 2:

Find the total charge inside a volume having volume charge density as $10z^2 \, e^{-0.1x} \sin \pi y \, C / m^3$. The volume is defined between $-2 \leq x \leq 2$, $0 \leq y \leq 1$ and $3 \leq z \leq 4$.

Solution : Given $\rho_v = 10 \, z^2 \, e^{-0.1x} \sin \pi y \, C/m^3$

Consider differential volume in cartesian system as,

$$dv = dx \, dy \, dz$$

∴ $dQ = \rho_v \, dv = 10 \, z^2 \, e^{-0.1x} \sin \pi y \, dx \, dy \, dz$

∴ $Q = \int\limits_{vol} \rho_v \, dv$

But now it becomes triple integration

$$\acute{Q} = \int\limits_{z=3} \int\limits_{y=0} \int\limits_{x=-2} 10 \, z^2 \, e^{-0.1x} \sin \pi y \, dx \, dy \, dz$$

$$= \int\limits_{z=3}^{4} \int\limits_{y=0}^{1} 10 \, z^2 \sin \pi y \left[\frac{e^{-0.1x}}{-0.1}\right]_{-2}^{2} dy \, dz$$

$$= \int\limits_{z=3}^{4} 10 \, z^2 \left[-\frac{\cos \pi y}{\pi}\right]_0^1 \left[\frac{e^{-0.2}}{-0.1} - \frac{e^{+0.2}}{-0.1}\right] dz$$

$$= 10 \left[\frac{z^3}{3}\right]_3^4 \left[\frac{-\cos \pi}{\pi} - \frac{-\cos 0}{\pi}\right] 4.0267$$

$$= 10 \left[\frac{4^3 - 3^3}{3}\right] \left[\frac{1}{\pi} + \frac{1}{\pi}\right] 4.0267 = 316.162 \text{ C}$$

10.Define Absolute potential. Explain with an example. [CO1-L1]
CONCEPT OF ABSOLUTE POTENTIAL:
- Absolute potentials are measured with respect to a specified reference position.
- The reference position is assumed to be at zero potential (ground pt).
- All the potentials of the circuits are measured with respect to ground which is treated to be zero potential difference.eg. Shielded cable.
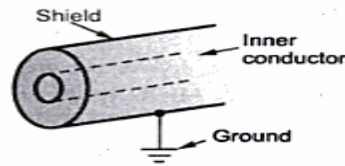
- The conductor is shielded as shown in the above figure.
- The outer shield is grounded to minimise the interference of the external signals with the signal carried by inner conductor.
- The outer shield is taken as reference.
- Consider potential difference V_{AB} due to movement of unit charge from B to A in a field of point charge Q

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

- Let the charge is moved from ∞ to point A

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{\infty}\right] = \frac{Q}{4\pi r_A\varepsilon_0}\,v$$

- The quantity represented by above equation is called potential of point A denoted as V_A.

$$V_A = \frac{Q}{4\pi r_A\varepsilon_0}\,v$$

- This is also called absolute potential of point A.
- Similarly absolute potential of point B is defined as

$$V_B = \frac{Q}{4\pi r_B\varepsilon_0}\,v$$

- This is work done in moving the point charge from ∞ to B.
- The potential difference can be expressed as the difference between the absolute potentials of two points.

$$V_{AB} = V_A - V_B$$

- The absolute potential at any point in an electric field is defined as the work done in unit test charge from the infinity to the point against the direction of the field.
- For spherical system the absolute potential is given by

$$V = \frac{Q}{4\pi r\varepsilon_0}\,v$$

- The reference point is at infinity.
- The potential is expressed mathematically as

$$V = -\int_B^A \vec{E} \cdot d\vec{L}$$

11.What is potential difference? [CO1-L1]
POTENTIAL DIFFERENCE:

Work done in moving unit charge from point B to point A in the field $\bar{E}$ is called potential difference.

Potential difference $V_{AB} = -\int_B^A \vec{E} \cdot d\vec{L}$

Unit of potential difference is

$$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

12.Derive potential difference for various configurations using various charges. [CO1-H1]
POTENTIAL DIFFERENCES FOR VARIOUS CONFIGURATIONS:
POTENTIAL DUE TO A POINT CHARGE:

Consider a point charge , located at the orgin of a spherical co-ordinate system, producing $\bar{E}$ radially in all the directions.
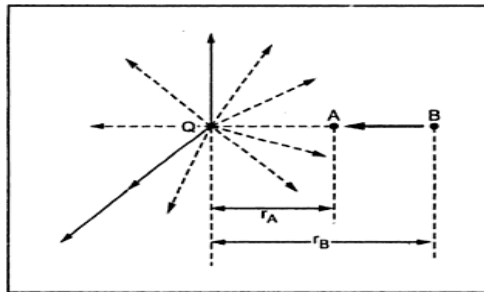

Fig.   **Potential due to a point charge Q**

The field $\bar{E}$ due to point charge is

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a}_r - - - - - -(1)$$

The differential length of the spherical system is

$$d\bar{L} = dr\,\bar{a}_r + rd\theta\bar{a}_\phi + r\sin\theta\,d\phi\bar{a}_\phi \quad ------(2)$$

The potential difference V_{AB} is

$$V_{AB} = -\int_B^A \bar{E} \cdot d\bar{L} \qquad \text{But } B \Rightarrow r_B \quad \text{and} \quad A \Rightarrow r_A$$
$$------(3)$$

Sub., equation 1 & 2 in 3, we get

$$V_{AB} = -\int_B^A \left(\frac{Q}{4\pi\varepsilon_0 r^2}\bar{a}_r\right)\cdot(dr\,\bar{a}_r + rd\theta\,\bar{a}_0 + r\sin\theta d\phi\bar{a}_\phi)$$

$$V_{AB} = -\int_B^{r_A} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

Taking the constant term outside,

$$V_{AB} = -\frac{Q}{4\pi\varepsilon_0}\int_B^{r_A} r^{-2}dr = \frac{-Q}{4\pi\varepsilon_0}\left[\frac{r^{-1}}{-1}\right]_B^{r_A}$$
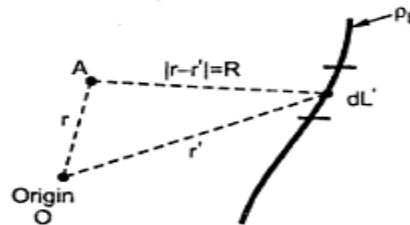
$$= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_B^A = \frac{-Q}{4\pi\varepsilon_0}\left[-\frac{1}{r_A}-\left(-\frac{1}{r_B}\right)\right]$$

$$\boxed{V_{AB} = -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r_A}+\frac{1}{r_B}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A}-\frac{1}{r_B}\right] \text{ V}}$$

when $r_B > r_A$, $\frac{1}{r_B} < \frac{1}{r_A}$ and V_{AB} is positive. This indicates the work is done by external source in moving unit charge from B to A.

POTENTIAL DUE TO A LINE CHARGE:

Consider a line charge having density ρ_L C/m as shown in figure
Consider differential length dL ' at a distance r' .

Differential charge on the length dL is given by,

$$dQ = \rho_L(r')\,dL' \qquad \text{... (1)}$$

where $\rho_L(r')$ = Line charge density at r'

Let the potential at A is to be determined. Then,

$$dV_A = \frac{dQ}{4\pi\varepsilon_0|r-r'|} = \frac{dQ}{4\pi\varepsilon_0 R} \qquad \text{... (2)}$$

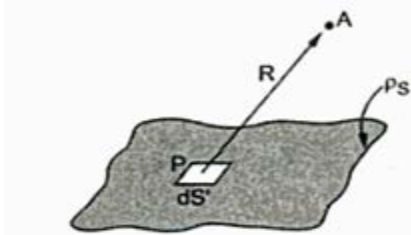The $R = |r-r'|$ indicates the distance of point A from the differential charge.

The dV_A is a differential potential at A. Hence the potential V_A can be obtained by integrating dV_A over the length over which line charge is distributed.

$$\therefore \qquad V_A = V(r) = \int_{Line}\frac{dQ}{4\pi\varepsilon_0 R} \qquad \text{and using (1),}$$

$$\therefore \qquad \boxed{V_A = V(r) = \int_{Line}\frac{\rho_L(r')\,dL'}{4\pi\varepsilon_0 R} \text{ V}} \qquad \text{... (3)}$$

POTENTIAL DUE TO A SURFACE CHARGE:

Consider a uniform surface charge having density ρ_S C/m as shown in figure

Consider differential surface area dS ' at a point P.
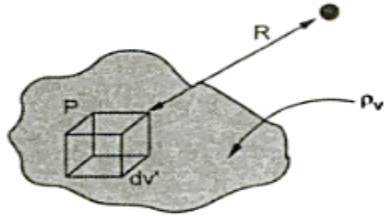The differential surface area can be expressed in

$$dQ = \rho_S(r')\,dS'$$

$$dV_A = \frac{dQ}{4\pi\varepsilon_0 R} = \frac{\rho_S(r')\,dS'}{4\pi\varepsilon_0 R}$$

where   R = Distance of point A from the differential charge

The total potential at A can be obtained by integrating dV_A over the given surface.

$$V_A = \int_S \frac{\rho_S(r')\,dS'}{4\pi\varepsilon_0 R} \ V$$

POTENTIAL DUE TO A VOLUME CHARGE:

Consider a uniform volume charge having density ρ_v C/m as shown in figure

Consider differential volume dV' at a point P with charge density $\rho_v(r')$

The differential charge can be expressed in

$$dQ = \rho_v(r')\,dv' \quad \dots (1)$$

$$dV_A = \frac{dQ}{4\pi\varepsilon_0 R} = \frac{\rho_v(r')\,dv'}{4\pi\varepsilon_0 R}$$

Where R = distance of point A from the diffential charge

The total potential at A can be obtained by integrating dV_A over the given volume.

$$V_A = \int_v \frac{\rho_v(r')\,dv'}{4\pi\varepsilon_0 R} \ V$$

LIST OF VARIOUS POTENTIALS:

Potential difference	$V_{AB} = -\int_B^A \vec{E}\cdot d\vec{L}$
Absolute potential due to point charge	$V_{AB} = \frac{Q}{4\pi\varepsilon_0 R}\ v$
Absolute potential due to line charge	$V_{AB} = \int_L \frac{\rho_L\,dL}{4\pi\varepsilon_0 R}\ v$
Absolute potential due to surface charge	$V_{AB} = \iint_S \frac{\rho_S\,dS}{4\pi\varepsilon_0 R}\ v$
Absolute potential due to volume charge	$V_{AB} = \iiint_V \frac{\rho_V\,dV}{4\pi\varepsilon_0 R}\ v$
If the reference is other than infinity	$V_{AB} = \frac{Q}{4\pi\varepsilon_0 R} + C\ v$
In all the expressions, R is the distance of point A from the charge Q or differential charge dQ	

13.What is electric dipole? Derive its expression. [CO1-H1]

ELECTRIC DIPOLE:

- Two point charges of equal magnitude but opposite sign separated by a very small distance is called an electric dipole.
- Consider an electric dipole as shown in the figure
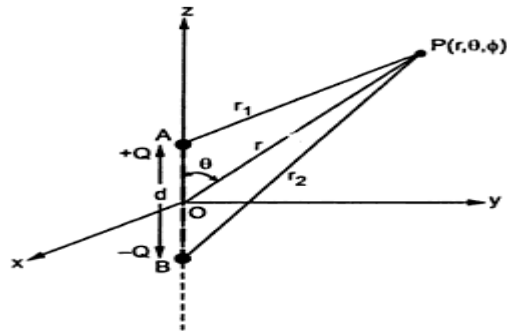
- The two point charges +Q and –Q are separated by a distance d.
- Consider a point P (r,Θ,Φ) in spherical coordinate system.
- E vector is find out due to an electric dipole at point P.
- Let O be the midpoint of AB.
- The distance of point P from A is r_1 while the distance of point P from B is r_2.
- The distance of point P from point O is r.
- d is very small compared to the distances r_1, r_2 and r.
- The coordinates of A are (0,0,+d/2) and that of B are (0,0,-d/2).
- To find E vector the potential V at point P due to an electric dipole.
- The E vector is given as
  $$\overline{E}=-\nabla V,$$
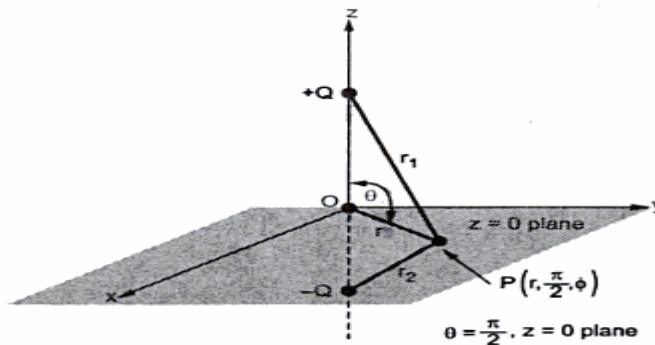
EXPRESSION OF E DUE TO AN ELECTRIC DIPOLE:
- The potential at P due to +Q is given by

$$V_1 = \frac{+Q}{4\pi r_1 \varepsilon_0} \, v$$

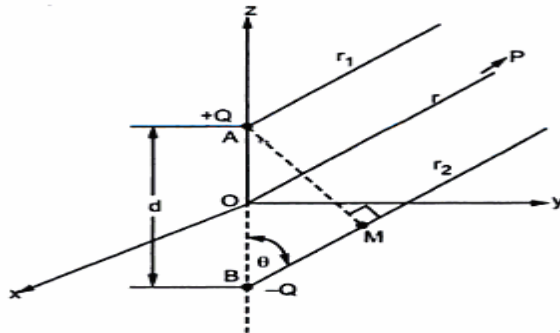- The potential at P due to -Q is given by

$$V_2 = \frac{-Q}{4\pi r_2 \varepsilon_0} \, v$$

- The total potential at point P is the vector sum of V_1 and V_2

$$V = V_1 + V_2$$

$$= \frac{+Q}{4\pi\varepsilon_0 \, r_1} - \frac{Q}{4\pi\varepsilon_0 \, r_2}$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 \, r_2}\right]$$

- Now point P is located in z=0 plane as shown in the above figure.
- Then $r_2 = r_1$. Hence V=0.
- Thus the entire z=0 plane ie xy plane is a zero potential surface.

- Now consider the point P is located far away from the electric dipole.
- Thus r_1, r_2 and r can be assumed to be parallel to each other as shown in the above figure.
- AM is drawn perpendicular to A on r_2.
- The angle made by r_1, r_2 and r with z axis is Θ as all are parallel.

$$\therefore \quad BM = AB \cos\theta$$

$$= d \cos\theta$$

$$PB = PM + BM$$

$$PA = PM \quad \text{as AM is perpendicular.}$$

$$PB = r_2, \quad PA = r_1$$

$$BM = PB - PM = r_2 - PM$$

$$PM = PA = r_1$$

$$BM = r_2 - r_1$$

$$r_2 - r_1 = d \cos\theta$$

As d is very small, $r_1 \approx r_2 \approx r$ hence $r_1 \, r_2 = r^2$

$$\therefore \qquad V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{d\cos\theta}{r^2}\right] \quad V$$

Now

$$\bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial\theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\bar{a}_\phi\right]$$

$$\therefore \qquad \frac{\partial V}{\partial r} = \frac{Q\,d\cos\theta}{4\pi\varepsilon_0}\left[\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right)\right] = \frac{Q\,d\cos\theta}{4\pi\varepsilon_0}\left[\frac{\partial}{\partial r}(r^{-2})\right]$$

$$= \frac{Q\,d\cos\theta}{4\pi\varepsilon_0}[-2\,r^{-3}] = \frac{-2Q\,d\cos\theta}{4\pi\varepsilon_0 \, r^3}$$

$$\frac{\partial V}{\partial \theta} = \frac{Q\,d}{4\pi\varepsilon_0\,r^2}[-\sin\theta] \quad \text{and} \quad \frac{\partial V}{\partial \phi} = 0$$
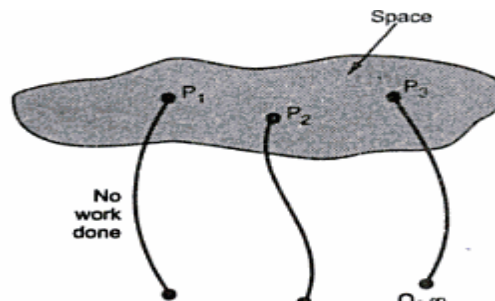
$$\overline{E} = -\left[\frac{-2Q\,d\cos\theta}{4\pi\varepsilon_0\,r^3}\,\overline{a}_r - \frac{Q\,d\sin\theta}{4\pi\varepsilon_0\,r^3}\,\overline{a}_\theta\right]$$

$$\boxed{\overline{E} = \frac{Q\,d}{4\pi\varepsilon_0\,r^3}[2\cos\theta\,\overline{a}_r + \sin\theta\,\overline{a}_\theta]}$$

- This is the electric field E vector at point P due to an electric dipole.

13.What is energy density? Explain energy density in electrostatic fields and expression for energy storage.
ENERGY DENSITY IN THE ELECTROSTATIC FIELDS:

- When a unit positive charge is moved from infinity to a point in field the work is done by the external source and the energy is expended.
- If the external source is removed then the unit positive charge will be subjected to a force exerted by a field and will be moved in the direction of force.
- To hold the charge at a point in a electrostatic field the external source has to do work.
- This energy gets stored in the form of potential energy when the test charge is hold at a point in the field.
- If the external source is removed the potential energy gets converted to kinetic energy.
- Consider an empty where there is no electric field.
- The charge Q_1 is moved from ∞ to a point in the space say P_1.There is no work because of absence of E.
    - The charge Q_2 is placed at a point P_2 in the space as shown in the figure.
    - Now there is an electric field due to Q_1 and Q_2 is required to move against the field of Q_1.

$$\text{Potential} = \text{Work done per unit charge}\left(\frac{W}{Q}\right)$$

$$\text{Work done} = \text{Potential V} \times \text{Charge Q}$$

$$\therefore \ \text{Work done to position } Q_2 \text{ at } P_2 = V_{2,1}\,Q_2$$

$$\text{where} \qquad V_{2,1} = \text{Potential at } P_2 \text{ due to } P_1$$

- Now let charge Q_3 is moved from ∞ to P_3.
- There are electric fields due to Q_1 and Q_2.

- Hence total work done is due to potential at P_3 due to charge at P_1 and potential at P_3 due to charge at P2.

  $\therefore$ Work done to position Q_3 at P_3 = $V_{3,1}$ Q_3 + $V_{3,2}$ Q_3

  Thus for charge Q_n to be placed at P_n, we can write,

  $\therefore$ Work done to position Q_n at P_n = $V_{n,1}$ Q_n + $V_{n,2}$ Q_n + ...

  Hence the total work done in positioning all the charges is,

  $$W_E = Q_2\ V_{2,1} + Q_3\ V_{3,1} + Q_3\ V_{3,2} + ...$$

- The total work done is nothing but the potential energy in the system of charges hence denoted as W_E.
- If charges are placed in reverse order we can write as

$$W_E = Q_3\ V_{3,4} + Q_2\ V_{2,3} + Q_2\ V_{2,4} + Q_1\ V_{1,2} + Q_1\ V_{1,3} + Q_1\ V_{1,4} + ...$$

In this expression Q_n is placed first, then Q_{n-1} ... then Q_4, Q_3, Q_2 and finally Q_1.

Adding

$$2\ W_E = Q_1\ (V_{1,2} + V_{1,3} + V_{1,4} + ... + V_{1,n})$$
$$+ Q_2\ (V_{2,1} + V_{2,3} + V_{2,4} + ... + V_{2,n})$$
$$+ Q_3\ (V_{3,1} + V_{3,2} + V_{3,4} + ... + V_{3,n}) + ...$$

- Each sum of the potentials is the total resultant potential due to all the charges except for the charge at the point at which potential is obtained.

$\therefore$ $V_{1,2} + V_{1,3} + V_{1,4} + ... + V_{1,n} = V_1$

This is potential at P_1 where Q_1 is placed due to all other charges $Q_2, Q_3, ... Q_n$.

Similarly, $V_{2,1} + V_{2,3} + V_{2,4} + ... + V_{2,n} = V_2$ and so on.

Using in the equation (6),

$$2\ W_E = Q_1\ V_1 + Q_2\ V_2 + Q_3\ V_3 + ...$$

$\therefore$
$$\boxed{W_E = \frac{1}{2} \sum_{m=1}^{n} Q_m\ V_m \quad J}$$

- This is the potential energy stored in the system of n point charges.
- Instead of point charges if the region is having continuous charge distributions then in the above equation the summation becomes integration.

For line charge ρ_L,	$W_E = \frac{1}{2} \int \rho_L\ dL\ V \quad J$
For surface charge ρ_S,	$W_E = \frac{1}{2} \int \rho_S\ dS\ V \quad J$
For volume charge ρ_v,	$W_E = \frac{1}{2} \int \rho_v\ dv\ V \quad J$

ENERGY STORED IN TERMS OF VECTOR D and E:

- Consider the volume charge distribution having uniform charge density ρ_v C/m^3.
- Hence the total energy stored is given by

$$W_E = \frac{1}{2} \int_{vol} \rho_v\ V\ dv$$

According to Maxwell's first equation,

$$\rho_v = \nabla \cdot \overline{D}$$

$$\therefore \qquad W_E = \frac{1}{2} \int_{vol} (\nabla \cdot \overline{D}) \, V \, dv$$

For any vector A and scalar V there is **vector identity,**

$$\nabla \cdot V\overline{A} = \overline{A} \cdot \nabla V + V(\nabla \cdot \overline{A})$$

$$\therefore \qquad (\nabla \cdot \overline{A}) V = \nabla \cdot V\overline{A} - \overline{A} \cdot \nabla V$$

$$W_E = \frac{1}{2} \int_{vol} (\nabla \cdot V\overline{D} - \overline{D} \cdot \nabla V) \, dv$$

$$W_E = \frac{1}{2} \int_{vol} (\nabla \cdot V\overline{D}) \, dv - \frac{1}{2} \int_{vol} \overline{D} \cdot \nabla V \, dv$$

- According to divergence theorem volume integral can be converted to closed surface integral if closed surface totally surrounds the volume.

We know that $V \propto \frac{1}{r}$ and $\overline{D} \propto \frac{1}{r^2}$ for point charge, $V \propto \frac{1}{r^2}$, $\overline{D} \propto \frac{1}{r^3}$ for dipoles and so on. So $V\overline{D}$ is proportional to at least $1/r^3$ while dS varies as r^2. Hence total integral varies as $1/r$. As surface becomes very large, $r \to \infty$ and $1/r \to 0$. Hence **closed surface integral is zero** in the equation

$$\therefore \qquad W_E = -\frac{1}{2} \int_{vol} \overline{D} \cdot \nabla V \, dv$$

But $\qquad \overline{E} = -\nabla V$

$$\therefore \qquad W_E = -\frac{1}{2} \int_{vol} \overline{D} \cdot (-\overline{E}) \, dv$$

$$\therefore \qquad \boxed{W_E = \frac{1}{2} \int_{vol} \overline{D} \cdot \overline{E} \, dv \ J}$$

$$\frac{1}{2} \int_{vol} (\nabla \cdot V\overline{D}) \, dv = \frac{1}{2} \oint (V\overline{D}) \cdot d\overline{S}$$

$$W_E = \frac{1}{2} \oint (V\overline{D}) \cdot d\overline{S} - \frac{1}{2} \int_{vol} \overline{D} \cdot \nabla V \, dv$$

$$\overline{D} = \varepsilon_0 \overline{E}$$

$$W_E = \frac{1}{2} \int_{vol} \varepsilon_0 \overline{E} \cdot \overline{E} \, dv \ J$$

$$\boxed{W_E = \frac{1}{2} \int_{vol} \varepsilon_0 E^2 \, dv \ J} \qquad \text{... as } \overline{E} \cdot \overline{E} = E^2$$

$$\therefore \qquad \boxed{W_E = \frac{1}{2} \int_{vol} \frac{D^2}{\varepsilon_0} \, dv \ J}$$

In a differential form,

$$d W_E = \frac{1}{2} \overline{D} \cdot \overline{E} \, dv$$

$$\therefore \qquad \boxed{\frac{d W_E}{dv} = \frac{1}{2} \overline{D} \cdot \overline{E} \ J/m^3}$$

- This is called energy density in the electric field having units J/m^3. If this integrated over volume we get total energy present.

$$W_E = \int_{vol} \left(\frac{d\,W_E}{dv}\right) dv$$

PROBLEM:

If $V = x - y + xy + 2z$ V, find $\bar{E}$ at (1, 2, 3) and the energy stored in a cube of side 2 m centered at the origin.

Solution :

$$\bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\,\bar{a}_x + \frac{\partial V}{\partial y}\,\bar{a}_y + \frac{\partial V}{\partial z}\,\bar{a}_z\right]$$

$$\frac{\partial V}{\partial x} = 1 + y, \quad \frac{\partial V}{\partial y} = -1 + x, \quad \frac{\partial V}{\partial z} = 2$$

$\therefore \qquad \bar{E} = -\left[(1+y)\bar{a}_x + (x-1)\bar{a}_y + 2\bar{a}_z\right]$

At (1,2,3), $\qquad \bar{E} = -\left[(1+2)\bar{a}_x + (1-1)\bar{a}_y + 2\bar{a}_z\right]$

$$= -3\bar{a}_x - 2\bar{a}_z \quad V/m$$

Now $\qquad W_E = \frac{1}{2}\int_{vol} \varepsilon_0 |\bar{E}|^2 \, dv$

$$|\bar{E}| = \sqrt{(1+y)^2 + (x-1)^2 + (2)^2}$$

$\therefore \qquad |E|^2 = (1+y)^2 + (x-1)^2 + (2)^2 = y^2 + 2y + x^2 - 2x + 6$

$$W_E = \frac{1}{2}\int \varepsilon_0 |\bar{E}|^2 \, dv \qquad \text{and } dv = dx \, dy \, dz$$

The cube is centered at the origin.

Thus x varies from -1 to $+1$, y from -1 to $+1$ and z from -1 to $+1$.

$\therefore \qquad W_E = \frac{\varepsilon_0}{2} \int_{z=-1}^{1} \int_{y=-1}^{1} \int_{x=-1}^{1} (y^2 + 2y + x^2 - 2x + 6) \, dx \, dy \, dz$

Changing limits from -1 to 1 to 0 to 1 of each integral, making it twice.

$\therefore \qquad W_E = \frac{\varepsilon_0}{2}(2 \times 2 \times 2) \int_{z=0}^{1} \int_{y=0}^{1} \int_{x=0}^{1} (y^2 + 2y + x^2 - 2x + 6) \, dx \, dy \, dz$

$$= 4\varepsilon_0 \int_{z=0}^{1} \int_{y=0}^{1} \left[xy^2 + 2xy + \frac{x^3}{3} - x^2 + 6x\right] dy \, dz$$

$$= 4\varepsilon_0 \left[xz\frac{y^3}{3} + zxy^2 + \frac{x^3}{3}yz - x^2yz + 6\,xyz\right]_0^1 = 0.2361 \text{ nJ}$$

14. Explain Gauss divergence theorem.[CO1-L1]
GAUSS DIVERGENCE THEOREM:

- From the gauss law we can write

$$Q = \oiint_S \vec{D} \cdot d\vec{S}$$

- While the charge enclosed in the volume is given by

$$Q = \iiint_V \rho_V \, dv$$

- But according to gauss law in the point form

$$\nabla \cdot \vec{D} = \rho_V$$

$$Q = \iiint_V (\nabla \cdot \vec{D}) \, dv$$

- By equating

$$\oiint_S \vec{D} \cdot d\vec{S} = \iiint_V (\nabla \cdot \vec{D}) \, dv$$

- The above equation is called divergence theorem.
- It is also called as Gauss-ostrogradsky theorem.
- The divergence theorem is applied to the flux density.
- Bothe the sides of the divergence theorem gives the net flux crossing the closed surface.

15. Explain briefly about rectangular co-ordinate system and its base and differential vector. [CO1-L2]

> ➤ The system has three coordinates axis namely x,y,z.
> ➤ These co-ordinates are mutually right angle to each other.
> ➤ The three axes intersect at a common point called origin of a system.

Representing a point:

> ➤ A point in rectangular co-ordinate system is located by 3 coordinates namely x, y & z.

For example P(3,-1,2)
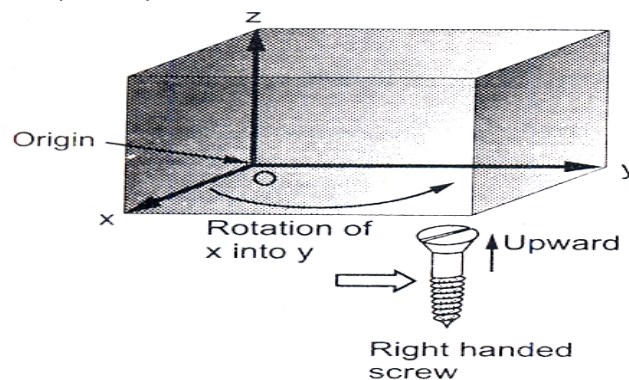
> ➤ The value may be positive or negative.

Unit vector or base vector:

➤ The base vector is a vector which is oriented along the direction of the axis.
➤ There are 3 base vectors in rectangular coordinates system.

➢ There are $\vec{a}_x + \vec{a}_y + \vec{a}_z$

$X = \vec{a}_x$, $Y = \vec{a}_y$, $Z = \vec{a}_z$

Position vector
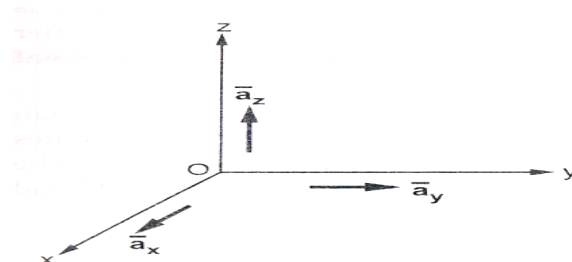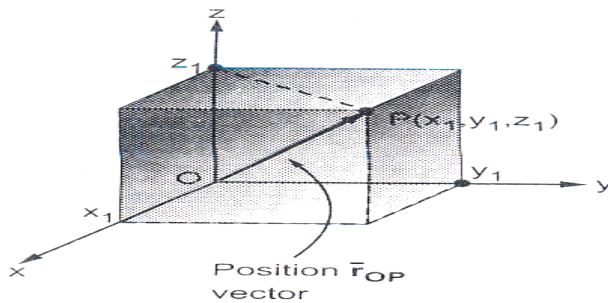

Position $\bar{r}_{OP}$ vector

    ➢ The position vector of point P is given as, $\vec{P} = x_1\vec{a}_x + y_1\vec{a}_y + z_1\vec{a}_z$

    ➢ Its magnitude is given by, $|\vec{P}| = \sqrt{x1^2 + y1^2 + z1^2}$

Distance vector:
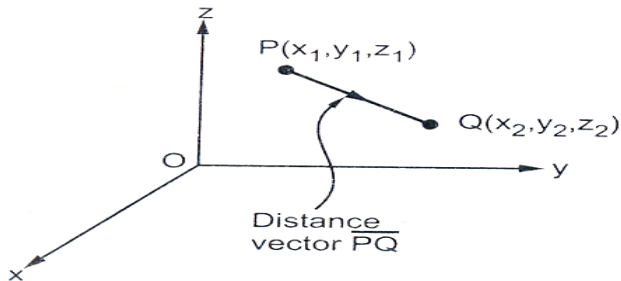
Distance vector $\overline{PQ}$

    ➢ The position vector of point P is given as
            $\vec{P} = x_1\vec{a}_x + y_1\vec{a}_y + z_1\vec{a}_z$
    ➢ The position vector of point Q is given as
            $\vec{Q} = x_2\vec{a}_x + y_2\vec{a}_y + z_2\vec{a}_z$
    ➢ The distance vector from P to Q is**,**
            $\overrightarrow{PQ} = \vec{Q} - \vec{P}$
            $\overrightarrow{PQ} = (x_2 - x_1)\vec{a}_x + (y_2 - y_1)\vec{a}_y + (z_2 - z_1)\vec{a}_z$

    ➢ The magnitude of the distance vector is given by,
$$|\overrightarrow{PQ}| = \left|\sqrt{(x2 - x1)\vec{a}x + (y2 - y1)\vec{a}y + (z2 - z1)\vec{a}z}\right|$$

Unit vectoris,

$$PQ = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}$$

$$= \frac{(x2-x1)\vec{a}x + (y2-y1)\vec{a}y + (z2-z1)\vec{a}z}{\left| \sqrt{(x2-x1)\vec{a}x + (y2-y1)\vec{a}y + (z2-z1)\vec{a}z} \right|}$$

Differential elements in cartesian coordinate system

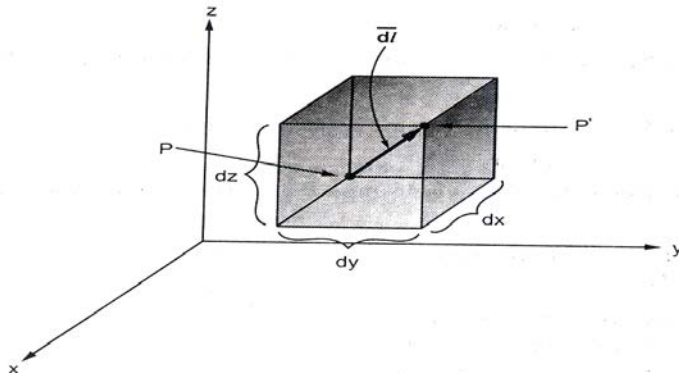➢ Consider a point P(x,y,z) in the rectangular coordinate system.
➢ Increase the coordinate to a differential length (dx, dy, dz) & a new cube is formed keeping the original point P as origin.
➢ Now P is the point located on new cube.

➢ The length of the new cube is given as (x+dx, y+dy , z+dz)
➢ The differential vector dl is given as, $\left| \vec{dl} \right| = \sqrt{dx^2 + dy^2 + dz^2}$
➢ The intersection of 3 new planes totally obtains 6 planes. This structure is known as parallelopiped.

Differential surface elements in cartesian system:
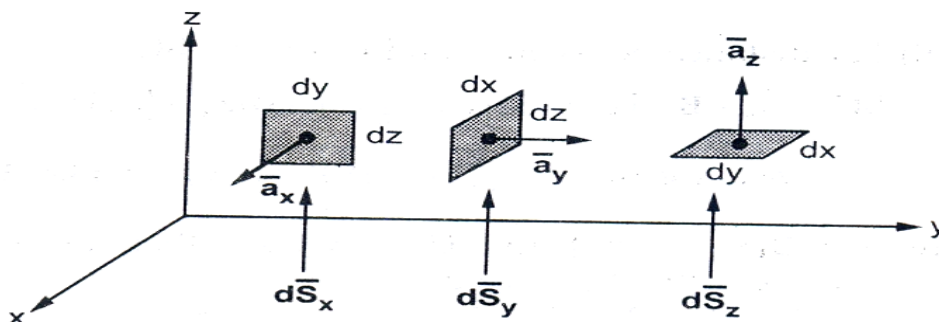
➢ The differential surface of x-plane is $\vec{ds_x}$=dydz $\vec{a}_x$
➢ The differential surface of y-plane is $\vec{ds_y}$=dxdz $\vec{a}_y$
➢ The differential surface of z-plane is $\vec{ds_z}$=dxdy $\vec{a}_z$

16 . Illustrate with neat diagram cylindrical co-ordinate system and its base and differential vector.[CO1-H2]

Cylindrical co-ordinate system

➢ The system has three coordinates axis namely r, φ ,z.

r plane:

➢ A cylinder of radius r is known as R-plane.

(a) r = constant
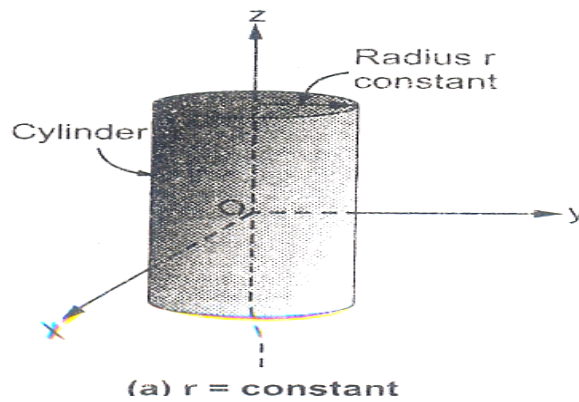
Ø plane:

➢ A vertical plane perpendicular to X-Y plane at angle φ with respect to X-axis.
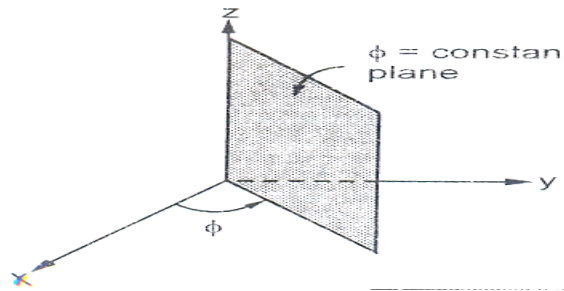
Z-plane:

➢ It is a horizontal plane which is parallel to X-Y plane.
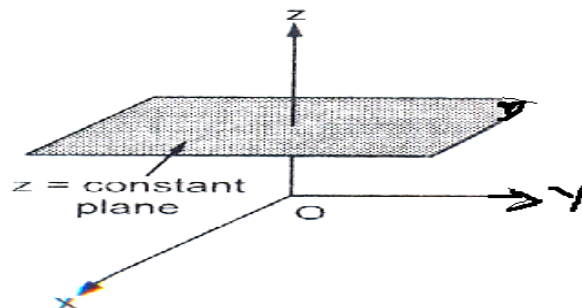
Representing a point P in cylindrical co-ordinate system:

➢ The 3 coordinates value lies in the range of

$0 \le r \le \infty$

$0 \le, \varphi \le 2\pi$

$-\infty \le z \le \infty$

➢ The point P can be represented as P(r, φ ,z).
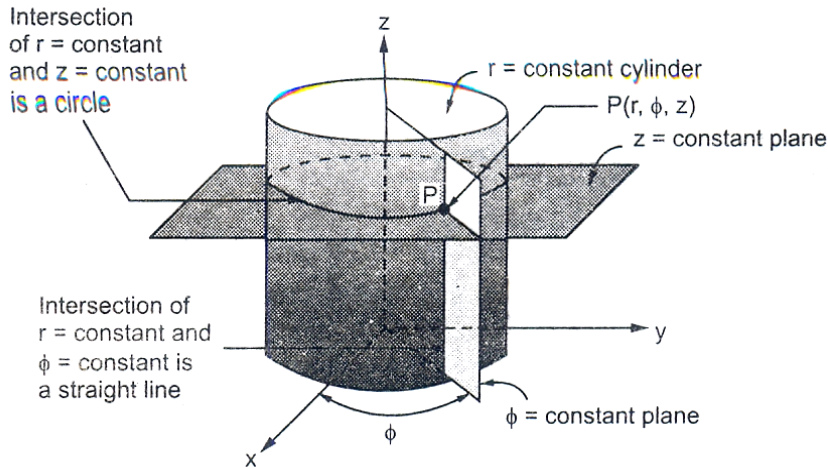
Unit vector:

➤ There are three unit vector $\vec{a}_r, \vec{a}_\varnothing, \vec{a}_z$
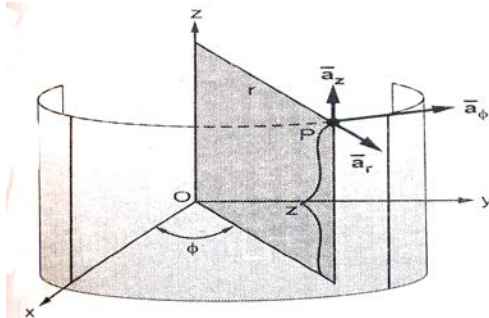
➤ Unit vector $\vec{a}_r$

       It lies in a plane parallel to X-Y

       It lies in a perpendicular to the cylinder.

➤ Unit vector $\vec{a}\,\varnothing$

       It is also parallel to X-Y plane.

       It points in the direction of increasing φ.

➤ Unit vector $\vec{a}_z$

       It lies parallel to Z axis.

Position vector

     ➤ The position vector of point P is given as,

$$\vec{P} = P_r\vec{a}_r + P_\varnothing\vec{a}\varphi + P_z\vec{a}_z$$

Differential elements in cylindrical coordinate system

     ➤ Consider a point P(r, φ ,z) in the cylindrical coordinate system.

     ➤ Increase each coordinate valuenby a differential length (dr, d φ, dz).

     ➤ Since Ø is not a straight line, the differential length of the arc is given as rd φ.

     ➤ The differential vector length is given as, $\left| \overrightarrow{dl} \right| = \sqrt{dr^2 + r^2 d\varphi^2 + dz^2}$
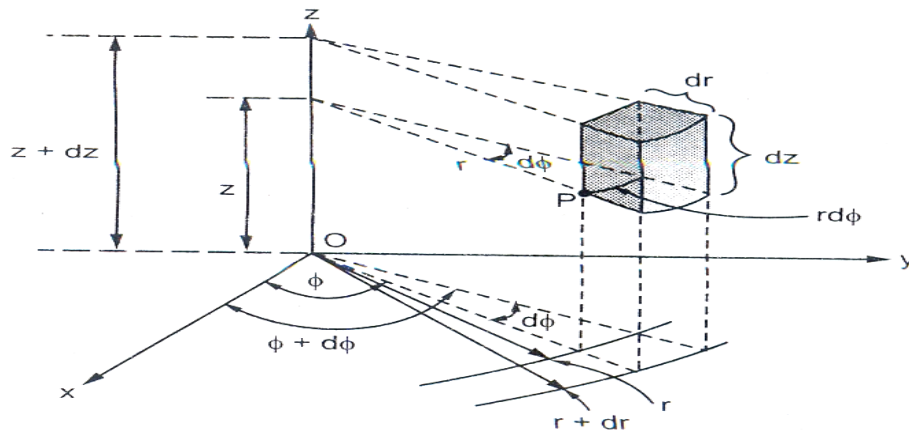
Differential volume:

  ➢ The Differential volume of the cylinder is,

       dV= dr rd φ dz

       dv= rdr d φ dz

Differential surface elements in cylindrical system: $\vec{a}_r, \vec{a}\,\varphi, \vec{a}_z$

The differential surface vector normal to r-plane is $\vec{ds_r}=$ rd φ dz$\vec{a}_r$

The differential surface vector normal to φ -plane is $\vec{ds_\varphi}$ =dr dz$\vec{a}\,\varphi$

The differential surface vector normal to z -plane is $\vec{ds_z}$ = rd φ dr$\vec{a}_z$
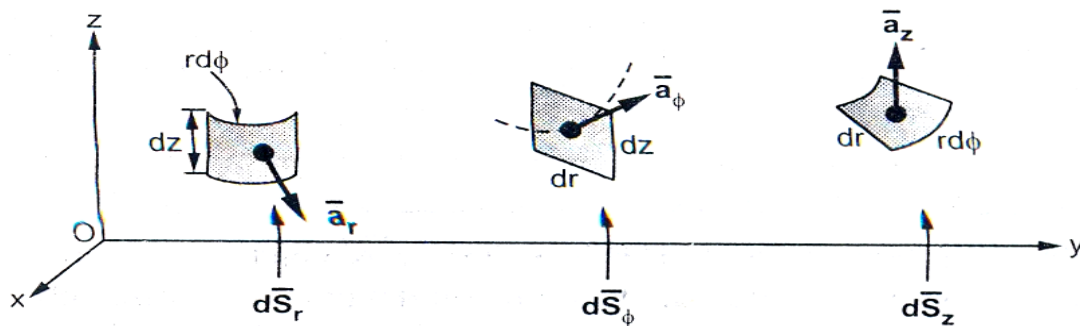
17. Derive the electric field due to infinite line charge with charge density ρ_L. [CO1-H2]

     Consider a infinite line charge whose charge density is ρ_L.

  ➢ The infinite line charge lies on Z- axis.

  ➢ The point P is on Y-axis at a distance r from the origin.
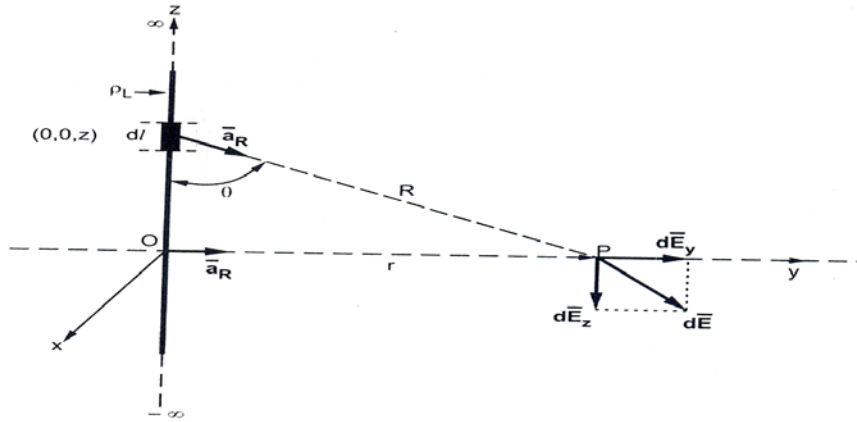
➤   The charge dQ is given as   dQ= ρ_L dl.

   ➤   since the differential elements lies on Z-axis

           dl=dz

           dQ= ρ_L.dz.

   ➤   the distance between dl and P is,

$$\vec{R}= \text{dl P} \qquad\qquad\qquad \vec{r}_p = r\vec{a}_y$$
$$\vec{R}= \vec{r}_p - \vec{r}\text{dl} \qquad\qquad\qquad \vec{r}\text{dl} = z\vec{a}_z$$
$$\vec{R} = r\vec{a}_y - z\vec{a}_z$$

   ➤   The unit vector

$$\vec{a}_R = \frac{\vec{R}}{|R|}$$

$$= \frac{r\vec{a}_y - z\vec{a}_z}{\sqrt{r^2 + z^2}}$$

The electric field of the differential elements is,

$$\overrightarrow{dE} = \frac{dQ}{4\pi\varepsilon R^2}\vec{a}_R$$

$$\overrightarrow{dE} = \frac{\rho_L.\,dz}{4\pi\varepsilon(\sqrt{r^2 + z^2})^2} \frac{r\vec{a}_y - z\vec{a}_z}{\sqrt{r^2 + z^2}}$$

Eliminating the Z- component , the above equation becomes,

$$\overrightarrow{dE} = \frac{\rho_L dz}{4\pi\varepsilon(\sqrt{r^2 + z^2})^2} \frac{r\vec{a}_y}{\sqrt{r^2 + z^2}}$$

BY integration , the total electric field is calculated as,

$$\vec{E} = \int_{-\infty}^{\infty} \frac{\rho_L dz}{4\pi\varepsilon(\sqrt{r^2 + z^2})^{3/2}} \, rdz \, \vec{a}_y$$

    z = r tanθ,

$$dz = r\sec^2\theta \, d\theta$$

for $z = -\infty$,

      $r\tan\theta = -\infty$,

$$\theta = \tan^{-1}(-\infty) = -\frac{\pi}{2} \text{ and}$$

for $z = +\infty$,

$$\theta = \tan^{-1}(\infty) = +\frac{\pi}{2}$$

Therefore

$$\vec{E} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\rho_L}{4\pi\varepsilon(r^2 + r^2\tan^2\theta)^{3/2}} \, r \times r \sec^2\theta \, d\theta \, \vec{a}_y$$

$$\vec{E} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\rho_L}{4\pi\varepsilon} \frac{r^3\sec^2\theta}{r^3\sec^3\theta} d\theta\vec{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{r\sec\theta} \, d\theta\vec{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon r} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta d\theta\vec{a}_y = \frac{\rho_L}{4\pi\varepsilon r} [\sin\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \vec{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon r} \left[\sin(\frac{\pi}{2}) - \sin(-\frac{\pi}{2}) \right] \vec{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon r} [1 - (-1)]\vec{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon r} 2 \, \vec{a}_y$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon r} 2 \, \vec{a}_y$$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon r} \, \vec{a}_y \, \text{v/m}}$$

➢ The above equation is the total electric field of infinite line charge.

18. **A circular disc of radius 'a'm charged uniformly with a charge density of ρ c/m2. Find the Electric field at a point 'h' m from the disc along its axis. [CO1-H2-Apr/May 2012].**
   Solution: electric field intensity due to circular disc:

> ➢ Consider a uniformly distributed charge over a circular disc having surface density ρs placed in X-Y plane.
> ➢ The charge dQ is given as $dQ = \rho_s ds$.

$$\text{Where ds} = r \, dr \, d\varphi$$

$$dQ = \rho_s \, r \, dr \, d\varphi$$

> ➢ the distance vector is, $\vec{R} = Q \, ds$ $\vec{R} = -r\vec{d_r} + h\vec{a_z}$
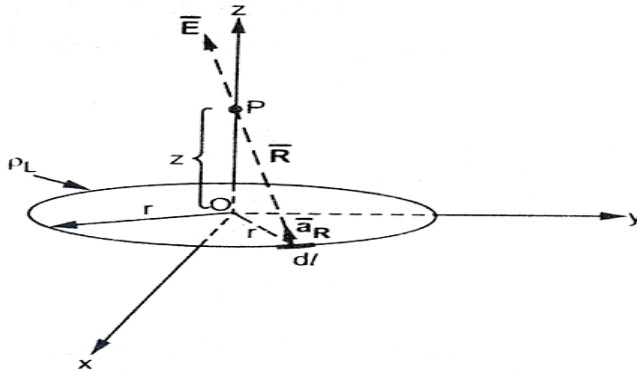
> ➢ The magnitude is $|\vec{R}| = \sqrt{r^2 + h^2}$
> ➢ The unit vector $\vec{a_R} = \dfrac{\vec{R}}{|R|}$

$$= \frac{-r\vec{d_r} + h\vec{a_z}}{\sqrt{r^2 + h^2}}$$

> ➢ The electric field intensity of the differential elements is,

$$\overrightarrow{dE} = \frac{dQ}{4\pi\varepsilon R^2}\vec{a_R}$$

$$\overrightarrow{dE} = \frac{\rho_s \, rdrd\varphi}{4\pi\varepsilon(\sqrt{r^2 + h^2})^2} \frac{-r\vec{d_r} + h\vec{a_z}}{\sqrt{r^2 + h^2}}$$

> ➢ Eliminating the r- component ,because the disc is placed in Z-axis, the above equation becomes,

$$\overrightarrow{dE} = \frac{\rho_s \, rdrd\varphi}{4\pi\varepsilon(\sqrt{r^2 + h^2})^2} \frac{h\vec{a_z}}{\sqrt{r^2 + h^2}}$$

$$\overrightarrow{dE} = \frac{\rho_s \, rdrd\varphi \; h\vec{a_z}}{4\pi\varepsilon(\sqrt{r^2 + h^2})^{3/2}}$$

> ➢ BY integration , the total electric field is calculated as,

$$\vec{E} = \int \overrightarrow{dE}$$

Take, $r^2 + h^2 = u^2$

2rdr = 2udu

when r = 0 , u1= h

r=a, u2= $\sqrt{a^2 + h^2}$

$$\vec{E} = \int_{\varphi=0}^{2\pi} \int_{u1}^{u2} \frac{\rho_s \, udu \, d\varphi \; h\vec{a_z}}{4\pi\varepsilon(u^2)^{3/2}}$$

$$= \int\limits_{\varphi=0}^{2\pi} \int\limits_{u1}^{u2} \frac{\rho_s}{4\pi\varepsilon} \frac{du}{u^2} \, d\varphi \, h\vec{a}_z$$

$$= \frac{\rho_s}{4\pi\varepsilon} \, [\varphi]_0^{2\pi} [-\frac{1}{u}]_{u1}^{u2}\vec{a}_z$$

$$= \frac{\rho_s}{4\pi\varepsilon} [2\pi] \, [-\frac{1}{u2} + \frac{1}{u1}] \, \vec{a}_z$$

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon} \left[\frac{-1}{\sqrt{a^2 + h^2}} + \frac{1}{h}\right] \vec{a}_z \text{ v/m}$$

The above equation is electric flux density of circular disc of radius R.

19. **Derive and prove the gauss law in differential and integral form.**
    (Or) state and prove gauss law [CO1-H1-April/May 2010 & Nov/Dec 2006]
    Gauss law:
    - Gauss law states that the electric flux passing through any closed surface is equal to the total charge enclosed by the surface.
    Mathematical representation of gauss law:
    - Consider an object of irregular shape as shown in the figure.
    - The total charge of the closed surface is Q.
    - Consider a differential surface ds at a point P
    - $\vec{a}_n$ vector is a normal vector to ds.
    - The differential surface is given by,
    $$\vec{ds} = ds \, \vec{a}_n$$
    - The flux density $\vec{D}$ makes an angle θ with the normal vector.
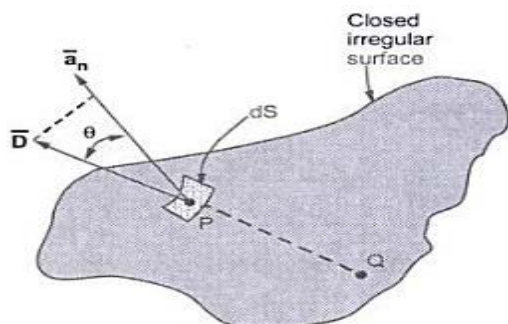
    - The flux dψ passing through ds is given by,
          dψ = Dn ds
          Dn = $|D|\cos\theta$
                dψ = $|D|\cos\theta$ ds
                    = $|D| \, ds \cos\theta$
       dψ = $\vec{D} \, \vec{ds}$
       ψ = ∫ dψ
       = $\oint \vec{D} \, \vec{ds}$
       ψ =Q

- Irrespective of the shape of the surface and charge distribution, the total flux is equal to the total charge enclosed by the surface.

20. Describe any two applications of gauss law [CO1-L2-Nov/Dec 2013]
   i) Point charge:

- Let a point charge Q is located at origin
- The D is a directed radially outward along with $\vec{a}_r$.
- Consider the differential surface ds.
- In spherical co-ordinate system,
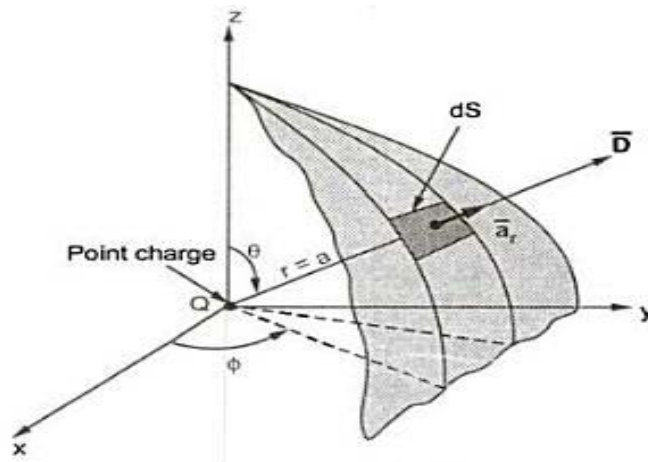
$$dS = r^2 \sin\theta \, d\theta \, d\phi = a^2 \sin\theta \, d\theta \, d\phi \quad (as \ r = a)$$

w.k.t, $\quad d\overline{S} = dS \, \overline{a}_n = a^2 \sin\theta \, d\theta \, d\phi \, \overline{a}_r$

- $\vec{D}$ due to point charge is,

$$\overline{D} = \frac{Q}{4\pi r^2}\overline{a}_r = \frac{Q}{4\pi a^2}\overline{a}_r \quad (as \ r = a)$$

$$\overline{D} \cdot d\overline{S} = |\overline{D}||d\overline{S}| \cos\theta'$$

$$|\overline{D}| = \frac{Q}{4\pi a^2}, \quad |d\overline{S}| = a^2 \sin\theta \, d\theta \, d\phi, \ \theta' = 0°$$

$$\overline{D} \cdot d\overline{S} = \frac{Q}{4\pi a^2}a^2 \sin\theta \, d\theta \, d\phi \cos 0°$$

$$= \frac{Q}{4\pi} \sin\theta \, d\theta d\phi$$

- The total flux is given by,

$$\psi = \oint_S \overline{D} \cdot d\overline{S} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi$$

$$= \frac{Q}{4\pi}[-\cos\theta]_0^\pi \, [\phi]_0^{2\pi} = \frac{Q}{4\pi}[-(-1)-(-1)] \, [2\pi]$$

$$\psi = Q \qquad\qquad\qquad\qquad \text{... Gauss's law is proved}$$

- Thus the flux is equal to the charge is proved for point charge.

ii) infinite line charge:

- Consider an infinite line charge with density ρ_L which lies on z-axis.
- The surface is cylindrical with radius r .
- The $\vec{D}$ is radially   outward along with $\vec{a}_r$.

The flux , $$\psi = \oint_S \overline{D} \cdot d\overline{S} : \qquad Q = \oint_S \overline{D} \cdot d\overline{S} \qquad [\psi = Q]$$
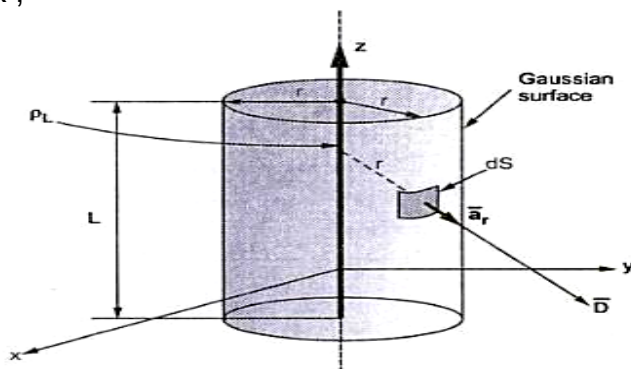
- The charge of the cylindrical is separated into three surfaces.

$$Q = \oint_{side} \overline{D} \cdot d\overline{S} + \oint_{top} \overline{D} \cdot d\overline{S} + \oint_{bottom} \overline{D} \cdot d\overline{S}$$

$$\overline{D} = D_r \, \overline{a}_r \quad \text{as has only radial component}$$

$$d\overline{S} = r \, d\phi \, dz \, \overline{a}_r \quad \text{normal to } \overline{a}_r \text{ direction.}$$

W.k.t $$\overline{D} \cdot d\overline{S} = D_r \, r \, d\phi \, dz \, (\overline{a}_r \cdot \overline{a}_r) = D_r \, r \, d\phi \, dz$$

- The top and bottom of the cylindrical is equal has no component in z- axis

$$\oint_{top} \overline{D} \cdot d\overline{S} = \oint_{bottom} \overline{D} \cdot d\overline{S} = 0$$

$$Q = \oint_{side} \overline{D} \cdot d\overline{S} = \oint_{side} D_r \, r \, d\phi \, dz$$

$$= \int_{z=0}^{L} \int_{\phi=0}^{2\pi} D_r r \, d\phi \, dz = r \, D_r [z]_0^L \, [\phi]_0^{2\pi}$$

$$Q = 2\pi \, r \, D_r \, L$$

$$D_r = \frac{Q}{2\pi r L}$$

W.k.t,

$$\overline{D} = D_r \bar{a}_r = \frac{Q}{2\pi rL} \bar{a}_r$$

$$\frac{Q}{L} = \rho_L \ C/m$$

$$\boxed{\overline{D} = \frac{\rho_L}{2\pi r} \bar{a}_r \ C/m^2}$$ ... Due to infinite line charge.

- Therefore,

$$\overline{E} = \frac{\overline{D}}{\varepsilon_0} = \frac{\rho_L}{2\pi \varepsilon_0 \ r} \bar{a}_r \ V/m$$

21. Derive the expression for potential due to an electric pole at any point 'P'. also, find electric field intensity at the same point. [CO1-H2-May/June 2012].

➤ Let point P is at distance r1 from +Q and r2 from −Q
➤ The centre point is 0 which is at a distance r from mid point to point P.
➤ The potential at point P, due to +Q is,

$$V1 = \frac{Q}{4\pi\epsilon r1}$$

➤ The potential at point P, due to -Q is,

$$V2 = -\frac{Q}{4\pi\epsilon r2}$$

Total potential is,
$$V = V_1 + V_2$$

$$= \frac{Q}{4\pi\epsilon r1} - \frac{Q}{4\pi\epsilon r2}$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{1}{r1} - \frac{1}{r2}\right]$$

From the diagram,

$$\cos\theta \quad = \frac{adj}{hyp} \quad = \frac{x}{l/2}$$

$$x = \frac{l}{2}\cos\theta$$

Here, $r_1 < r_2$

Assume ,
$$r_1 = r\text{-}x \rightarrow r - \frac{l}{2}\cos\theta$$

$$r_1 = r\text{+}x \rightarrow r + \frac{l}{2}\cos\theta$$

sub , r_1 & r_2 in V

$$= \frac{Q}{4\pi\epsilon}\left[\frac{1}{r - \frac{l}{2}\cos\theta} - \frac{1}{r + \frac{l}{2}\cos\theta}\right]$$

$$=$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{r + \frac{l}{2}\cos\theta - r + \frac{l}{2}\cos\theta}{(r - \frac{l}{2}\cos\theta)(r \mp \frac{l}{2}\cos\theta)}\right]$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{2\frac{l}{2}\cos\theta}{(r^2 - \frac{l^2}{4}\cos^2\theta)}\right]$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{l\ \cos\theta}{(r^2 - \frac{l^2}{4}\cos^2\theta)}\right]$$

Since r>> l , l is neglected,

$$V = \frac{Q}{4\pi\epsilon}\left[\frac{l\ \cos\theta}{r^2}\right]$$

$$V = \frac{Q\ l\ \cos\theta}{4\pi\epsilon r^2}$$

We take $Q\ l = $ m

➢ The   potential   at   P   due   to   the   dipole   is,

$$V = \frac{m\ \cos\theta}{4\pi\epsilon r^2}$$

➢ The electric field intensity is given as,

$$E = \frac{V}{d}$$

$$E = \frac{m\ \cos\theta}{4\pi\epsilon r^2 d}$$

22. Prove that divergence of a curl of a vector is zero, using stoke's theorem. [CO1-L2-May/June 2012].

Solution:

➢ Consider a vector A

➢ Curl of A = $\nabla \times A$

➢ Divergence = $\nabla . (\nabla \times A)$

$$\nabla . (\nabla \times A) = 00 \qquad\qquad \text{-------(1)}$$

➢ Let us evaluate integral of both sides over a volume

$$\int \nabla . (\nabla \times A)dv = \int \infty\ dv \qquad \text{-------(2)}$$

➢ Apply divergence theorem,

$$\int \nabla . (\nabla \times A)dv = \int \infty\ ds \qquad \text{--------(3)}$$

Where, s = closed surface enclosing the given volume

Sub eqn (3) in (2) we get**,**

$$\int (\nabla \times A)dv = \int \infty \, dv$$

➢ As per stoke's theorem applied to the closed surface produces zero

$$\int \infty \, dv = 0 \quad , \text{ hence } \infty = 0$$

Substitute this in eqn (1)

$$\nabla \cdot (\nabla \times A) = 0$$

Hence it is proved.

23. Div D = ρ using gauss law. [CO1-L2-Apr/May 2012].
Solution: Prove

- Gauss law states that the electric flux passing through any closed surface is equal to the total charge enclosed by the surface.
- Mathematical representation of gauss law;

$$\psi = \oint \vec{D} \, \vec{ds}$$

the divergence of electric flux density is,

$$\text{div } D = \lim \frac{\oint \vec{D} \, \vec{ds}}{\Delta v}$$

$$= \frac{\partial Dx}{\partial x} + \frac{\partial Dy}{\partial y} + \frac{\partial Dz}{\partial z}$$

From gauss law,

$$\psi = Q = \oint \vec{D} \, \vec{ds}$$

$$\frac{Q}{\Delta v} = \frac{\oint \vec{D} \, \vec{ds}}{\Delta v}$$

$$\lim \frac{Q}{\Delta v} = \lim \frac{\oint \vec{D} \, \vec{ds}}{\Delta v}$$

$$\text{since } \lim \frac{Q}{\Delta v} = \rho_v$$

$$\text{div } D = \rho$$

24. If V = $\left[2x^2y + 20z - \frac{4}{x^2+y^2}\right]$ volts. Find $\vec{E}$ and $\vec{D}$ at point P (6,-2.5,3). [CO1-L2]
Solution:

$$\Delta V = -\vec{E}$$

$$\vec{E} = -\Delta V$$

$$= -\left[\frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}\right] V$$

$$= -\left[\frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}\right]\left[2x^2y + 20z - \frac{4}{x^2+y^2}\right]$$

$$= -\frac{\partial}{\partial x}(2x^2y + 20z - \frac{4}{x^2+y^2})\vec{a_x} - \frac{\partial}{\partial y}(2x^2y + 20z - \frac{4}{x^2+y^2})\vec{a_y} - \frac{\partial}{\partial z}(2x^2y20z - \frac{4}{x^2+y^2})\vec{a_z}$$

$$= -[4xy - 4(-1)(x^2 + y^2)^{-2}(2x)]\vec{a_x} - [2x^2 - 4(-1)(x^2 + y^2)^{-2}(2y)]\vec{a_y} - [20]\vec{a_z}$$

$$= -[4xy + \frac{4(2x)}{(x^2+y^2)^2}]\vec{a_x} - [2x^2 + \frac{4(2y)}{(x^2+y^2)^2}]\vec{a_y} - [20]\vec{a_z}$$

At point (6, -2.5, 3)

$$= -\left[4(6)(2.5) + \frac{4(2\times 6)}{(6^2 + -2.5^2)^2}\right]\overrightarrow{a_x} - \left[2(6)^2 + \frac{4(2\times 6)}{(6^2 + -2.5^2)^2}\right]\overrightarrow{a_y} - [20]\overrightarrow{a_z}$$

$$= -\left\{\left[-60 + \frac{48}{(36+6.25)^2}\right]\overrightarrow{a_x} + \left[72 + \frac{20}{(78.25)^2}\right]\overrightarrow{a_y} + 20\overrightarrow{a_z}\right\}$$

$$= -\left\{\left[-60 + \frac{48}{6123}\right]\overrightarrow{a_x} + \left[72 + \frac{20}{6123}\right]\overrightarrow{a_y} + 20\overrightarrow{a_z}\right\}$$

$$= -\left\{[-60 + 7.839 \times 10^{-3}]\overrightarrow{a_x} + [72 + 3.266 \times 10^{-3}]\overrightarrow{a_y} + 20\overrightarrow{a_z}\right\}$$

$$\vec{E} = 59.992\,\overrightarrow{a_x} - 72.0036\,\overrightarrow{a_y} - 20\overrightarrow{a_z}$$

$$\vec{D} = \varepsilon_0 \vec{E}$$

$$\vec{D} = 5.31169 \times 10^{-10}\,\overrightarrow{a_x} - 6.378 \times 10^{-10}\,\overrightarrow{a_y} - 1.777 \times 10^{-10}\,\overrightarrow{a_z}$$

25. A regular tetrahedron has vertices at P1 (2,0,0), P2 (-1,-√3,0), P3(-1, -√3, 0) and P4(0,0,2√2) charges of 1mC are located at each other of the 4 vertices. If the configuration is located in free space, find the magnitude of force on each charge. [CO1-H2]

Force Q1 Q4

$$\vec{F} = \frac{Q_1 Q_4}{4\pi\varepsilon_0 |\vec{R}_{14}|^2}\,|\vec{a}_{14}|$$

$$\vec{R}_4 = \vec{r}_4 - \vec{r}_1$$
$$= [0\,\overrightarrow{a_x} + 0\overrightarrow{a_y} + 2\sqrt{2}\,\overrightarrow{a_z}] - [2\overrightarrow{a_x}]$$
$$= -2\overrightarrow{a_x} + 2\sqrt{2}\,\overrightarrow{a_z}$$

$$|\vec{R}_{14}| = \sqrt{4+8} = \sqrt{12}$$

$$\vec{a}_R = \frac{\vec{R}_{14}}{|\vec{R}_{14}|}$$
$$= \frac{-2\overrightarrow{a_x} + 2\sqrt{2}\,\overrightarrow{a_z}}{\sqrt{12}}$$

$$\vec{F} = \frac{Q_1 Q_4}{4\pi\varepsilon_0(\sqrt{12})^2} \cdot \left(\frac{-2\overrightarrow{a_x} + 2\sqrt{2}\,\overrightarrow{a_z}}{\sqrt{12}}\right)$$
$$= \frac{(1\times 10^{-3})(1\times 10^{-3})}{4\pi\times 8.854\times 10^{-12}\times 12} \cdot \frac{-2\overrightarrow{a_x} + 2\sqrt{2}\,\overrightarrow{a_z}}{\sqrt{12}}$$
$$= \frac{1\times 10^{-6}}{4622.756\times 10^{-12}} \cdot (-2\overrightarrow{a_x} + 2\sqrt{2}\,\overrightarrow{a_z})$$

$$=$$

$$= 2.163 \times 10^{-4} \times 10^6 \cdot (-2\overrightarrow{a_x} + 2\sqrt{2}\,\overrightarrow{a_z})$$

$$=$$

$$= 216.3 \times (-2\overrightarrow{a_x} + 2\sqrt{2}\,\overrightarrow{a_z})$$

$$=$$

$$= -432.6\,\overrightarrow{a_x} + 611.446\overrightarrow{a_z}$$

Force Q2 Q4

$$\vec{F} = \frac{Q_2 Q_4}{4\pi\varepsilon_0 |\vec{R}_{14}|^2} |\vec{a}_{24}|$$

$$\vec{R}_{24} = \vec{r}_4 - \vec{r}_2$$
$$= 2\sqrt{2}\vec{a_z} - (-\vec{a_x} + \sqrt{3}\vec{a_y})$$
$$= \vec{a_x} - \sqrt{3}\vec{a_y} + 2\sqrt{2}\,\vec{a_z}$$
$$|\vec{R}_{24}| = \sqrt{1 + 3 + 8} = \sqrt{12}$$

$$\vec{a}_R = \frac{\vec{R}_{24}}{|\vec{R}_{24}|}$$

$$\vec{F} = \frac{(1\times10^{-3})(1\times10^{-3})}{4\pi\times8.854\times10^{-12}\times12} \cdot \frac{\vec{a_x} - \sqrt{3}\vec{a_y} + 2\sqrt{2}\,\vec{a_z}}{\sqrt{12}}$$
$$= 216.3\left(\vec{a_x} - \sqrt{3}\vec{a_y} + 2\sqrt{2}\vec{a_z}\right)$$
$$= 216.3\vec{a_x} - 374.489\vec{a_y} - 611.446\vec{a_z}$$

Force Q3 Q4

$$\vec{F} = \frac{Q_3 Q_4}{4\pi\varepsilon_0 |\vec{R}_{34}|^2} |\vec{a}_{34}|$$

$$\vec{R}_{34} = \vec{r}_4 - \vec{r}_3$$
$$= 2\sqrt{2}\,\vec{a_z} - (\vec{a_x} - \sqrt{3}\vec{a_y})$$
$$|\vec{R}_{34}| = \sqrt{1 + 3 + 8} = \sqrt{12}$$

$$\vec{a}_R = \frac{\vec{R}_{34}}{|\vec{R}_{34}|}$$

$$= \frac{\vec{a_x} + \sqrt{3}\vec{a_y} + 2\sqrt{2}\vec{a_z}}{\sqrt{12}}$$

$$\vec{F} = \frac{(1\times10^{-3})(1\times10^{-3})}{4\pi\times8.854\times10^{-12}\times12} \cdot \frac{\vec{a_x} + \sqrt{3}\vec{a_y} + 2\sqrt{2}\,\vec{a_z}}{\sqrt{12}}$$
$$= 216.3\left(\vec{a_x} + \sqrt{3}\vec{a_y} + 2\sqrt{2}\vec{a_z}\right)$$
$$= 216.3\vec{a_x} + 374.489\vec{a_y} + 611.446\vec{a_z}$$

According to Superposition principle,

Total force $= F_1 + F_2 + F_3$
$$= 1835.528\,\vec{a_x}$$

26. The electric flux density is given as $D = \frac{r}{4}$ nC/m2 in free space

Calculate,

    i)   **The electric field intensity at r = 0.25m**
    ii)   **The total charge within a sphere of r = 0.25m**
    iii)   **The total flux leaving the sphere of r = 0.35m [CO1-L3]**

Solution:

Electric field intensity:

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{\frac{r}{4}\,nC}{8.854 \times 10^{-12}}$$

$$= \frac{0.25}{4}\,nC \times \frac{1}{8.854 \times 10^{-12}}$$

$$= \frac{0.0625 \text{ nC}}{8.854 \times 10^{-12}} = \frac{7.058 \times 10^{-3} \times 10^{-9}}{10^{-12}}$$

$$\vec{E} = 7.058 \text{ V/m}$$

$$\vec{E} = 7.058 \, \vec{a_r} \text{V/m}$$

$D = \frac{r}{4} \text{ nC/cm2}$

$$\vec{ds} = r^2 \sin\theta \, d\theta \, d\varphi$$

$$\text{W. K. T,} \quad Q = \int \vec{D} . \vec{ds}$$

$$= \iint \left(\frac{r}{4}\right) \vec{a_r} . (r^2 \sin\theta \, d\theta \, d\varphi) \, \vec{a_r}$$

$$= \int_0^{2\pi} \int_0^{\pi} \frac{r^3}{4} \sin\theta \, d\theta \, d\varphi \quad [\text{Since } \vec{a_r} . \vec{a_r} = 1]$$

$$= \frac{r^3}{4} \int_0^{2\pi} [-\cos\theta]_{\pi}^{0} \, d\varphi$$

$$= \frac{r^3}{4} \int_0^{2\pi} [-\cos\pi + \cos 0] \, d\varphi$$

$$= \frac{r^3}{4} \int_0^{2\pi} [-(-1) + 1] \, d\varphi$$

$$= \frac{r^3}{4} . 2 \int_0^{2\pi} d\varphi$$

$$= \frac{r^3}{4} [\varphi]_0^{2\pi}$$

$$Q = \pi r^3$$

$$Q/_{r = 0.25m} = \pi(0.25)^3$$

$$= 0.015625 \text{ m}^3 \times 3.14$$

$$Q = 0.042875 \text{ C}$$

$$\psi = Q = \pi r^3$$

$$= \pi(0.35)^3$$

$$= \pi \times 0.042875$$

$$\psi = 0.1346 \text{ C}$$

27. Given $A = (Y \cos a_x)\vec{a_x} + (Y + e^x)\vec{a_z}$. Find $\nabla \times A$ at the origin. [CO1-L2]

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Y \cos a_x & 0 & Y + e^x \end{vmatrix}$$

$$= a_x \left[\frac{\partial}{\partial y} (Y + e^x)\right] + a_y \left[\frac{\partial}{\partial z} (Y \cos a_x) - \frac{\partial}{\partial x} (Y + e^x)\right] + a_z \left[-\frac{\partial}{\partial y} (Y \cos a_x)\right]$$

$$= a_x[0] + a_y \left[\frac{\partial}{\partial z} (0) - (Y - e^{\infty})\right] + a_z[0]$$

$$\nabla \times A = -a_y(Y + e^x)$$

28. If two vectors are expressed in cylindrical co-ordinates as $= 2a_x + \pi a_y + a_z$;

$B = -a_x + \frac{3\pi}{2}a_y - 2a_z$. **Compute a unit vector perpendicular to the plane containing A & B. [CO1-H2]**

Solution:

By taking spherical co-ordinate system of radius R, we can obtain the area. Assuming an elemental area ds and the normal is actually radially outwards.

So , $ds = r^2 \sin\theta \, d\varphi \, d\theta$

$$= R^2 \sin\theta \, d\theta \, d\varphi$$

As φ varies from 0 to ∞.

Area $= R^2 \int_0^\infty \int_0^\pi \sin\theta \, d\theta \, d\varphi$

$$= R^2 \int_0^\infty [-\cos\theta]_0^\pi \, d\varphi$$

$= -R^2 \int_0^\infty [\cos\pi - \cos 0] \, d\varphi$

$$= -R^2 \int_0^\infty [-1-1] \, d\varphi$$

$$= -R^2 \int_0^\infty 2 \, d\varphi$$

$$= R^2[2\varphi]_0^\infty$$

$$= R^2[2(\infty) - 0]$$

$$= 2\infty R^2$$

29. An infinitely long uniform line charge is located at y =3, z = 5, if ρ1 = 30 nC/m. Find field intensity E at (i) Origin   (ii) P(0, 6, 1) and P(5, 6, 1). [CO1-L2]

Given:

ρ_n = 30 nC/m,

Line charge at y = 3, z = 5.

Solution:

$$\vec{E} = \frac{\rho_n}{2\pi\varepsilon_0 \, r} \, \vec{a_y} \, V/m$$

$\vec{E}$ at origin

Distance,　　$r = \sqrt{(3-0)^2 + (5-0)^2}$

$$= \sqrt{9 + 25}$$

$$= \sqrt{34} m$$

$$\vec{E} = \frac{30 \, n}{2\pi \times 8.854 \times 10^{-12} \times \sqrt{34}} = 92.48 \, V/m$$

$\vec{E}$ at P(0, 6, 1)

Distance,　　$r = \sqrt{(3-6)^2 + (5-1)^2}$

$$= \sqrt{9 + 16} = \sqrt{25} = 5m$$

$$\vec{E} = \frac{30 \, n}{2\pi \times 8.854 \times 10^{-12} \times 5} = 107.85 \, V/m$$

E at P(5, 6, 1)

Distance,　　$r = \sqrt{(3-6)^2 + (5-1)^2}$

$$= \sqrt{9 + 16} = \sqrt{25} = 5m$$

$$\vec{E} = \frac{30\,n}{2\pi \times 8.854 \times 10^{-12} \times 5} = 107.85 \text{ V/m}$$

30. Given the two points A (x = 2, y = 3, z = -1) & B $= (r = 4, \theta = 25^{\circ}, \varphi = 120°)$. Find the spherical co-ordinate of A and Cartesian co-ordinate of B. [CO1-L2]

Solution:

A (x = 2, y = 3, z = -1)

$r = \sqrt{x^2 + y^2 + z^2}$

$$r = \sqrt{4 + 9 + 1} = \sqrt{14} = 3.741$$

$$\theta = \cos^{-1}\left(\frac{z}{r}\right)$$

$$= \cos^{-1}\left(\frac{-1}{3.741}\right) = 105.5$$

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

$= \tan^{-1}\left(\frac{3}{2}\right)$

$= 56.36$

$$B = (r = 4, \theta = 25^{\circ}, \varphi = 120°)$$

$$x = r \sin\theta \cos\varphi$$

$$= 4 \sin 25^{\circ} \cos 120°$$

$$= -0.845$$

$$y = r \sin\theta \sin\varphi$$

$$= 4 \sin 25^{\circ} \sin 120°$$

$$= 1.463$$

$$z = r \cos\theta$$

$$= 4 \cos 25^{\circ}$$

$$= 3.625$$

31. Find curl $\vec{H}$, if $\vec{H} = (2\rho \cos\varphi\vec{a_\rho} - 4\rho \sin\varphi\vec{a_\varphi} + 3\vec{a_z})$. [CO1-L1]

Solution:

$$\vec{H} = (2\rho \cos\varphi\vec{a_\rho} - 4\rho \sin\varphi\vec{a_\varphi} + 3\vec{a_z})$$

.

$$\nabla \times \vec{H} = \frac{1}{\rho}\begin{vmatrix} \vec{a_\rho} & \rho\vec{a_\varphi} & \vec{a_z} \\ \partial/\partial\rho & \partial/\partial\varphi & \partial/\partial z \\ 2\rho \cos\varphi & -4\rho \sin\varphi & 3 \end{vmatrix}$$

$$= \frac{1}{\rho}\left\{\vec{a_\rho}\left[\frac{\partial}{\partial\varphi}(3) + \frac{\partial}{\partial z}(4\rho \sin\varphi)\right] - \rho\vec{a_\varphi}\left[\frac{\partial}{\partial\rho}(3) - \frac{\partial}{\partial z}(2\rho \cos\varphi)\right]\right.$$
$$\left. + \vec{a_z}\left[\frac{\partial}{\partial\rho}(-4\rho \sin\varphi) - \frac{\partial}{\partial\varphi}(2\rho \cos\varphi)\right]\right\}$$

$$= \frac{1}{\rho} \{ \vec{a_\rho}[0 + 0] - \rho\vec{a_\varphi}[0 - 0] + \vec{a_z}[-4 \sin\varphi - 2\rho(-\sin\varphi)] \}$$

$$= \frac{1}{\rho} \{ \vec{a_z}[-4 \sin\varphi - 2\rho(-\sin\varphi)] \}$$

$$= \frac{1}{\rho} [-4 \sin\varphi \ \vec{a_z} + 2\rho \sin\varphi \ \vec{a_z}]$$

$$= \frac{-4 \sin\varphi \ \vec{a_z}}{\rho} + 2 \sin\varphi \ \vec{a_z}$$

$$\nabla \times \vec{H} = \left[\frac{-4 \sin\varphi}{\rho} + 2 \sin\varphi \right] \vec{a_z}$$

Unit – II

Conductors and dielectrics

Part – A

1. What is relative dielectric constant? [CO2-L1-Apr/May2010]

The quantity X_e+1 is defined as relative permittivity or dielectric constant of the dielectric material

$E_r=X_e+1$

X_e-electric susceptibility of the material

2. Derive Poisson equation from gauss law? [CO2-H1]

Gauss law in point from is given by

$$\nabla \cdot \overline{D} = \rho_v$$

this is Poisson equation

3. What is meant by dielectric breakdown? [CO2-L1-May/June2006]

As the electric field applied to dielectric increases sufficiently, due to the force exerted on the molecules, the electrons in the dielectric become free. Under such large electric field the dielectric become conducting due to presence of large number of free electron, this condition is called dielectric breakdown

4.Define dielectric strength of material given its unit? [CO2-L1-Arp/May2010]

The minimum value of the applied electric field at which the dielectric breakdown is called dielectric strength of the material its unit is V/M

5. Write the continuity equation? [CO2-L1-Nov/Dec2008]

The continuity equation of the current is based on the principal of conversation of charge

consider a closed surface (s) with a current density J, then the total current(I) crossing the surface S is given by

$$\nabla.\vec{J} = -\frac{\partial \rho}{\partial t}$$

This is the integral form of continuity equation of the current
Rate of decrease of charge inside the closed surface

6.Give energy &energy density in a capacitor? [CO2-L1-April/May 2015]
Energy stored:
Energy stored in the capacitor is known as energy storage
Energy density:
It is define as the energy stored per unit volume as volume tends to zero

7.What is displacement current ?

In dielectrics there can be flow of charger under the influence of the electric field intensity such a current is called the displacement current or convection current the flowing across the capacitor through the dielectric

separating its plates is an example of convection current

8.Give the expression for the capacitance of parallel plate capacitor with two dielectric medium? [CO2-L1-Apr/May2008 & April/May2015]

The capacitance of parallel plate capacitor with two dielectric medium is given by

$$C = \frac{A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} + \frac{d_3}{\varepsilon_3} + \dots + \frac{d_n}{\varepsilon_n}} \qquad \dots (7)$$

Where A-> area of cross section

9.What is a homogenous material? [CO2-L1]

The material which has constant E is known as homogenous material.

10.Brief about boundary conditions for electric fields? [CO2-L1]

The conditions existing at the boundary of the media, when the fields panes form one medium to other are called as boundary conditions.
Two situation of boundary conditions is
    (i)Boundary between conductor & free space
    (ii) Boundary between two dielectrics with different properties

11.Define capacitance. [CO2-L1-Apr/May 2011]

A system which has two conducting surfaces carrying equal and opposite charges separated by a dielectric is called capacitive system which gives rise to capacitance.

12.What is polarization? [CO2-L1]

If the electric field is applied to a insulator, the ions shift the positions against the atomic forces.The shift in positive and negative charges is in opposite direction and these charges are like small electric dipoles.The process due to which separation of bound charges results to produce electric dipoles under the influence of electric field is called as polar

13.What is point form of ohms law? [CO2-L1]

The relationship between vector J and E can also be expressed in terms of conductivity of the material.Thus for a metallic conductor

$$\vec{J} = \sigma\vec{E}$$

$$\sigma - \text{Conductivity of the material}$$

The conductivity is measured in mhos per meter (ü/m).

14.Give some properties of conductors. [CO2-L1]

1.Under static conditions no charge and no electric field can exist at any point within the conducting material.
2.The charge can exist on the surface of the conductor giving rise to surface charge density.
Within a conductor the charge density is always zero.
3.The charge distribution on the surface depends on the shape of the surface.
4.The conductivity of an ideal conductor is infinite.
5.The conductor surface is an equipotential surface.

15.What is polarization? [CO2-L1]

1.If the electric field is applied to a insulator, the ions shift the positions against the atomic forces.

2.The shift in positive and negative charges is in opposite direction and these charges are like small electric dipoles.

3.The process due to which separation of bound charges results to produce electric dipoles under the influence of electric field is called as polarization.

16.What are the properties of dielectric materials? [CO2-L1]

T1.he dielectrics do not contain any free charges but contain bound charges.

2.Bound charges are under the internal molecular and atomic forces and cannot contribute to the conduction.

3.When subjected to an electric field vector E , the bound charges shift their relative positions. Due to this, small electric dipoles get induced inside the dielectric. This is called polarization.

4.Due to polarization the dielectrics can store energy.

5.Due to polarization, the flux density of the dielectric increases by amount equal to the polarization.

6.The induced dipoles produce their own electric field and align in the direction of the applied electric field.

7.When polarization occurs, the volume charge density is formed inside the dielectric while the surface charge density is formed over the surface of the dielectric.

8.The electric field inside and outside the dielectric gets modified due to the induced electric dipoles.

17. What is steady current? [CO2-L1-May/June 2014]

For steady current which are not the function of time

$$\frac{\partial \rho_v}{\partial t} = 0$$

and hence

$$\nabla \cdot \vec{J} = 0$$

Steady current
The rate of flow of charge remains constant with time.

18.What is method of images? [CO2-L1]

The method of images, introduced by Lord Kelvin in 1848, is commonly used to determine $V, \bar{E}, \bar{D}$, and ρ_s due to charges in the presence of conductors.

By this method, we avoid solving Poisson's or Laplace's equation but rather utilize the fact that a conducting surface is an equipotential.

19. Define capacitance. [CO2-L1]

A system which has two conducting surfaces carrying equal and opposite charges separated by a dielectric is called capacitive system which gives rise to capacitance.

The potential difference between M1 and M2 is V_{12} and the capacitance of two material is given by

$$\therefore \qquad C = \frac{Q}{V_{12}} \qquad \qquad \text{... (1)}$$

In general, $\qquad C = \frac{Q}{V} \qquad \qquad$ ... (2)

Where Q=charge in coulombs
V=potential difference in volts

20.What is energy density? [CO2-L1]

The energy density is defined as the energy stored per unit volume as volume tends to zero.
The energy storage is given as,

$$\therefore \qquad W_E = \frac{1}{2} \epsilon \int_{Vol} |\overline{E}|^2 \, dv$$

$$\therefore \qquad W_E = \frac{1}{2} \epsilon |\overline{E}|^2 \ J/m^3 = \text{Energy density}$$

Using $\qquad |\overline{D}| = \epsilon |\overline{E}|$ we can write,

$$\boxed{W_E = \frac{1}{2} \frac{|\overline{D}|^2}{\epsilon} = \frac{1}{2} |\overline{D}||\overline{E}| \ J/m^3}$$

Part – B

1.What is conductor? Give its characteristics. [CO2-L1]
CONDUCTORS:

- Under the effect of electric field the available free electrons start moving.
- The moving electrons strike the adjacent atoms and rebound in the random directions. This is called drifting of the electrons.
- After some time the electrons attain the constant average velocity called drift velocity V_d.
- The current constituted due to the drifting of such electrons in metallic conductors is called drift current.
- The drift velocity is directly proportional to the applied electric field.

$$\overline{v}_d \ \propto \ \overline{E} \qquad \qquad \text{... (1)}$$

- The constant of proportionality is called mobility of the electrons in a given material and denoted as μ_e.It is positive for the electrons.

$$\boxed{\overline{v}_d = -\mu_e \overline{E}} \qquad \qquad \text{... (2)}$$

- The velocity of the electrons is against the direction of electric field

$$\mu \text{ (mobility)} = \frac{velocity}{field} = \frac{m/s}{V/m} = \frac{m^2}{V\text{-}s}$$

- Thus the mobility is measured in square meters per volt-second (m^2/V-s).

- The typical values of mobility are 0.0012 for aluminium,0.0032 for copper etc.
- Acccording to relation between vector J and V, we can write,

$$\bar{J} = \rho_v \, \bar{v} \qquad\qquad\qquad\qquad \text{... (3)}$$

- But in the material, the no of protons and electrons are same and it is always electrically neutral.
- Hence $\rho_v = 0$ for the neutral materials.
- The drift velocity is the velocity of free electrons ,hence the above relation can be expressed as

$$\boxed{\bar{J} = \rho_e \bar{v}_d} \qquad\qquad\qquad \text{... (4)}$$

ρ_e = Charge density due to free electrons

- The charge density ρ_c can be obtained as the product of number of free electrons/m_3 and the charge e on one electron.
- Thus $\rho_c = ne$ where n is the number of free electrons per m^3
- Sub 2 in 4 we get

$$\boxed{\bar{J} = -\rho_e \mu_e \bar{E}} \qquad\qquad\qquad \text{... (5)}$$

2. Define point form of ohms law. [CO2-L1]
POINT FORM OF OHMS LAW:
- The relationship between vector J and E can also be expressed in terms of conductivity of the material.
- Thus for a metallic conductor

$$\vec{J} = \sigma\vec{E}$$

- The conductivity is measured in mhos per meter (ü/m).
- The equation 6 is called point form of ohms law

3. What is resistance of the conductor? Illustrate with the neat diagram. [CO2-L1]
RESISTANCE OF A CONDUCTOR:
- Consider voltage V is applied to a conductor of length L having uniform cross section S as shown in the figure.
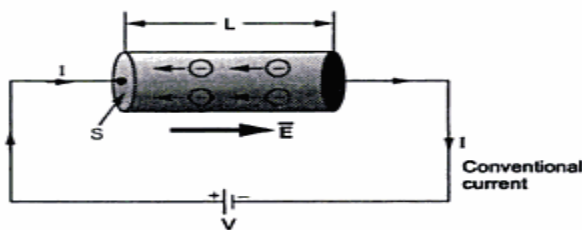
- The direction of vector E is same as the direction of conventional current, which is opposite to the flow of electrons.
- The electric field applied is uniform and its magnitude is given by,

$$E = \frac{V}{L} \qquad \text{... (8)}$$

- The conductor has uniform cross section S and hence we can write

$$I = \int_S \bar{J} \cdot d\bar{S} = JS \qquad \text{... (9)}$$

- The current direction is normal to the surface S

$$J = \frac{I}{S} = \sigma E \qquad \text{... (10)}$$

Sub 8 in 10

$$J = \frac{\sigma V}{L} \qquad \text{where } \sigma = \text{Conductivity of the material} \qquad \text{... (11)}$$

$$V = \frac{JL}{\sigma} = \frac{IL}{\sigma S} = \left(\frac{L}{\sigma S}\right)I \qquad \text{... (12)}$$

$$\boxed{R = \frac{V}{I} = \frac{L}{\sigma S}} \qquad \text{... (13)}$$

- Thus the ratio of potential difference between the two ends of the conductors to the current flowing through it is resistance of the conductor.
- The equation 12 is nothing but the ohms law in its normal form given by V=IR.
- The equation is true for the uniform fields and resistance is measured in ohms (Ω).
- For nonuniform fields the resistance R is defined as the ratio V to I,where V is the potential difference between the two specified equipotential surfaces and I is the current crossing the more positive surface of the two.
- Mathematically the resistance for nonuniform fields is given by,

$$\boxed{R = \frac{V_{ab}}{I} = \frac{-\int_b^a \bar{E} \cdot d\bar{L}}{\int_S \bar{J} \cdot d\bar{S}} = \frac{-\int_b^a \bar{E} \cdot d\bar{L}}{\int_S \sigma \bar{E} \cdot d\bar{S}}} \qquad \text{... (14)}$$

- The resistance can also be expressed as,

$$\boxed{R = \frac{L}{\sigma S} = \frac{\rho_c L}{S} \, \Omega} \qquad \text{... (15)}$$

$$\rho_c = \frac{1}{\sigma} = \text{resistivity of the conductor in } \Omega\text{-m}$$

4. Give the properties of conductors. [CO2-L1]

PROPERTIES OF CONDUCTORS:
- Under static conditions no charge and no electric field can exist at any point within the conducting material.
- The charge can exist on the surface of the conductor giving rise to surface charge density.
- Within a conductor the charge density is always zero.
- The charge distribution on the surface depends on the shape of the surface.
- The conductivity of an ideal conductor is infinite.

- The conductor surface is an equipotential surface.

5. Give the characteristics of dielectric material and how the polarization is performed in dielectric materials. [CO2-L1]

DIELECTRIC MATERIALS:
- Dielectric materials do not have free charges.
- The charges in dielectrics are bound by the finite forces and hence called as bound charges.
- They cannot contribute to the conduction process.
- But if subjected to the electric field, they shift their relative positions against then normal molecular and atomic forces.
- This shift in relative positions allows the dielectric to store the energy.
- In an electric field vector E the shift in positive and negative charges are in opposite direction which acts like small electric dipoles.
- These electric dipoles produce an electric field which opposes the externally applied electric field.

POLARISATION:
- If the electric field is applied to a insulator, the ions shift the positions against the atomic forces.
- The shift in positive and negative charges is in opposite direction and these charges are like small electric dipoles.
- The process due to which separation of bound charges results to produce electric dipoles under the influence of electric field is called as polarization.

CLASSIFICATION OF ATOMS:
      There are two types of atom
- Unpolarised atom
- Polarised atom

UNPOLARISED ATOM:
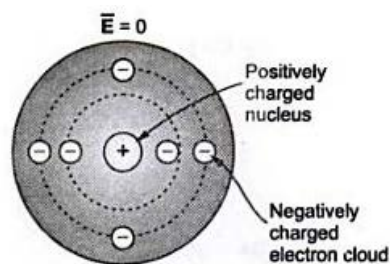
- Consider an atom of Dielectric as shown in the figure
- It consists of positive charge nucleus and negative charges revolving around it.
- The no. of + charge is equal to no of –ve charge and hence it is electrically neutral.
- The EF is zero. This is called as an unpolarized atom.

      POLARISED ATOM:
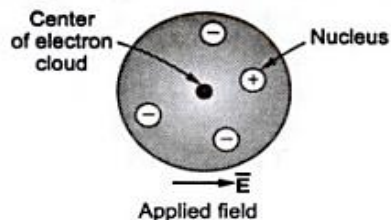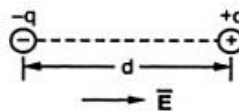
(a) Polarized atom   (b) Equivalent dipole

- When the EF is applied to an Unpolarised atom the charges will be distributed.
- The positive charge experiences a force $\bar{F} = Q\bar{E}$
- The negative charge experiences a force $\bar{F} = -Q\bar{E}$
- These two charges combine to form a electric dipole.This atom is called polarized atom.

MATHEMATICAL EXPRESSION FOR POLARISATION:

The electric dipole moment due to the polarization is given as

$$\boxed{\bar{p} = Q\bar{d}}$$ ... (1)

Q = Magnitude of one of the two charges
$\bar{d}$ = Distance vector from negative to positive charge
n = Number of dipoles per unit volume
Δv = Total volume of the dielectric
N = Total dipoles = $n\,\Delta v$

- The total dipole moment is to be obtained using superposition principle as

$$\bar{P}_{total} = Q_1\bar{d}_1 + Q_2\bar{d}_2 + ... + Q_n\bar{d}_n = \sum_{i=1}^{n\Delta v} Q_i\bar{d}_i$$

- If dipoles are randomly oriented P_{total} is zero but if dipoles are aligned in the direction of applied vector E then P_{total} has a significant value.
- The polarization vector P is defined as the total dipole moment per unit volume.

$$\boxed{\bar{P} = \lim_{\Delta v \to 0} \frac{\sum_{i=1}^{n\Delta v} Q_i\bar{d}_i}{\Delta v}}$$

- It is measured in coulombs per square meter (C/m^2).
- The unit of polarization and flux density vector D is same and hence polarization increases the electric flux density.
- Hence the flux density in a dielectric is

$$\boxed{\bar{D} = \varepsilon_0\bar{E} + \bar{P}}$$

- For isotropic and linear medium vector P and E are parallel to each other at every point and related as

$$\overline{P} = \chi_e \, \varepsilon_0 \, \overline{E}$$

χ_e = Dimensionless quantity called **electric susceptibility** of the material.

- Sub vector P in above equation

$$\overline{D} = \varepsilon_0 \, \overline{E} + \chi_e \, \varepsilon_0 \, \overline{E}$$
$$\overline{D} = (\chi_e + 1)\varepsilon_0 \, \overline{E}$$
$$\overline{D} = \varepsilon \, \overline{E}$$
$$\varepsilon = \varepsilon_R \, \varepsilon_0$$

$$\varepsilon_R = \chi_e + 1$$

- € Permittivity of the dielectric medium.

PROPERTIES OF DIELECTRIC MATERIALS:

- The dielectrics do not contain any free charges but contain bound charges.
- Bound charges are under the internal molecular and atomic forces and cannot contribute to the conduction.
- When subjected to an electric field vector E , the bound charges shift their relative positions. Due to this, small electric dipoles get induced inside the dielectric. This is called polarization.
- Due to polarization the dielectrics can store energy.
- Due to polarization, the flux density of the dielectric increases by amount equal to the polarization.
- The induced dipoles produce their own electric field and align in the direction of the applied electric field.
- When polarization occurs, the volume charge density is formed inside the dielectric while the surface charge density is formed over the surface of the dielectric.
- The electric field inside and outside the dielectric gets modified due to the induced electric dipoles.

6. What is electric current and explain its related terms? [CO2-L1]

 ELECTRIC CURRENT

- The current is defined as the rate of flow of charge and it is measured in amperes.
- The current is due to the motion of positive charge and the conventional current is due to the flow of negative charge.
- The current and conventional current is opposite in direction.

 DRIFT VOLTAGE

- The current which exists in the conductor due to the drifting of electrons under the Influence of the applied voltage is called drift voltage.

 CURRENT DENSITY

- It is defined as the current passing through the unit surface area when the surface is normal to the direction of current. It is measured in A/m^2.

DISPLACEMENT CURRENT

There will be flow of charge under the influence of electric field intensity is called
displacement current

7.Derive the relationship between I and $\vec{J}$ with the suitable equations. [CO2-H2]

RELATION BETWEEN I and $\vec{J}$
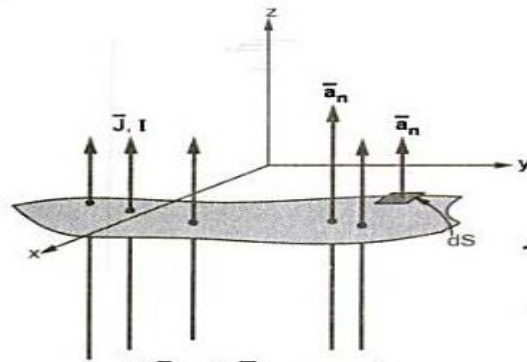
- Consider a surface S and I is passing through it.
- $\vec{J}$ is normal to the surface S.
- Consider a decrement surface ds and normal vector $\overrightarrow{an}$ is normal to it.

The $\overrightarrow{ds}$ and $\vec{J}$is given as

$$d\bar{S} = dS\ \bar{a}_n \quad \text{while } \bar{J} = J\ \bar{a}_n \qquad \text{... (1)}$$

The differential current is given as

$$\therefore \qquad dI = \bar{J} \cdot d\bar{S} \quad \text{(dot product)} \qquad \text{... (2)}$$

$\vec{J}$ and $\overrightarrow{ds}$ are perpendicular

$$dI = J\ \bar{a}_n \cdot dS\ \bar{a}_n = J\ dS \qquad \text{... (3)}$$

$$I = \int_S J\ dS \qquad \text{... (4)}$$

8.Derive the relationship between $\vec{J}$ and $\overrightarrow{\text{þv}}$ with the suitable equations. [CO2-H2]

RELATION BETWEEN $\vec{J}$ and þ$_v$
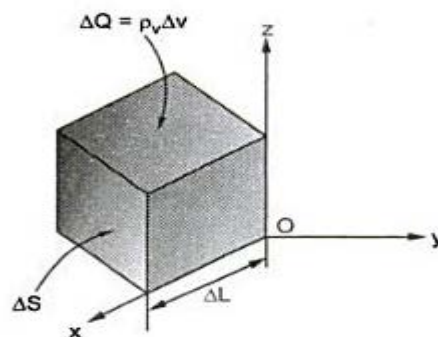
The elementary volume is given as

$$\Delta v = \Delta S \, \Delta L \qquad \text{... (7)}$$

The elementary charge is given as

$$\Delta Q = \rho_v \, \Delta S \, \Delta L \qquad \text{... (8)}$$

The elementary current is

$$\Delta I = \frac{\Delta Q}{\Delta t} \qquad \text{... (9)}$$

now, $\Delta Q = \rho_v \, \Delta S \Delta x$

$$\Delta I = \rho_v \, \Delta S \frac{\Delta x}{\Delta t} \qquad \text{.. (10)}$$

$$\frac{\Delta x}{\Delta t} = \text{velocity in x direction i.e. } v_x$$

$$\Delta I = \rho_v \, \Delta S v_x \qquad \text{... (11)}$$

$$\Delta I = \bar{J} \Delta S$$

$$\boxed{\bar{J} = \rho_v \, \bar{v}} \qquad \text{... (13)}$$

9. What is continuity equation? Derive its equation. [CO2-L1-Apr/May 2012]

CONTINUITY EQUATION:

- The continuity equation of the current is based on the principle of conservation of charge.
- The principle states that the charges can neither be created nor be destroyed.
- Consider a closed surface S with current density vector J then the total current I passing the closed surface S is

$$\boxed{I = \oint_S \bar{J} \cdot d\bar{S}} \qquad \text{... (1)}$$

- According to principle of conservation if a amount of flux gets outside the surface the equal amount of charge is decreased inside the surface.

- The rate of decrease is given as $-\dfrac{dQ_i}{dt}$

$$I = \oint_S \bar{J} \cdot d\bar{S} = -\frac{dQ_i}{dt} \qquad \text{... (2)}$$

- By divergence theorem

$$\therefore \quad \boxed{\oint_{S} \bar{J} \cdot d\bar{S} = \int_{vol} (\nabla \cdot \bar{J})\, dv} \qquad \qquad \text{...(3)}$$

$$\therefore \quad -\frac{d\,Q_i}{dt} = \int_{vol} (\nabla \cdot \bar{J})\, dv \qquad \qquad \text{... (4)}$$

But $\quad Q_i = \int_{vol} \rho_v\, dv \qquad \qquad \text{... (5)}$

$$\therefore \quad \int_{vol} (\nabla \cdot \bar{J})\, dv = -\frac{d}{dt}\left[\int_{vol} \rho_v\, dv\right] = -\int_{vol} \frac{\partial \rho_v}{\partial t}\, dv \qquad \text{... (6)}$$

$$\therefore \quad \int_{vol} (\nabla \cdot \bar{J})\, dv = \int_{vol} -\frac{\partial \rho_v}{\partial t}\, dv \qquad \qquad \text{... (7)}$$

$$\therefore \quad (\nabla \cdot \bar{J})\Delta v = -\frac{\partial \rho_v}{\partial t}\Delta v \qquad \qquad \text{... (8)}$$

$$\therefore \quad \boxed{\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}} \qquad \qquad \text{... (9)}$$

- The above equation is the continuity equation

10. What is steady current? [CO2-L1]
Steady current:
- For steady current which are not the function of time
  $\partial \rho_v / \partial t = 0$
- and hence

$$\boxed{\nabla \cdot \bar{J} = 0 \quad \text{(Steady current)}}$$

- The rate of flow of charge remains constant with time.

11. Explain boundary conditions for electric fields between conductor and free space and two different dielectric medias. [CO2-H1-Nov/Dec 2012 & April/May 2015]
BOUNDARY CONDITIONS:
- When an electric field passes from one medium to other medium, it is important to study the conditions at the boundary between the two media. The conditions existing at the boundary of the two media when field passes from one medium to other are called boundary conditions. Depending upon the nature of the media, there are two situations of the boundary conditions,
- I. Boundary between conductor and free space.

- 2. Boundary between two dielectrics with different properties.
- The free space is nothing but a dielectric hence first case is nothing but the boundary between conductor and a dielectric. For studying the boundary conditions, the Maxwell's equations for electrostatics are required.

$$\oint \overline{E} \cdot \overline{dL} = 0 \quad \text{and} \quad \oint \overline{D} \cdot d\overline{S} = Q$$

- Similarly the field intensity t is required to be decomposed into two components namely tangential to the boundary and normal to the boundary

$$\therefore \qquad \boxed{\overline{E} = \overline{E}_{tan} + \overline{E}_N}$$

- Similar decomposition is required for flux density D as well.
- Let us study the various cases of boundary conditions in detail.

Boundary Conditions between Conductor and Free Space

- Consider a boundary between conductor and free space. The conductor is ideal having infinite conductivity Such conductors are copper, silver etc. having conductivity of the order of 10" S/m andcan be treated ideal. For ideal conductors it is known that,
- 1. The field intensity inside a conductor is zero and the flux density inside a conductor is zero.
- 2. No charge can exist within a conductor. The charge appears on the surface in the form at
- surface charge density.
- 3. The charge density within the conductor is zoro.
- Consider the conductor free space boundary as shown in the Fig.
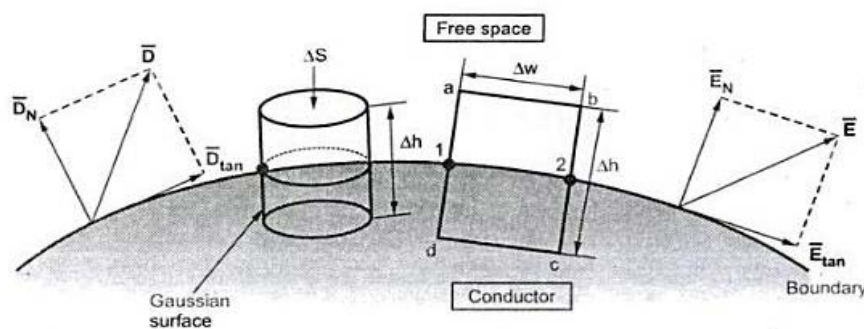
E at the Boundary
- Let E be the electric field intensity, in the direction shown in the Fig. angle with the boundary.
- This E can be resolved into two components :
- I. The component tangential to the surface Ctun
- 2. The component normal to the surface (EN It is known that,

$$\oint \bar{E} \cdot d\bar{L} = 0 \qquad \text{... (1)}$$

- Consider a rectangular closed path abcda as shown in the It is traced in clockwise direction as
- a-b-c-d-a and hence ∫E.dl can be divided into four parts.

$$\oint \bar{E} \cdot d\bar{L} = \int_a^b \bar{E} \cdot d\bar{L} + \int_b^c \bar{E} \cdot d\bar{L} + \int_c^d \bar{E} \cdot d\bar{L} + \int_d^a \bar{E} \cdot d\bar{L} = 0 \qquad \text{... (2)}$$

- Now the portion c-d is in the conductor $E = 0$ hence the corresponding integral is zero.
- As the width Aw is very small, E. over it can be assumed constant and hence can be taken out of integration.

$$\therefore \quad \int_a^b \bar{E} \cdot d\bar{L} = \bar{E} \int_a^b d\bar{L} = \bar{E}(\Delta w) \qquad \text{... (4)}$$

- But Aw is along tangential direction to the boundary in which $\bar{E} = \bar{E}_{tan}$.

$$\therefore \quad \int_a^b \bar{E} \cdot d\bar{L} = E_{tan}(\Delta w) \quad \text{where} \quad E_{tan} = \left| \bar{E}_{tan} \right| \qquad \text{... (5)}$$

- Now b-c is parallel to the normal component

$$\text{Let } E_N = \left| \bar{E}_N \right|.$$

- Over the small height Δh, E_N can be assumed constant and can be taken out of integration.

$$\therefore \quad \int_b^c \bar{E} \cdot d\bar{L} = \bar{E} \int_b^c d\bar{L} = E_N \int_b^c d\bar{L} \qquad \text{... (6)}$$

- But out of b-c, b-2 is in free space and 2-c is in the conductor where 1= 0. 2

$$\bar{E} = 0.$$

$$\therefore \quad \int_b^c d\bar{L} = \int_b^2 d\bar{L} + \int_2^c d\bar{L} = \frac{\Delta h}{2} + 0 = \frac{\Delta h}{2} \qquad \text{... (7)}$$

$$\therefore \quad \int_b^c \bar{E} \cdot d\bar{L} = E_N \left(\frac{\Delta h}{2} \right) \qquad \text{... (8)}$$

- Similarly for path d-a, the condition is same as for the path b-c, only direction is opposite.

$$\therefore \quad \int_d^a \bar{E} \cdot d\bar{L} = -E_N \left(\frac{\Delta h}{2} \right) \qquad \text{... (9)}$$

- Substituting (4), (8) and (9) in (3) we get,

$$\therefore \; E_{tan} \, \Delta w + E_N \left(\frac{\Delta h}{2}\right) - E_N \left(\frac{\Delta h}{2}\right) = 0 \qquad \qquad \text{... (10)}$$

$$\therefore \qquad E_{tan} \, \Delta w = 0 \qquad \qquad \text{But } \Delta w \neq 0 \text{ as finite}$$

$$\therefore \qquad \boxed{E_{tan} = 0} \qquad \qquad \text{... (11)}$$

- Thus the tangential component of the electric field intensity is zero at the boundary between conductor and free space.

$$\text{Now} \qquad \qquad \overline{D} = \varepsilon_0 \, \overline{E} \quad \text{for free space}$$

$$\therefore \qquad \boxed{D_{tan} = \varepsilon_0 \, E_{tan} = 0} \qquad \qquad \text{... (12)}$$

- Thus the tangential component of electric flux density is zero at the boundary between conductor and free space.

   D_N at the Boundary

- select a closed Gaussian surface in the form of right circular cylinder as shown in the Fig.
- Its height is Δh and is placed in such a way that $\Delta h/2$ is in the conductor and remaining $\Delta h/2$ is in the free space. Its axis is in the normal direction to the surface.

$$\text{According to Gauss's law,} \; \oint_S \overline{D} \cdot d\overline{S} = Q$$

- The surface integral must be evaluated over three sur(aces,
- i) Top, ii) Bottom and iii) Lateral.
- Let the area of top and bottom is same equal to ΔS.

$$\therefore \quad \int_{top} \overline{D} \cdot d\overline{S} + \int_{bottom} \overline{D} \cdot d\overline{S} + \int_{lateral} \overline{D} \cdot d\overline{S} = Q \qquad \qquad \text{... (13)}$$

- The bottom surface is in the conductor where

   $$\overline{D} = 0$$

   hence corresponding integral is zero.
- The top surface is in the free space and we are interested in the boundary condition hence top surface can be shifted at the boundary with Ah

$$\therefore \quad \int_{top} \overline{D} \cdot d\overline{S} + \int_{lateral} \overline{D} \cdot d\overline{S} = Q \qquad \qquad \text{... (14)}$$

- The lateral surface area $2\pi \, r \, \Delta h$ where r is the radius of the cylinder. But as $\Delta h \rightarrow$ 0, this area reduces to zero and corresponding integral is zero.
- While only component of D present is the normal component having magnitude DN,

$$\therefore \qquad \int_{top} \overline{D} \cdot d\overline{S} = D_N \int_{top} d\overline{S} = D_N \, \Delta S \qquad \qquad \text{... (15)}$$

From Gauss's law,

$$\therefore \qquad D_N \, \Delta S = Q \qquad \qquad \text{... (16)}$$

- But at t. boundary, the charge exists in the form of surface charge density ps C / m2.

$$\therefore \qquad Q = \rho_S \, \Delta S \qquad \qquad \text{... (17)}$$

- Equating (16) and (17),

$$\therefore \qquad D_N \, \Delta S = \Delta S$$

$$\therefore \qquad \boxed{D_N = \rho_S} \qquad \qquad \text{... (18)}$$

- Thus the flux leaves the surface normally and the normal component, of flux density is equal to the surface charge density.

$$\therefore \qquad D_N = \varepsilon_0 \, E_N = \rho_S \qquad \qquad \text{... (19)}$$

$$\therefore \qquad \boxed{E_N = \frac{\rho_S}{\varepsilon_0}} \qquad \qquad \text{... (20)}$$

Boundary Conditions between Conductor and Dielectric
- The free space is a dielectric with $\epsilon = \epsilon_0$
- Thus if the boundary is between conductor and dielectric with

$\varepsilon = \varepsilon_0 \varepsilon_r.$

$$\therefore \qquad \boxed{\begin{aligned} E_{tan} &= D_{tan} = 0 \\ D_N &= \rho_S \\ E_N &= \frac{\rho_S}{\varepsilon} = \frac{\rho_S}{\varepsilon_0 \varepsilon_r} \end{aligned}} \qquad \begin{aligned} &\text{... (21)} \\ &\text{... (22)} \\ &\text{... (23)} \end{aligned}$$

and

Boundary Conditions between Two Perfect Dielectrics
- Let us consider the boundary between two perfect dielectrics. One dielectric has permittivity ϵ_1 while the other has permittivity ϵ_2. The interface is shown in the Fig.
- The E and D are to be obtained again by resolving each into two components, tangential to the boundary, and normal to the surface.
- Consider a closed path abcda rectangular in shape having elementary height Δh and elementary width Δw, as shown in the Fig.
- It is placed in such a way that $\Delta h/2$ is in dielectric 1 while the remaining is dielectric 2. Let us evaluate the integral by tracing it in clockwise direction as a-b-c-d-a.
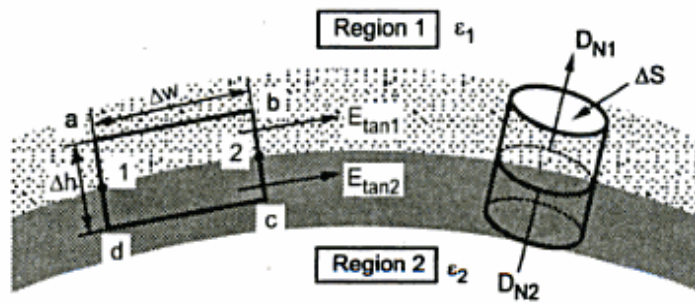
$$\oint \overline{E} \cdot d\overline{L} = 0 \qquad \text{... (1)}$$

$$\therefore \int_a^b \overline{E} \cdot d\overline{L} + \int_b^c \overline{E} \cdot d\overline{L} + \int_c^d \overline{E} \cdot d\overline{L} + \int_d^a \overline{E} \cdot d\overline{L} = 0 \qquad \text{... (2)}$$

Now $\qquad \overline{E}_1 = \overline{E}_{1t} + \overline{E}_{1N} \qquad \text{... (3)}$

and $\qquad \overline{E}_2 = \overline{E}_{2t} + \overline{E}_{2N} \qquad \text{... (4)}$

- Both E_1 and E_2 in the respective dielectrics have both the components, tangential and normal

  Let $\qquad \left| \overline{E}_{1t} \right| = E_{tan1}, \qquad \left| \overline{E}_{2t} \right| = E_{tan2}$

  $\qquad \left| \overline{E}_{1N} \right| = E_{1N}, \qquad \left| \overline{E}_{2N} \right| = E_{2N}$

- Now for the rectangle to be reduced at the surface to analyse boundary $\Delta h \to 0$. As $\Delta h \to 0$,

  $$\int_b^c \text{ and } \int_d^a$$

- Becomes zero. Hence eq. 2 becomes

  $$\int_a^b \overline{E} \cdot d\overline{L} + \int_c^d \overline{E} \cdot d\overline{L} = 0 \qquad \text{... (5)}$$

- Now integral a to b

  $$\therefore \qquad \int_a^b \overline{E} \cdot d\overline{L} = E_{tan\,1} \int_a^b d\overline{L} = E_{tan\,1} \, \Delta w \qquad \text{... (6)}$$

- Now integral c to d

  $$\therefore \qquad \int_c^d \overline{E} \cdot d\overline{L} = -E_{tan\,2} \, \Delta w \qquad \text{... (7)}$$

- Add all the integrals

  $$E_{tan\,1} \, \Delta w - E_{tan\,2} \, \Delta w = 0 \qquad \text{... (8)}$$

  $$\therefore \qquad \boxed{E_{tan\,1} = E_{tan\,2}} \qquad \text{... (9)}$$

- Thus the tangential component of field intensity at the boundary in both the dielectrics remain same.
- The relation between D and E is

$$\overline{D} = \varepsilon \overline{E} \qquad \qquad \text{... (10)}$$

$$D_{\tan 1} = \varepsilon_1 E_{\tan 1} \quad \text{and} \quad D_{\tan 2} = \varepsilon_2 E_{\tan 2} \qquad \text{... (11)}$$

$$\therefore \quad \frac{D_{\tan 1}}{\varepsilon_1} = \frac{D_{\tan 2}}{\varepsilon_2} \qquad \qquad \text{... (12)}$$

•

$$\therefore \quad \boxed{\frac{D_{\tan 1}}{D_{\tan 2}} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \qquad \qquad \text{... (13)}$$

D at Boundary:
• The equation for finding out the flux density is

$$\therefore \qquad \oint \overline{D} \cdot d\overline{S} = Q \qquad \qquad \text{... (14)}$$

• The Gaussian surface is divided into 3 regions

$$\therefore \left[\int_{top} + \int_{bottom} + \int_{lateral} \right] \overline{D} \cdot d\overline{S} = Q \qquad \text{... (15)}$$

• As $\Delta h \rightarrow 0$

$$\text{But} \quad \int_{lateral} \overline{D} \cdot d\overline{S} = 0 \quad \text{as} \quad \Delta h \rightarrow 0 \qquad \text{... (16)}$$

$$\therefore \int_{top} \overline{D} \cdot d\overline{S} + \int_{bottom} \overline{D} \cdot d\overline{S} = Q \qquad \text{... (17)}$$

• The flux leaving normal to the boundary is normal to the top and bottom surfaces

$$\therefore |\overline{D}| = D_{N1} \text{ for dielectric 1 and } D_{N2} \text{ for dielectric 2.}$$

• Top surface:

$$\therefore \qquad \int_{top} \overline{D} \cdot d\overline{S} = D_{N1} \int_{top} d\overline{S} = D_{N1} \Delta S \qquad \text{... (18)}$$

• Bottom surface:

$$\therefore \qquad \int_{bottom} \overline{D} \cdot d\overline{S} = -D_{N2} \int_{bottom} d\overline{S} = -D_{N2} \Delta S \qquad \text{... (19)}$$

• Adding the integrals

$$\therefore D_{N1} \Delta S - D_{N2} \Delta S = Q \qquad \qquad \text{... (20)}$$

$$\text{But} \qquad \qquad Q = \rho_S \Delta S \qquad \qquad \text{... (21)}$$

$$\therefore \qquad D_{N1} - D_{N2} = \rho_S \qquad \qquad \text{... (22)}$$

• For ideal dielectric

$$\therefore \qquad\qquad \rho_S = 0$$

$$\therefore \qquad\qquad D_{N1} - D_{N2} = 0$$

$$\therefore \qquad\qquad \boxed{D_{N1} = D_{N2}} \qquad\qquad\qquad ... (23)$$
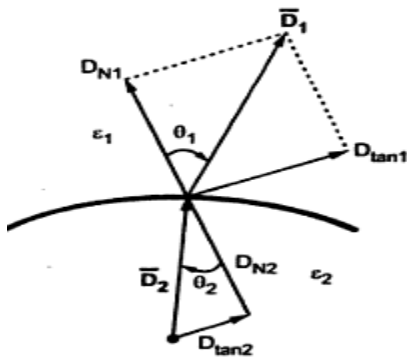
- The normal component of flux density vector D is continuous at the boundary between two perfect dielectric.

$$\text{Now} \quad D_{N1} = \varepsilon_1 E_{N1} \quad \text{and} \quad D_{N2} = \varepsilon_2 E_{N2}$$

$$\therefore \qquad\qquad \frac{D_{N1}}{D_{N2}} = \frac{\varepsilon_1 E_{N1}}{\varepsilon_2 E_{N2}} = 1 \qquad\qquad ... \text{ as } D_{N1} = D_{N2}$$

$$\therefore \qquad\qquad \boxed{\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}} \qquad\qquad ... (24)$$

- Refraction of $\vec{D}$

$$\text{Let} \qquad |\vec{D_1}| = D_1 \quad \text{and} \quad |\vec{D_2}| = D_2$$

$$\therefore \qquad D_{N1} = D_1 \cos \theta_1 \qquad\qquad ... (25)$$

$$\text{and} \qquad D_{N2} = D_2 \cos \theta_2 \qquad\qquad ... (26)$$

$$\text{But} \qquad D_{N1} = D_{N2}$$

$$\therefore \quad D_1 \cos \theta_1 = D_2 \cos \theta_2 \qquad\qquad ... (27)$$

$$\therefore \qquad\qquad \frac{D_1 \sin \theta_1}{D_2 \sin \theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{D_{\tan 1}}{D_{\tan 2}} \qquad\qquad ... (28)$$

$$\text{Now} \qquad \tan \theta_1 = \frac{D_{\tan 1}}{D_{N1}} \quad \text{and} \quad \tan \theta_2 = \frac{D_{\tan 2}}{D_{N2}}$$

$$\therefore \qquad\qquad \frac{\tan \theta_1}{\tan \theta_2} = \frac{D_{\tan 1}}{D_{\tan 2}} \frac{D_{N2}}{D_{N1}} \qquad\qquad ... \text{ But } D_{N2} = D_{N1}$$

$$\therefore \quad \frac{\tan \theta_1}{\tan \theta_2} = \frac{D_{\tan 1}}{D_{\tan 2}} = \frac{\varepsilon_1}{\varepsilon_2} \qquad \dots (29)$$

$$\therefore \quad \boxed{\frac{\tan \theta_1}{\tan \theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \qquad \dots (30)$$

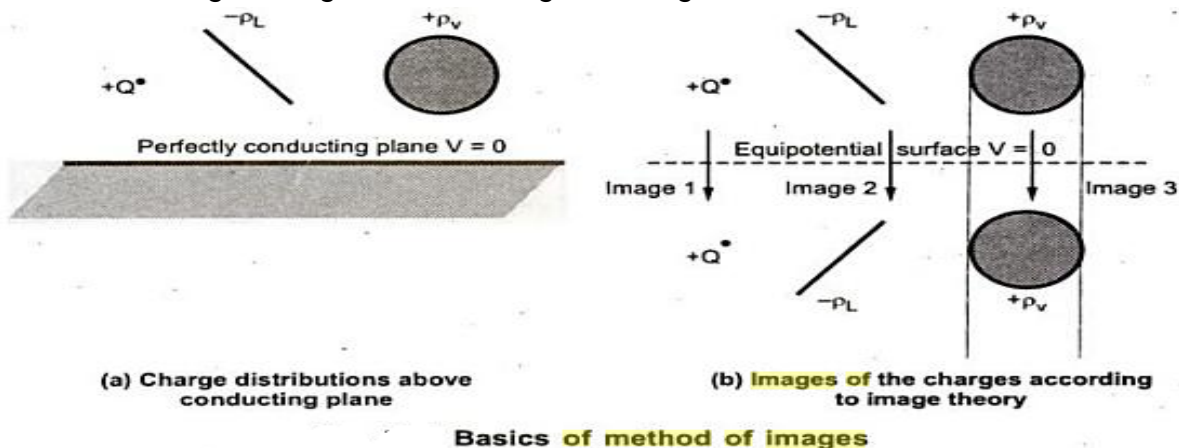- This is called Law of refraction.

12. What is image theory? Explain how the images are formed using image theory. [CO2-L1]

METHOD OF IMAGES:

- The method of images, introduced by Lord Kelvin in 1848, is commonly used to determine $V, \bar{E}, \bar{D}$, and ρ_s due to charges in the presence of conductors.
- By this method, we avoid solving Poisson's or Laplace's equation but rather utilize the fact that a conducting surface is an equipotential.

IMAGE THEORY:

- Consider a dipole field.
- The plane exists midway between the 2 charges, is a zero potential infinite plane.
- The conductor is an equipotential surface at a potential V=0 and vector E is normal to the surface.
- Out of dipole only positive charge is considered then fields at all points in upper half of plane are same.
- An image of the charge at a symmetrical location below the plane is formed.
- Such an image is negative of the original charge.

(a) Charge distributions above conducting plane

(b) Images of the charges according to image theory

Basics of method of images

- Typical examples of point, line, and volume charge configurations are portrayed in Figure (a), and their corresponding image configurations are in Figure (b).
- In applying the image method, two conditions must always be satisfied:
    1. The image charge(s) must be located in the conducting region.
   2. The image charge(s) must be located such that on the conducting surface(s) the potential is
zero or constant.

- The first condition is necessary to satisfy Poisson's equation, and the second condition

ensures that the boundary conditions are satisfied. Let us now apply the image theory to two specific problems

METHOD OF IMAGES IN POINT CHARGE:

- Consider a point charge Q placed at a distance h from a perfect conducting plane of infinite extent as in Figure a. The image configuration is in Figure b.

(a) Original charge and conducting plane   (b) **Method of images**
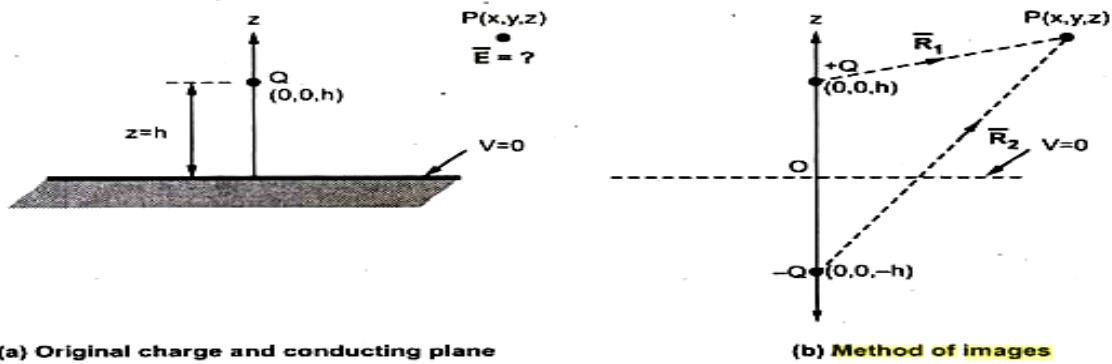
- The electric field at point $P(x, y, z)$ is given by,

$$\overline{E} = \overline{E}_1 + \overline{E}_2 \qquad \text{where } \overline{E}_1 \text{ due to } + Q, \ \overline{E}_2 \text{ due to } - Q$$

$$= \frac{Q}{4\pi\varepsilon_0 R_1^2}\,\overline{a}_{R1} + \frac{-Q}{4\pi\varepsilon_0 R_2^2}\,\overline{a}_{R2}$$

Now
$$\overline{R}_1 = (x-0)\,\overline{a}_x + (y-0)\,\overline{a}_y + (z-h)\,\overline{a}_z$$

∴
$$\overline{a}_{R1} = \frac{\overline{R}_1}{|\overline{R}_1|} = \frac{(x)\overline{a}_x + (y)\overline{a}_y + (z-h)\overline{a}_z}{\sqrt{x^2 + y^2 + (z-h)^2}}$$

And
$$\overline{R}_2 = (x-0)\,\overline{a}_x + (y-0)\,\overline{a}_y + [z-(-h)]\,\overline{a}_z$$

∴
$$\overline{a}_{R2} = \frac{\overline{R}_2}{|\overline{R}_2|} = \frac{x\,\overline{a}_x + y\,\overline{a}_y + (z+h)\,\overline{a}_z}{\sqrt{x^2 + y^2 + (z+h)^2}}$$

$$\overline{E} = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{[x\,\overline{a}_x + y\,\overline{a}_y + (z-h)\,\overline{a}_z]}{\left(x^2 + y^2 + (z-h)^2\right)^{3/2}} - \frac{[x\,\overline{a}_x + y\,\overline{a}_y + (z+h)\,\overline{a}_z]}{\left(x^2 + y^2 + (z+h)^2\right)^{3/2}}\right\}$$

- It should be noted that when $z = 0$, E has only the z-component, confirming that E is normal to the conducting surface.

Let us obtain **potential at P (x, y, z)**.

The potential due to the point charge is given by,

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

$$V_P = V_1 + V_2 = \frac{Q}{4\pi\varepsilon_0 R_1} + \frac{-Q}{4\pi\varepsilon_0 R_2}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

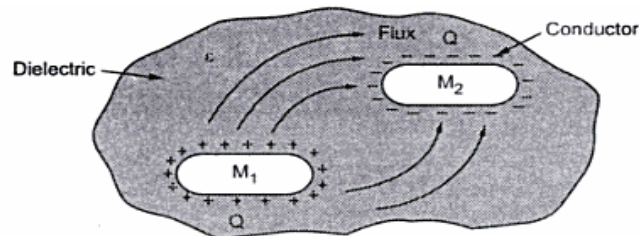$$R_1 = |\bar{R_1}| = \sqrt{x^2 + y^2 + (z-h)^2}$$

$$R_2 = |\bar{R_2}| = \sqrt{x^2 + y^2 + (z+h)^2}$$

$$V_P = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{[x^2 + y^2 + (z-h)^2]^{1/2}} - \frac{1}{[x^2 + y^2 + (z+h)^2]^{1/2}}\right]$$

- At $z=0$, $V_p=0$ V which confirms that surface of the conductor is equipotential surface with V=0.
- The total charge induced on the conducting surface also can be obtained from Gauss law.

13. Define capacitance and give its equation. [CO2-L1]
CAPACITANCE:

- Consider two conducting materials M1 and M2 which are placed in a dielectric medium having permittivity €
- The conductor M1 carries + charge and M2 carries – charge.
- The charges are equal and opposite. Therefore the total charge of the system is zero.

Definition:

A system which has two conducting surfaces carrying equal and opposite charges separated by a dielectric is called capacitive system which gives rise to capacitance.

The potential difference between M1 and M2 is V_{12} and the capacitance of two material is given by

$\therefore$ $$C = \frac{Q}{V_{12}}$$ ... (1)

In general, $$C = \frac{Q}{V}$$ ... (2)

Where Q=charge in coulombs

V=potential difference in volts

The capacitance is measured in farads (F)

$$1 \text{ farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$$

The charge is given as

$$Q = \oint_S \overline{D} \cdot d\overline{S} = \oint_S \varepsilon_0 \varepsilon_r \overline{E} \cdot d\overline{S} = \oint_S \varepsilon \overline{E} \cdot d\overline{S}$$

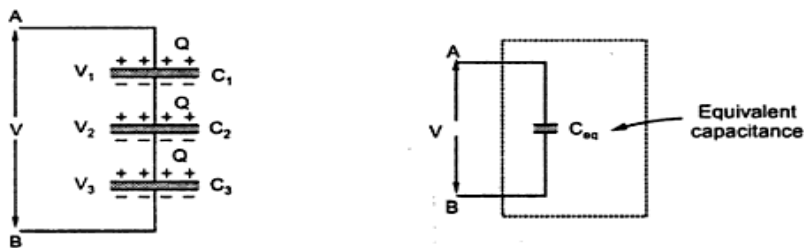V is the work done in moving the charge from negative to positive

$$V = -\int_L \overline{E} \cdot \overline{dL} = -\int_- \overline{E} \cdot \overline{dL}$$

The capacitance can be expressed as

$$C = \frac{Q}{V} = \frac{\oint_S \varepsilon \overline{E} \cdot d\overline{S}}{-\int_+ \overline{E} \cdot \overline{dL}} \quad F$$ ... (3)

14. Derive capacitance in series. [CO2-H2]
CAPACITORS IN SERIES:

- Consider 3 capacitors connected in series as shown in the figure. Voltage V is applied to it.
- The charge on the upper plate will be produced as negative form to the lower plate due to Connection leads.

The charge of the 3 capacitors is equal to

$$Q = C_1 V_1 = C_2 V_2 = C_3 V_3$$

Giving, $$V_1 = \frac{Q}{C_1}; \quad V_2 = \frac{Q}{C_2}; \quad V_3 = \frac{Q}{C_3}$$

The equivalent capacitance is

$$C_{eq} = \frac{Q}{V} \quad \text{or} \quad V = \frac{Q}{C_{eq}}$$

But,
$$V = V_1 + V_2 + V_3$$

$$\therefore \quad \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$
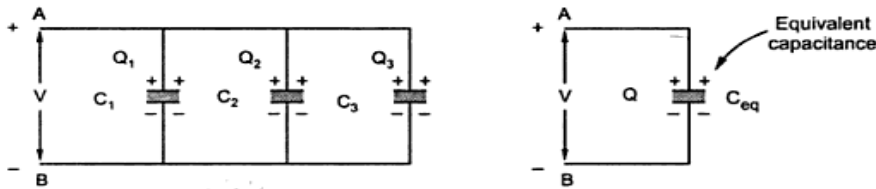
$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\boxed{\text{For 'n' capacitors in series, } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$$

If the three capacitors is replaced by a equivalent capacitor the 3 capacitance will be added.

15.Derive capacitance in parallel. [CO2-H1-Apr/May 2012]
CAPACITORS IN PARALLEL:

When capacitors are in parallel the charge becomes different and voltage remains same.

$$\boxed{Q_1 = C_1 V, \quad Q_2 = C_2 V, \quad Q_3 = C_3 V}$$

The total charge is given by,

$$Q = Q_1 + Q_2 + Q_3$$
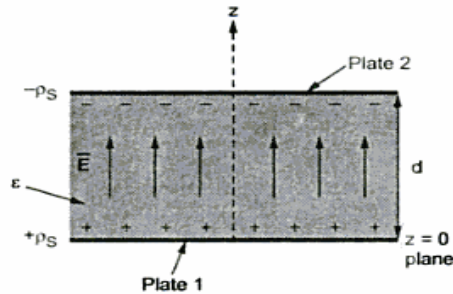$$= C_1 V + C_2 V + C_3 V = (C_1 + C_2 + C_3) V \qquad \dots (1)$$

The equivalent capacitor is replaced for,

$$Q = C_{eq} V \qquad \dots (2)$$

Comparing 1 and 2

$$C_{eq} = C_1 + C_2 + C_3$$

$$\boxed{\text{For 'n' capacitors in parallel, } C_{eq} = C_1 + C_2 + \dots + C_n}$$

16.Explain parallel plate capacitor and give its equation[CO2-L1-April/May 2015]
PARALLEL PLATE CAPACITOR:

- A parallel plate capacitor consists of 2 metallic parallel plates separated by a distance D.
- It is filled with dielectric permittivity €
- The lower plate carries positive charge with surface density $+\rho_s$ at Z=0.
- The lower plate carries positive charge with surface density $-\rho_s$ at Z=d.

The charge on any one of the plate is

$$Q = \rho_s A \quad C \qquad \text{... (1)}$$

Where A= Area of cross section of the plates
Assume plate 1 be infinite sheet of charge

$$\bar{E_1} = \frac{\rho_s}{2\varepsilon} \bar{a}_N = \frac{\rho_s}{2\varepsilon} \bar{a}_z \quad V/m \qquad \text{... (2)}$$

The electric field of plate 2 is

$$\bar{E_2} = \frac{-\rho_s}{2\varepsilon} \bar{a}_N = \frac{-\rho_s}{2\varepsilon} (-\bar{a}_z) \quad V/m \qquad \text{... (3)}$$

The total electric field of the plates

$$\bar{E} = \bar{E_1} + \bar{E_2} = \frac{\rho_s}{2\varepsilon} \bar{a}_z + \frac{\rho_s}{2\varepsilon} \bar{a}_z = \frac{\rho_s}{\varepsilon} \bar{a}_z \qquad \text{... (4)}$$

The potential difference is given as,

$$V = -\int_{-}^{+} \bar{E} \cdot d\bar{L} = -\int_{upper}^{lower} \frac{\rho_s}{\varepsilon} \bar{a}_z \cdot d\bar{L}$$

Write dL in terms of Cartesian,

$$d\bar{L} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$$

$$V = -\int_{z=d}^{z=0} \frac{\rho_s}{\varepsilon} \bar{a}_z \cdot [dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z]$$

$$= -\int_{z=d}^{z=0} \frac{\rho_s}{\varepsilon} dz = -\frac{\rho_s}{\varepsilon} [z]_d^0 = -\frac{\rho_s[-d]}{\varepsilon}$$
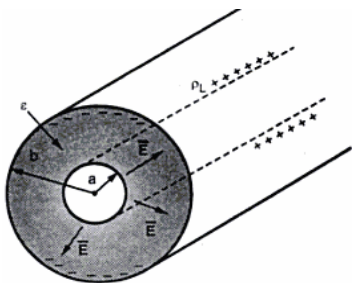
$$\therefore \quad V = \frac{\rho_s d}{\varepsilon} \quad V$$

The capacitance of the parallel plate is given as

$$C = \frac{Q}{V} = \frac{\rho_S A}{\frac{\rho_S d}{\varepsilon}} = \frac{\varepsilon A}{d} \; F \qquad \qquad \cdots (5)$$

$$\boxed{C = \frac{\varepsilon_0 \varepsilon_r A}{d} \; F} \qquad \qquad \cdots (6)$$

17.Derive capacitance of a coaxial cable. [CO2-H1]
CAPACITANCE OF A COAXIAL CABLE:

- Let us consider a coaxial cable as shown in the figure
- Let a be the inner radius and b be the outer radius
- The inner and outer conductors are separated by permittivity €
- The length of the cable is L
- The inner conductor has surface density $+\rho_L$ and the outer conductor has surface
  density $-\rho_L$

  The charge on any one of the conductor is

$$\therefore \qquad Q = \rho_L \times L \qquad \cdots (1)$$

The electric field of coaxial cable is

$$\overline{E} = \frac{\rho_L}{2\pi\varepsilon \, r} \overline{a}_r \qquad \qquad \cdots (2)$$

The potential difference V is given as

$$\therefore \qquad V = -\int \overline{E} \cdot d\overline{L} = -\int_{r=b}^{r=a} \frac{\rho_L}{2\pi\varepsilon r} \overline{a}_r \cdot dr\, \overline{a}_r$$

$$= -\frac{\rho_L}{2\pi\varepsilon} [\ln r]_b^a = -\frac{\rho_L}{2\pi\varepsilon} \ln\left[\frac{a}{b}\right]$$

$$\therefore \qquad V = \frac{\rho_L}{2\pi\varepsilon} \ln\left[\frac{b}{a}\right] V \qquad \qquad \cdots (3)$$

$$\therefore \qquad C = \frac{Q}{V} = \frac{\rho_L \times L}{\frac{\rho_L}{2\pi\varepsilon} \ln\left[\frac{b}{a}\right]}$$

$$\therefore \qquad \boxed{C = \frac{2\pi\varepsilon L}{\ln\left[\frac{b}{a}\right]} \; F} \qquad \qquad \cdots (4)$$

18.What is spherical capacitor ?Derive its capacitance. [CO2-L1]

SPHERICAL CAPACITOR:
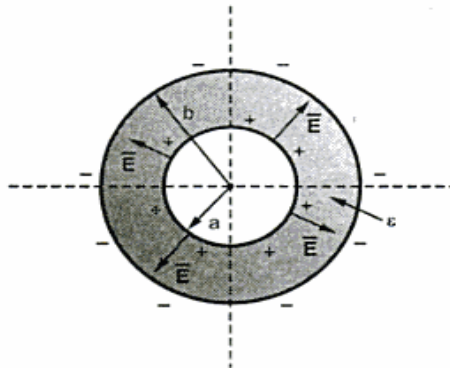
- Consider 2 shells of radius a and b as shown in the fig
- The radius of outer sphere is b and inner sphere is a.
- The region between 2 spheres is filled with permittivity ε
- The inner sphere has positive charge(+Q) and the outer sphere has negative charge(-Q).

  The electric field of 2 spheres is given as

$$\bar{E} = \frac{Q}{4\pi\varepsilon r^2}\bar{a}_r \ V/m \qquad \text{... (1)}$$

  The potential difference V is given as

$$\therefore \quad V = -\int_{-}^{+} \bar{E}\cdot d\bar{L} = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2}\bar{a}_r \cdot d\bar{L} \qquad \text{... (2)}$$

Now $\qquad d\bar{L} = dr\,\bar{a}_r \qquad\qquad$ ... in radial direction

$$\therefore \quad V = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2}\bar{a}_r \cdot dr\,\bar{a}_r = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2}\,dr$$

$$= -\frac{Q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_{r=b}^{r=a} = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r}\right]_{r=b}^{r=a}$$

$$\therefore \quad V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{a}-\frac{1}{b}\right] V \qquad \text{... (3)}$$

Now $\qquad C = \frac{Q}{V} = \dfrac{Q}{\dfrac{Q}{4\pi\varepsilon}\left[\dfrac{1}{a}-\dfrac{1}{b}\right]}$

$$\therefore \quad \boxed{C = \frac{4\pi\varepsilon}{\left[\dfrac{1}{a}-\dfrac{1}{b}\right]}\ F} \qquad \text{... (4)}$$
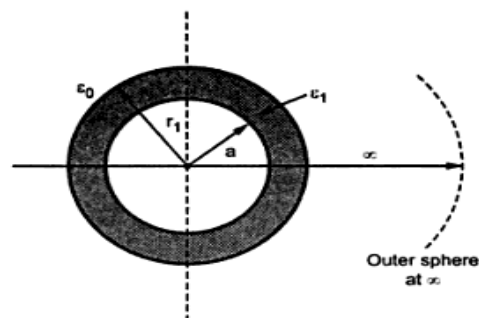
19. Derive capacitance of single isolated sphere. [CO2-H2]

CAPACITANCE OF SINGLE ISOLATOR SPHERE:

- The inner radius is a and the outer radius is b becomes infinity.
  Then the capacitance is given as,

$$\therefore \quad C = \frac{4\pi\varepsilon}{\left[\dfrac{1}{a} - \dfrac{1}{\infty}\right]} \quad \text{but} \quad \frac{1}{\infty} = 0$$

$$\therefore \quad \boxed{C = 4\pi\varepsilon a \ F} \qquad \qquad \dots (5)$$

ISOLATED SPHERE COATED WITH DIELECTRIC

Outer sphere
at ∞

The isolated sphere is divided into 2 limits

$$a < r < r_1, \quad \varepsilon = \varepsilon_1$$

$$r > r_1, \quad \varepsilon = \varepsilon_0.$$

The potential difference V is given as

Now for $a < r < r_1$,

$$\bar{E}_1 = \frac{Q}{4\pi\varepsilon_1 r^2}\bar{a}_r \qquad \qquad \dots (7)$$

Now for $r_1 < r < \infty$,

$$\bar{E}_2 = \frac{Q}{4\pi\varepsilon_0 r^2}\bar{a}_r \qquad \qquad \dots (8)$$

while $d\bar{L} = dr\,\bar{a}_r$ ...as $\bar{E}_1$ and $\bar{E}_2$ are in radial direction.

$$\therefore \quad V = -\int_{r=\infty}^{r=r_1} \frac{Q}{4\pi\varepsilon_0 r^2}\bar{a}_r \cdot dr\,\bar{a}_r - \int_{r=r_1}^{r=a} \frac{Q}{4\pi\varepsilon_1 r^2}\bar{a}_r \cdot dr\,\bar{a}_r$$
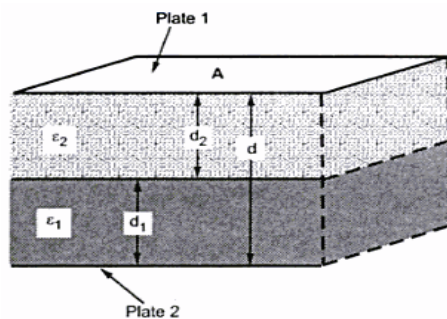
$$= -\frac{Q}{4\pi}\left[\frac{1}{\varepsilon_0}\int_{r=\infty}^{r=r_1}\frac{1}{r^2}dr + \frac{1}{\varepsilon_1}\int_{r=r_1}^{r=a}\frac{1}{r^2}dr\right]$$

$$= -\frac{Q}{4\pi}\left[\frac{1}{\varepsilon_0}\left(-\frac{1}{r}\right)_{r=\infty}^{r=r_1} + \frac{1}{\varepsilon_1}\left(-\frac{1}{r}\right)_{r=r_1}^{r=a}\right]$$

$$= -\frac{Q}{4\pi}\left[\frac{1}{\varepsilon_0}\left(-\frac{1}{r_1}+\frac{1}{\infty}\right)+\frac{1}{\varepsilon_1}\left(-\frac{1}{a}+\frac{1}{r_1}\right)\right]$$

$$= \frac{Q}{4\pi}\left[\frac{1}{\varepsilon_0}\left(\frac{1}{r_1}\right)+\frac{1}{\varepsilon_1}\left(\frac{1}{a}\right)-\frac{1}{\varepsilon_1}\left(\frac{1}{r_1}\right)\right]$$

$$V = \frac{Q}{4\pi}\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a}-\frac{1}{r_1}\right)+\frac{1}{\varepsilon_0 r_1}\right]V \qquad\qquad \text{... (9)}$$

$$\therefore \qquad C = \frac{Q}{V}=\frac{Q}{\frac{Q}{4\pi}\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a}-\frac{1}{r_1}\right)+\frac{1}{\varepsilon_0 r_1}\right]}$$

$$\therefore \qquad \boxed{C = \frac{4\pi}{\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a}-\frac{1}{r_1}\right)+\frac{1}{\varepsilon_0 r_1}\right]}F} \qquad\qquad \text{... (10)}$$

20. Explain composite capacitor and derive its capacitance. [CO2-L1]
COMPOSITE PARALLEL PLATE CAPACITOR

- Let us consider a composite parallel plate capacitor with 2 plates (upper and lower plate)
- The space d_1 is filled with dielectric ε_1 and the space d_2 is filled with dielectric ε_2.

The charge Q is given as

$$Q = \rho_S \times A$$

The voltage for 2 dielectric is given as

$$V_1 = E_1 d_1$$
$$V_2 = E_2 d_2$$

$$V = V_1 + V_2 = E_1 d_1 + E_2 d_2 \qquad\qquad \text{... (1)}$$

Where

$$D_1 = \varepsilon_1 E_1 \qquad \text{and} \qquad D_2 = \varepsilon_2 E_2$$

Sub in 1

$$V = \frac{D_1}{\varepsilon_1}d_1 + \frac{D_2}{\varepsilon_2}d_2 \qquad\qquad \text{... (2)}$$

Where

$$\rho_S = D_1 = D_2 \qquad\qquad \text{... (3)}$$

Sub in 2

$\therefore$ $\qquad$ $V = \rho_s \left[\dfrac{d_1}{\varepsilon_1} + \dfrac{d_2}{\varepsilon_2} \right]$ $\qquad\qquad$ ... (4)

The capacitance of the composite plate capacitor is

Now $\qquad$ $C = \dfrac{Q}{V} = \dfrac{Q}{\rho_s \left[\dfrac{d_1}{\varepsilon_1} + \dfrac{d_2}{\varepsilon_2} \right]}$

But $\qquad$ $Q = \rho_s \times A$

$\therefore$ $\qquad$ $C = \dfrac{\rho_s A}{\rho_s \left[\dfrac{d_1}{\varepsilon_1} + \dfrac{d_2}{\varepsilon_2} \right]} = \dfrac{A}{\dfrac{d_1}{\varepsilon_1} + \dfrac{d_2}{\varepsilon_2}}$ $\qquad$ ... (5)

$\therefore$ $\qquad$ $C = \dfrac{1}{\dfrac{d_1}{\varepsilon_1 A} + \dfrac{d_2}{\varepsilon_2 A}} = \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}}$ $\qquad$ ... (6)

where $\qquad$ $C_1 = \dfrac{\varepsilon_1 A}{d_1}$ $\qquad$ and $\qquad$ $C_2 = \dfrac{\varepsilon_2 A}{d_2}$

$\therefore$ $\qquad$ $C = \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}}$

$$\boxed{C = \dfrac{A}{\dfrac{d_1}{\varepsilon_1} + \dfrac{d_2}{\varepsilon_2} + \dfrac{d_3}{\varepsilon_3} + \dots + \dfrac{d_n}{\varepsilon_n}}} \qquad \text{... (7)}$$

This is the equation for composite plate capacitor

21. Explain dielectric boundary normal to the plates. [CO2-L1]
DIELECTRIC BOUNDARY NORMAL TO THE PLATES
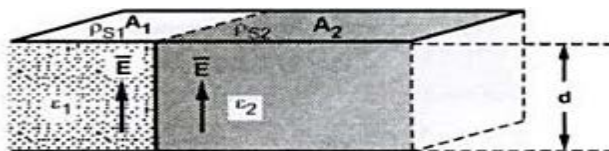
- Let us consider a composite capacitor with dielectric boundary normal to the plates.
- The dielectric ε_1 is in area A_1 and dielectric ε_2 is in area A_2.

The electric field is given as

$\qquad$ $E = \dfrac{V}{d}$ $\qquad\qquad$ ... (8)

$$\therefore \qquad E_{tan1} = E_{tan1} = E_1 = E_2 = \frac{V}{d} \qquad \qquad \cdots (9)$$

Now $\qquad D_1 = \varepsilon_1 E_1 \qquad$ and $\qquad D_2 = \varepsilon_2 E_2$

$$\therefore \qquad D_1 = \frac{\varepsilon_1 V}{d} \qquad$$ and $\qquad D_2 = \frac{\varepsilon_2 V}{d} \qquad \qquad \cdots (10)$$

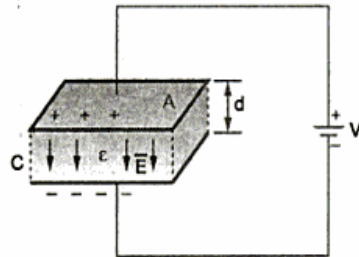The charge on the capacitor is

$$\therefore \qquad Q = Q_1 + Q_2 = \rho_{S1} A_1 + \rho_{S2} A_2 = D_1 A_1 + D_2 A_2 \qquad \cdots (11)$$

$$\therefore \qquad Q = \frac{\varepsilon_1 V A_1}{d} + \frac{\varepsilon_2 V A_2}{d} \qquad \qquad \cdots (12)$$

Now $\qquad C = \dfrac{Q}{V} = \dfrac{\dfrac{\varepsilon_1 V A_1}{d} + \dfrac{\varepsilon_2 V A_2}{d}}{V}$

$$\therefore \qquad \boxed{C = \frac{\varepsilon_1 A_1}{d} + \frac{\varepsilon_2 A_2}{d} = C_1 + C_2} \qquad \qquad \cdots (13)$$

22.Derive Energy stored & energy density of a capacitor. [CO2-L1-Nov/Dec 2013]
ENERGY STORED IN A CAPACITOR

- We know that capacitor store energy
- Consider a parallel plate capacitor as shown in the figure
- Voltage V is applied to it
  The electric field is given as

$$\therefore \qquad \bar{E} = \frac{V}{d} \bar{a}_N \qquad \qquad \cdots (1)$$

The energy stored is given by

$$W_E = \frac{1}{2} \int_{vol} \bar{D} \cdot \bar{E} \, dv$$

$$= \frac{1}{2} \int_{vol} \varepsilon \bar{E} \cdot \bar{E} \, dv \qquad \text{but} \quad \bar{E} \cdot \bar{E} = |\bar{E}|^2$$

$$= \frac{1}{2} \int_{vol} \varepsilon |\bar{E}|^2 \, dv \qquad \text{but} \quad |\bar{E}| = \frac{V}{d}$$

$$= \frac{1}{2} \varepsilon \frac{V^2}{d^2} \int_{vol} dv \qquad \text{but} \quad \int_{vol} dv = \text{Volume} = A \times d$$

$$= \frac{1}{2} \varepsilon \frac{V^2 A d}{d^2} = \frac{1}{2} \frac{\varepsilon A}{d} V^2$$

$$\boxed{W_E = \frac{1}{2} C V^2 \ J}$$

ENERGY DENSITY

- The energy density is defined as the energy stored per unit volume as volume tends to zero.
- The energy storage is given as,

$$\therefore \quad W_E = \frac{1}{2}\varepsilon \int_{Vol} |\overline{E}|^2 \, dv$$

$$\therefore \quad W_E = \frac{1}{2}\varepsilon |\overline{E}|^2 \ J/m^3 = \text{Energy density}$$

Using $\quad |\overline{D}| = \varepsilon |\overline{E}|$ we can write,

$$W_E = \frac{1}{2}\frac{|\overline{D}|^2}{\varepsilon} = \frac{1}{2}|\overline{D}||\overline{E}| \ J/m^3$$

23. Give equations for Poisons and Laplace equation. [CO2-L1-May/June 2014]

POISSONS AND LAPLACE EQUATION:

- To solve some electrostatic problems using boundary conditions ,Laplace and poisons equations are derived.
- From the Gauss law in point form poisons equation can be derived as below.

$$\nabla \cdot \overline{D} = \rho_v \qquad \qquad \text{... (1)}$$

$\overline{D}$ = Flux density and ρ_v = Volume charge density

- Foe homogenous, isotropic and linear medium flux density and electric field intensity are directly proportional.

$$\overline{D} = \varepsilon \overline{E} \qquad \qquad \text{... (2)}$$

$$\nabla \cdot \varepsilon \overline{E} = \rho_v \qquad \qquad \text{... (3)}$$

- From the gradient relationship

$$\overline{E} = -\nabla V \qquad \qquad \text{... (4)}$$

- Sub 4 in 3

$$\nabla \cdot \varepsilon(-\nabla V) = \rho_v \qquad \qquad \text{... (5)}$$

$$\nabla \cdot \nabla V = -\frac{\rho_v}{\varepsilon} \qquad \qquad \text{... (6)}$$

- Now $\Delta.\Delta$ operation is called del squared operation and denoted as Δ^2.

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \qquad \qquad \text{... (7)}$$

- The equation 7 is called poisons equation.
- In a certain region volume charge density is zero ρ_v =o which is true for dielectric medium, then the poisons equation becomes

$$\boxed{\nabla^2 V = 0} \quad \text{(For charge free region)} \qquad \qquad \text{... (8)}$$

- This special case of poisons equation is called Laplace equation.

- The Δ^2 operation is called Laplacian of V.
- Equation 5 should be used as poisons equation for inhomegenous medium.

Δ^2 OPERATION IN DIFFERENT COORDINATE SYSTEMS:

- The potential V can be expressed in any of the 3 coordinate systems as

$$V(x, y, z), V(r, \phi, z) \quad \text{or} \quad V(r, \theta, \phi).$$

In **Cartesian co-ordinate system,**

$$\nabla V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z$$

$$\therefore \quad \nabla \cdot \nabla V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$$

$$\therefore \quad \boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0} \quad \text{... (9)}$$

- Equation 9 is the Laplace equation in Cartesian form.

In **cylindrical co-ordinate system,**

$$\boxed{\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} = 0} \quad \text{... (10)}$$

- Equation 10 is the Laplace equation in Cartesian form.

In **spherical co-ordinate system,**

$$\boxed{\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0} \quad \text{... (11)}$$

- Equation 11 is the Laplace equation in Cartesian form.

24.Prove that Laplace equation is a Uniqueness theorem. [CO2-L3]

UNIQUENESS THEOREM:

- Here the Laplace equation is proved as uniqueness theorem.
- Assume the Laplace equation has two solutions V_1 and V_2 (both are function of the coordinates of the system used).
- These solutions may satisfy Laplace equation, therefore

$$\nabla^2 V_1 = 0 \quad \text{and} \quad \nabla^2 V_2 = 0 \quad \text{... (1)}$$

- Both the solutions should satisfy Boundary conditions also.
- At the boundary, the potentials at different points are same due to equipotential surface.

$$V_1 = V_2 \quad \text{... (2)}$$

- Let the difference between two solutions is V_d.

$$V_d = V_2 - V_1 \quad \text{... (3)}$$

- Using Laplace equation for V_d

$$\nabla^2 V_d = \nabla^2 (V_2 - V_1) = 0 \qquad \text{... (4)}$$

$$\therefore \quad \nabla^2 V_2 - \nabla^2 V_1 = 0 \qquad \text{... (5)}$$

- On the boundary $V_d = 0$ from the equations 2 and 3.
- Now the divergence theorem states that

$$\int_{vol} \nabla \cdot \overline{A} \, dv = \oint_S \overline{A} \cdot d\overline{S} \qquad \text{... (6)}$$

Let $\overline{A} = V_d \nabla V_d$ and from vector algebra,

$$\nabla \cdot (\alpha \overline{B}) = \alpha (\nabla \cdot \overline{B}) + \overline{B} \cdot (\nabla \alpha)$$

Now use this for $\nabla \cdot (V_d \nabla V_d)$ with $\alpha = V_d$ and $\nabla V_d = \overline{B}$.

$$\therefore \quad \nabla \cdot (V_d \nabla V_d) = V_d (\nabla \cdot \nabla V_d) + \nabla V_d \cdot (\nabla V_d)$$

But $\nabla \cdot \nabla = \nabla^2$ hence,

$$\therefore \quad \nabla \cdot (V_d \nabla V_d) = V_d \nabla^2 V_d + \nabla V_d \cdot \nabla V_d \qquad \text{... (7)}$$

Using (4),

$$\nabla \cdot (V_d \nabla V_d) = \nabla V_d \cdot \nabla V_d \qquad \text{... (8)}$$

To use this in (6), let $\overline{A} = V_d \nabla V_d$ hence

$$\nabla \cdot V_d \nabla V_d = \nabla \cdot \overline{A} = \nabla V_d \cdot \nabla V_d$$

$$\int_{vol} \nabla V_d \cdot \nabla V_d \, dv = \oint_S V_d \nabla V_d \cdot d\overline{S} \qquad \text{... (9)}$$

But $V_d = 0$ on boundary, hence right hand side of equation (9) is zero.

$$\therefore \quad \int_{vol} \nabla V_d \cdot \nabla V_d \, dv = 0 \qquad \text{... (10)}$$

- This is volume integral to be evaluated on the volume enclosed by the boundary.
- It is known that

$$\overline{C} \cdot \overline{C} = |\overline{C}|^2,$$

$$\therefore \quad \int_{vol} |\nabla V_d|^2 \, dv = 0 \quad \text{as } \nabla V_d \text{ is vector.} \qquad \text{... (11)}$$

- Now integration can be zero under 2 conditions.
- 1. The quantity under integral sign is zero.
- 2. The quantity is positive in some regions and negative in other regions by equal amount and hence zero.
- But square term can be negative in any region hence, quantity under integral must be zero.

$$|\nabla V_d|^2 = 0$$

i.e. $\qquad \nabla V_d = 0 \qquad \text{... (12)}$

- As the gradient of $V_d = V_2 - V_1$ is zero means $V_2 - V_1$ is constant and not changing with any coordinates.
- But considering boundary it can be proved that $V_2 - V_1 = $ constant $=$ zero.

$$V_2 = V_1 \qquad \qquad \text{... (13)}$$

- This proves that both the solutions are equal and cannot be different.
- Thus uniqueness theorem can be stated as
- 1. If the solution of Laplace equation satisfy the boundary condition then that solution is unique by whatever method it is obtained.
- 2. The solution of Laplace equation gives the field which is unique satisfying the same boundary conditions, in a given region.

PROCEDURE FOR SOLVING LAPLACE EQUATION:

- 1.Solve the Laplace equation using the method of integration.
- 2.Determine the constants applying the boundary conditions given or known for the region.
- 3.Then vector E can be obtained for potential field V using gradient operation - ΔV.
- 4.For homogenous medium vector D can be given as ϵE.
- 5.At the surface $p_s = D_N$ hence once vector D is known the normal component D_N to the surface is known.Hence the charge induced on the conductor surface can be obtained as

$$Q = -\int \rho_s dS.$$

- 6.Once the charge induced Q is known and potential V is known then the capacitance C of the system can be obtained.

PROBLEM 1:

Determine whether or not the following potential fields satisfy the Laplace's equation :

a) $V = x^2 - y^2 + z^2$ b) $V = r \cos\phi + z$ c) $V = r \cos\theta + \phi$

Sol. :   a) $V = x^2 - y^2 + z^2$

$$\therefore \quad \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\partial^2}{\partial x^2}[x^2 - y^2 + z^2] + \frac{\partial^2}{\partial y^2}[x^2 - y^2 + z^2] + \frac{\partial^2}{\partial z^2}[x^2 - y^2 + z^2]$$

$$= \frac{\partial}{\partial x}[2x] + \frac{\partial}{\partial y}[-2y] + \frac{\partial}{\partial z}[2z] = 2 - 2 + 2 = 2$$

So $\nabla^2 V \neq 0$

Hence field V **does not satisfy Laplace's equation.**

b) $V = r \cos\phi + z$

In cylindrical co-ordinate system,

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$$

$$\frac{\partial V}{\partial r} = \frac{\partial}{\partial r}[r\cos\phi + z] = \cos\phi$$

$$\frac{\partial V}{\partial \phi} = \frac{\partial}{\partial \phi}[r\cos\phi + z] = -r\sin\phi$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}[r \cos \phi + z] = 1$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}[r \cos \phi] = \frac{1}{r}\cos \phi$$

$$\frac{1}{r^2}\left[\frac{\partial^2 V}{\partial \phi^2}\right] = \frac{1}{r^2}\left[\frac{\partial}{\partial \phi}(-r \sin \phi)\right] = -\frac{r \cos \phi}{r^2} = -\frac{\cos \phi}{r}$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{\partial}{\partial z}[1] = 0$$

$$\nabla^2 V = \frac{1}{r}\cos \phi - \frac{\cos \phi}{r} + 0 = 0$$

- So this field satisfies Laplace equation.

c) $V = r \cos \theta + \phi$

In spherical system,

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin \theta}\frac{\partial}{\partial \theta}\left(\sin \theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2 \theta}\frac{\partial^2 V}{\partial \phi^2}$$

$$r^2 \frac{\partial V}{\partial r} = r^2 \frac{\partial}{\partial r}[r \cos \theta + \phi] = r^2(\cos \theta)$$

$$\sin \theta \frac{\partial V}{\partial \theta} = \sin \theta \frac{\partial}{\partial \theta}[r \cos \theta + \phi] = \sin \theta[-r \sin \theta] = -r \sin^2 \theta$$

$$\frac{1}{r^2 \sin^2 \theta}\frac{\partial^2 V}{\partial \phi^2} = \frac{1}{r^2 \sin^2 \theta}\frac{\partial^2}{\partial \phi^2}[r \cos \theta + \phi] = \frac{1}{r^2 \sin^2 \theta}\frac{\partial}{\partial \phi}[1] = 0$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 \cos \theta] + \frac{1}{r^2 \sin \theta}\frac{\partial}{\partial \theta}(-r \sin^2 \theta)$$

$$= \frac{1}{r^2}2r \cos \theta + \frac{1}{r^2 \sin \theta}[-r 2 \sin \theta \cos \theta] = \frac{2}{r}\cos \theta - \frac{2}{r}\cos \theta$$

$$= 0$$

- So this field satisfies Laplace equation.

PROBLEM 2:

Verify that the potential field given below satisfies the Laplace's equation.
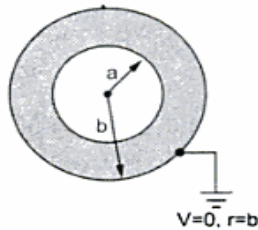$V = 2x^2 - 3y^2 + z^2$ **[V.T.U. March - 2001]**

Sol. :   Given field is in cartesian system,

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{\partial^2}{\partial x^2}[2x^2 - 3y^2 + z^2] + \frac{\partial^2}{\partial y^2}[2x^2 - 3y^2 + z^2] + \frac{\partial^2}{\partial z^2}[2x^2 - 3y^2 + z^2]$$

$$= \frac{\partial}{\partial x}[4x] + \frac{\partial}{\partial y}[-6y] + \frac{\partial}{\partial z}[2z] = 4 - 6 + 2 = 0$$

25.Give any two applications of Poisson and Laplace equation. [CO2-L1]
APPLICATIONS OF POISSON AND LAPLACE EQUATION:
CAPACITANCE OF A SPHERICAL CAPACITOR USING LAPLACE EQUATION:

V=0, r=b

- Consider concentric conductors as shown in the above figure.
- At r=b and V=0 hence the outer sphere is shown at zero potential.
- The field intensity vector E will be only in radial direction hence V is changing only in radial direction as the radial distance r and not the function of θ and Φ.
- According to Laplace equation

$$\nabla^2 V = 0$$

$$\therefore \quad \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0 \quad \text{... as V is function of r only}$$

$$\therefore \quad \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

Integrate, $r^2 \frac{\partial V}{\partial r} = \int 0 + C_1 = C_1$ ... (1)

$$\therefore \quad \frac{\partial V}{\partial r} = \frac{C_1}{r^2} = C_1 r^{-2}$$

Integrate, $V = \int C_1 r^{-2} dr + C_2 = \frac{C_1 r^{-1}}{-1} + C_2$

$$\therefore \quad V = -\frac{C_1}{r} + C_2 \quad \text{... (2)}$$

Use the boundary conditions,

$V = 0$ at $r = b$ and $V = V_0$ at $r = a$

$$\therefore \quad 0 = -\frac{C_1}{b} + C_2 \quad \text{and} \quad V_0 = -\frac{C_1}{a} + C_2$$

Subtracting the two equations,

$$-V_0 = -\frac{C_1}{b} - \left(-\frac{C_1}{a}\right)$$

$$\therefore \quad -V_0 = C_1 \left[\frac{1}{a} - \frac{1}{b}\right]$$

$$C_1 = \frac{-V_0}{\left[\frac{1}{a} - \frac{1}{b}\right]} = \frac{V_0}{\left[\frac{1}{b} - \frac{1}{a}\right]}$$

$$C_2 = \frac{C_1}{b} = \frac{V_0}{b\left[\frac{1}{b} - \frac{1}{a}\right]}$$

$$V = -\frac{V_0}{r\left[\frac{1}{b}-\frac{1}{a}\right]} + \frac{V_0}{b\left[\frac{1}{b}-\frac{1}{a}\right]} V$$

- This is the potential field in the region between 2 spheres.

$$\bar{E} = -\nabla V = -\frac{\partial V}{\partial r}\bar{a}_r = -\frac{\partial}{\partial r}\left[\frac{-V_0}{r\left(\frac{1}{b}-\frac{1}{a}\right)}\right]\bar{a}_r \qquad \dots C_2 \text{ is not function of } \dot{r}$$

$$= \frac{V_0}{\left(\frac{1}{b}-\frac{1}{a}\right)}\frac{\partial}{\partial r}\left(\frac{1}{r}\right)\bar{a}_r = \frac{-V_0}{\left(\frac{1}{b}-\frac{1}{a}\right)r^2}\bar{a}_r \quad V/m$$

$$\bar{D} = \varepsilon\bar{E} = \frac{-\varepsilon V_0}{\left(\frac{1}{b}-\frac{1}{a}\right)r^2}\bar{a}_r = \frac{\varepsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)r^2} \quad C/m^2$$

- As per the boundary conditions between conductor and dielectric the vector D is always normal to the surface D_N.

$$\rho_s = \left|\bar{D}_N\right| = \left|\bar{D}\right| = \frac{\varepsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)r^2} \quad C/m^2$$

$$Q = \text{Total charge on the surface of sphere of radius } r$$

$$= \frac{\varepsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)r^2} \times \text{ surface area of sphere of radius } r$$

$$= \frac{\varepsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)r^2} \times 4\pi r^2 = \frac{4\pi\varepsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)} \quad C$$

$$C = \frac{Q}{V} \quad \text{where } V = \text{potential between two spheres}$$

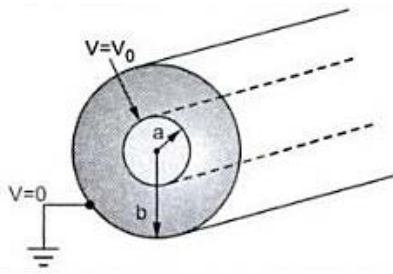$$V = V_0 = \text{potential difference between two spheres}$$

$$\boxed{C = \frac{Q}{V_0} = \frac{4\pi\varepsilon V_0}{\left(\frac{1}{a}-\frac{1}{b}\right)} = \frac{4\pi\varepsilon}{\left(\frac{1}{a}-\frac{1}{b}\right)} \quad F}$$

- This is the capacitance of a spherical capacitor using Laplace transform.

CAPACITANCE OF A COAXIAL CABLE USING LAPLACE EQUATION:
- Consider a coaxial cable as shown in the figure below.

- Consider cylindrical coordinate system.
- The field intensity vector E is in radial direction from inner to outer cylinder hence V is a function of r only and not the function of Φ and z.
- Using Laplace equation

$$\nabla^2 V = 0$$

$$\therefore \quad \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \qquad\qquad \dots V = f(r) \text{ only}$$

$$\therefore \quad \frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$$

Integrate, $\quad r\dfrac{\partial V}{\partial r} = \displaystyle\int 0 + C_1 = C_1 \qquad\qquad \dots (1)$

$$\therefore \quad \frac{\partial V}{\partial r} = \frac{C_1}{r}$$

Integrate, $\quad V = \displaystyle\int \frac{C_1}{r} + C_2 = C_1[ln\, r] + C_2 \qquad\qquad \dots (2)$

- Using boundary conditions

$V = 0$ at $r = b$ and $V = V_0$ at $r = a$,

$$0 = C_1 \, ln(b) + C_2 \quad \text{and} \quad V_0 = C_1 \, ln(a) + C_2$$

Subtracting, $\quad -V_0 = C_1\{ln(b) - ln(a)\} = C_1\left\{ln\left(\dfrac{b}{a}\right)\right\}$

$$\therefore \quad C_1 = \frac{-V_0}{ln\left(\dfrac{b}{a}\right)} = \frac{V_0}{ln\left(\dfrac{a}{b}\right)}$$

and $\quad C_2 = -C_1 \, ln(b) = \dfrac{-V_0 \, ln(b)}{ln\left(\dfrac{a}{b}\right)}$

$$\therefore \quad \boxed{V = \frac{V_0}{ln\left(\dfrac{a}{b}\right)}\, ln(r) - \frac{V_0 \, ln(b)}{ln\left(\dfrac{a}{b}\right)} \quad V}$$

Now $\qquad\qquad \bar{E} = -\nabla V = -\dfrac{\partial V}{\partial r}\,\bar{a}_r$

$$= -\frac{\partial}{\partial r}\left[\frac{V_0 \, ln(r)}{ln\left(\dfrac{a}{b}\right)}\right]\bar{a}_r \qquad\qquad \dots C_2 \text{ is not function of } r$$

$$\overline{E} = -\frac{V_0}{\ln\left(\frac{a}{b}\right)}\left[\frac{\partial}{\partial r}\ln(r)\right]\overline{a}_r = -\frac{V_0}{r\ln\left(\frac{a}{b}\right)}\overline{a}_r \quad V/m$$

$$\overline{D} = \varepsilon\overline{E} = \frac{-V_0\,\varepsilon}{r\ln\left(\frac{a}{b}\right)}\overline{a}_r = \frac{V_0\,\varepsilon}{r\ln\left(\frac{b}{a}\right)}\overline{a}_r \quad C/m^2$$

- Now vector D is existing normal to the surface as per the boundary conditions.

$$\overline{D} = \overline{D}_N = \frac{V_0\,\varepsilon}{r\ln\left(\frac{b}{a}\right)}\overline{a}_r$$

$$\rho_S = \left|\overline{D}_N\right| = \frac{V_0\,\varepsilon}{r\ln\left(\frac{b}{a}\right)} \quad C/m^2$$

ρ_S exists on entire surface area of inner cylinder.

$$\therefore \qquad Q = \rho_S \times \text{Surface area of inner cylinder}$$

$$= \frac{V_0\,\varepsilon}{r\ln\left(\frac{b}{a}\right)}\times 2\pi r\times L = \frac{V_0\,\varepsilon\,2\pi L}{\ln\left(\frac{b}{a}\right)} \quad C$$

- The potential difference between 2 cylinders is V_0. Thus $V=V_0$.

$$C = \frac{Q}{V} = \frac{\dfrac{V_0\,\varepsilon\,2\pi L}{\ln\left(\frac{b}{a}\right)}}{V_0}$$

$$C = \frac{2\pi\varepsilon L}{\ln\left(\frac{b}{a}\right)} \quad F$$

- The capacitance per unit length ie L=1m is

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)} \quad F/m$$

Unit III

Static magnetic fields

Part-A

1. State Biot–savart law. [CO3-L1-Nov/Dec 2006 & May/June 2006,2013]

Biot-savart law is defined as, the magnetic field intensity $d\overrightarrow{H}$ produced at a point P, due to a differential current element $Id\overrightarrow{L}$ is

- Proportional to the product of the current I and differential length dL
- Proportional to sine of the angle between element and line joining point P to the element.

- Inversely proportional to square of the distance R between point P and the element.

$$\text{i.e.,}\ \overrightarrow{dH} \propto \frac{IdLsin\theta}{R^2}$$

$$\overrightarrow{dH} = \frac{kIdLsin\theta}{R^2}\ (A/m)$$

Where,   k = Constant of proportionally $k = \frac{1}{4\pi}$

2. Define magnetic flux. [CO3-L1]

The total number of lines of force existing in a particular magnetic field is called magnetic flux which is denoted as ϕ.

3. Define magnetic flux density. [CO3-L1]

The magnetic flux in weber passing through unit area in a plane at right angles to the direction of flux is called magnetic flux density.

It is denoted as $\vec{B}$ is a vector quantity.

$$\vec{B} = \frac{\varphi}{S}\quad \text{Wb/m}^2\ \text{(or) tesla (T)}$$

4. Define Magnetic Field Intensity? [CO3-L1]

It is defined as the force experienced by a unit north pole of one weber strength, when placed at that point.

It is measured in newtons/weber (N/Wb) or amperes per metre (A/m)

It is denoted as $\vec{H}$

5. What is the relation between Magnetic Flux Density and Magnetic Field Intensity? [CO3-L1-April/May 2015]

The magnetic flux density B is related to the magnetic field intensity H through the permeability of the medium (μ).

$$\vec{B} = \mu\vec{H} = \mu_0\mu_r\vec{H}$$

Where, μ_0 = permeability of free space = $4\pi \times 10^{-7}$, μ_r = Relative permeability

6. Define magnetic scalar and vector potential. [CO3-L1-May/June 2013 & April/May 2015]

The scalar magnetic potential V_m can be defined for source free region where $\vec{J}$ is zero.

It is defined as,

$$V_{m\ a,b} = -\int_a^b \vec{H}\ \overrightarrow{dL}\ \text{for specified path.}$$

The vector potential in a magnetic field whose curl gives the magnetic flux density is called vector magnetic potential of that field. It is denoted as $\vec{A}$ and measured in (Wb/m). It has to satisfy the equation,

$$\vec{B} = \nabla \times \vec{A}$$

Similarly $\vec{J}$ can be obtained from $\vec{A}$ as,

$$\vec{J} = \frac{1}{\mu_0}\left[\nabla \times \nabla \times \vec{A}\right] = \frac{1}{\mu_0}\left[\nabla(\vec{A}.\nabla - \nabla^2\vec{A})\right]$$

7. State Ampere's Circuital law. [CO3-L1-May/June 2007 & Nov/Dec 2007]

The Ampere's Circuital law states that the line integral of Magnetic Field Intensity $\vec{H}$ around a closed path is exactly equal to the direct current 'I' enclosed by that path.

Mathematical representation of Ampere's Circuital law,

$$\oint \vec{H}\,\vec{dL} = I$$

8. State the point form of Ampere's Circuital law. [CO3-L1]

The point form of Ampere's Circuital law is,

9. A ferrite material has $\mu_r = 50, B = 0.05\,T$.Find $\vec{H}$. [CO3-L1]
Given:

$$\mu_r = 50$$
$$B = 0.05\,T$$

To find: $\vec{H}$?

Formula: $\vec{B} = \mu_0\mu_r\vec{H}$

Solution:

$$\vec{H} = \frac{\vec{B}}{\mu_0\mu_r} = \frac{0.05}{4 \times 3.14 \times 10^{-7} \times 50} = 796.17$$

10. Two long parallel conductors are separated by 2cm in air carrying currents 10A in opposite direction. Find force per meter length of conductor. State the nature of force. [CO3-L1]

Given: $I_1 = I_2 = 10A$

$$d = 2 \times 10^{-2}m$$
$$\mu = \mu_0 = \mu_r = 4\pi \times 10^{-7}$$

To find: $F/L = ?$

Solution:

$$F/L = \frac{\mu I_1 I_2}{2\pi d} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{4\pi \times 10^{-7} \times 10 \times 10}{2 \times 3.14 \times 2 \times 10^{-2}} = 1m\,(N/m)$$

$$F/_L = 1 \times 10^{-3} \ N/m$$

11. State the expression of $\vec{H}$. [CO3-L1]

(i) Due to infinitely long straight conductor along z- axis carrying I.
(ii) Due to conductor of finite length.
(iii) At the centre of circular conductor.
(iv) On the axis of a circular loop, at a distance x.

Ans $\vec{H} = \frac{I}{2\pi r} \vec{a_\varphi} \ A/m$

$$\vec{B} = \mu \vec{H} = \frac{\mu I}{2\pi r} \vec{a_\varphi} \ Wb/m^2$$

(i) $\vec{H} = \frac{I}{4\pi r} [\sin \alpha_2 - \sin \alpha_1] \vec{a_\varphi} \ A/m$

$$\vec{B} = \mu \vec{H} = \frac{\mu I}{4\pi r} [\sin \alpha_2 - \sin \alpha_1] \vec{a_\varphi} \ Wb/m^2$$

(ii) $\vec{H} = \frac{I \sin 90^\circ}{2R} \vec{a_N} = \frac{I}{2R} \vec{a_N} A/m$

$$\vec{B} = \frac{\mu_0 I}{2R} \vec{a_N} Wb/m^2$$

(iii) $\vec{H} = \frac{I_r{}^2}{2(r^2+z^2)^{3/2}} \vec{a_z} A/m$

12. State the expression of $\vec{H}$ due to infinite sheet of current. [CO3-L1]

In general, for an infinite sheet of current density $\vec{K} \ A/m$ we can write,

$$\vec{H} = \frac{1}{2}\vec{K} \times \vec{a_n}$$

Where, $\vec{a_n}$ = unit vector normal from the current sheet to the point at which $\vec{H}$ is to be

obtained.

13. State Laplace equation for scalar magnetic potential. [CO3-L1]

$$\nabla^2 V_m = 0 \quad for \ \vec{H} = 0$$

This is Laplace's equation for scalar magnetic potential. This is similar to the Laplace's equation for scalar electric potential $\nabla^2 V = 0$.

14. State Poisson's equation for Magnetic Field. [CO3-L1]

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

This is the Poisson's equation for magneto static field, where $\vec{A}$ is the vector magnetic potential and $\vec{J}$ is the current density.

15. What is the difference between Scalar and Vector Magnetic Potential? [CO3-H1]

Scalar Magnetic Potential	Vector Magnetic Potential
1. It is denoted as V_m	**1.** It is denoted as $\vec{J}$
2. $\nabla \times \nabla V = 0$; V is a scalar	**2.** $\nabla \cdot (\nabla \times \vec{A}) = 0$; $\vec{A}$ = Vector
3. $V_{m\ a,b} = -\int_a^b \vec{H}\,\overrightarrow{dL}$	**3.** $\vec{B} = \nabla \times \vec{A}$
4. Laplace equation, $\nabla^2 V_m = 0$; for $\vec{J} = 0$	**4. Poisson equation,** $\nabla^2 \vec{A} = -\mu_0 \vec{J}$

16. What is the significance of Biot-savart law? [CO3-H1]

Biot-savart law is used to obtain magnetic field intensity $\vec{H}$ and the magnetic flux density $\vec{B}$

Biot-savart law is given by,

$$\overrightarrow{dH} = \frac{kIdLsin\theta}{4\pi R^2} (A/m)$$

In vector form,

$$\overrightarrow{dH} = \frac{\overrightarrow{IdL} \times \overrightarrow{a_{R12}}}{4\pi \left|\overrightarrow{R_{12}}\right|^2} (A/m)$$

Where, IdL = differential current element

θ = Angle between line joining point P to the element.

$\overrightarrow{a_{R12}}$ = unit vector along $\overrightarrow{R_{12}}$ (distance vector).

17. A steady current of 'I' flows in a conductor bent in the form of a square loop of side 'a' meters. Find the magnetic field intensity at the centre of the loop. [CO3-H1]

Solution:

Since, $\vec{H} = \frac{I}{4\pi l}[\cos\alpha - \cos\beta]$

$$\vec{H} = \frac{I}{4\pi\,a/2}\left[\cos 45^\circ - \cos 135^\circ\right]$$

$$\vec{H} = \frac{I}{2\pi a}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{I}{\pi a}$$

18. State the applications of amperes circuital law. [CO3-L1]

It can be used to obtain

- Due to infinitely long straight conductor along z- axis carrying I.
- Due to conductor of finite length.
- At the centre of circular conductor.
- On the axis of a circular loop, at a distance x

Part-B

1. State Biot–savart law.[CO3-L1-Nov/Dec 2006, May/June 2006,2013]

Biot-savart law is defined as, the magnetic field intensity $d\vec{H}$ produced at a point P, due to a differential current element $Id\vec{L}$ is

- Proportional to the product of the current I and differential length dL
- Proportional to sine of the angle between element and line joining point P to the element.
- Inversely proportional to square of the distance R between point P and the element.

$$\text{i.e,} \overrightarrow{dH} \propto \frac{IdLsin\theta}{R^2}$$

$$\overrightarrow{dH} = \frac{kIdLsin\theta}{R^2}(A/m)$$

Where,   k = Constant of proportionally

$$k = \frac{1}{4\pi}$$

Biot-savart law is used to obtain magnetic field intensity $\vec{H}$ and the magnetic flux density $\vec{B}$

Biot-savart law is given by,

$$\overrightarrow{dH} = \frac{kIdLsin\theta}{4\pi R^2}(A/m)$$

In vector form,

$$\overrightarrow{dH} = \frac{\overrightarrow{IdL} \times \overrightarrow{a_{R12}}}{4\pi \left|\overrightarrow{R_{12}}\right|^2}(A/m)$$

Where, IdL = differential current element

$\theta =$ Angle between line joining point P to the element.

$\overrightarrow{a_{R12}}$ = unit vector along $\overrightarrow{R_{12}}$ (distance vector).
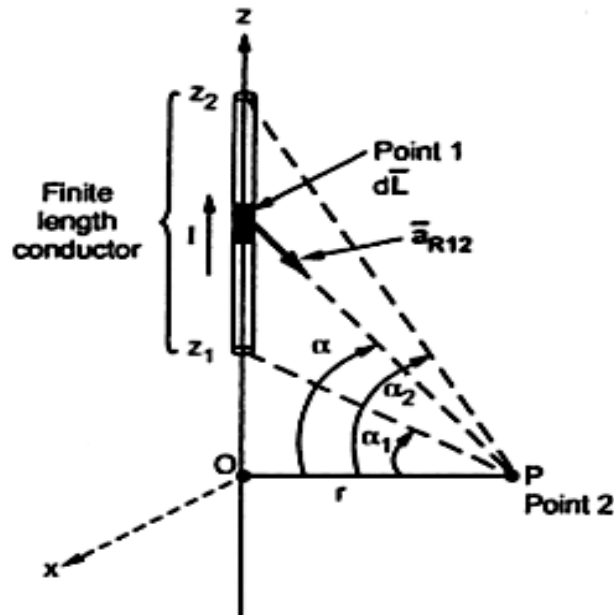
2. Explain about Magnetic field intensity.[CO3-L2]

Magnetic field intensity:

      The magnetic field intensity at any point in the magnetic field is defined as the force experienced by a unit north pole of one weber strength , when placed at that point. It is measured in newtonms/weber (N/Wb) or amperes per metre (A/m).
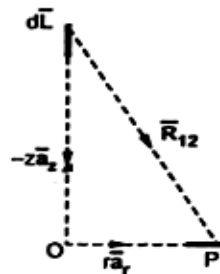
      It is denoted as $\overline{H}$

3.   Find the magnetic field intensity at a point P due to a finite straight conductor, carrying a current I.   [CO3-L1-Nov/Dec 2011, April/May 2015]

      Consider the conductor of finite length placed along z axis. It carries a direct current I. One end of the conductor is z_1 and the other end is z_2

      The perpendicular distance of point P from z- axis is 'h' as shown in figure. Consider a differential element dL along z axis, at a distance 'z' from the origin, which is given as $\overrightarrow{dL} = dz\vec{a}_z$

The distance vector from point P to the current element is given as,

$$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$$
$$= (r\vec{a}_r) - (z\vec{a}_z) = r\vec{a}_r - z\vec{a}_z$$

$$|\vec{R}_{12}| = \sqrt{(r)^2 + (-z)^2}$$

The unit vector $\vec{a}_{R12} = \dfrac{\vec{R}_{12}}{|\vec{R}_{12}|}$

$\vec{a}_{R12} = \dfrac{r\vec{a}_r - z\vec{a}_z}{\sqrt{r^2 + z^2}}$

$$d\vec{L} \times \vec{a}_{R12} = \begin{vmatrix} \vec{a}_r & \vec{a}_\varphi & \vec{a}_z \\ 0 & 0 & dz \\ \dfrac{r}{\sqrt{r^2 + z^2}} & 0 & \dfrac{-z}{\sqrt{r^2 + z^2}} \end{vmatrix}$$

$$= \vec{a}_r[0 - 0] - \vec{a}_\varphi\left[0 - \frac{r}{\sqrt{r^2 + z^2}}\right] + \vec{a}_z[0 - 0]$$

$$= \frac{1}{\sqrt{r^2 + z^2}}\left[rdz\vec{a}_\varphi\right]$$

$I\,(d\vec{L} \times \vec{a}_{R12}) = \dfrac{Irdz\,\vec{a}_\varphi}{\sqrt{r^2 + z^2}}$

According to Biot-savart law $d\vec{H}$ at point P is,

$d\vec{H} = \dfrac{Id\vec{L} \times \vec{a}_{R12}}{4\pi R_{12}^2}$

$$= \frac{Irdz\vec{a}_\varphi}{4\pi\sqrt{r^2 + z^2} + (\sqrt{r^2 + z^2})^2}$$

$$= \frac{Irdz\vec{a}_\varphi}{4\pi(r^2 + z^2)^{3/2}}$$

Integrating $d\vec{H}$ over the full length of the conductor 'z',

$$\Rightarrow \vec{H} = \int_{z_1}^{z_2} \frac{Irdz\,\vec{a}_\varphi}{4\pi(r^2 + z^2)^{3/2}}$$

$Put\, z = r\tan\alpha, \qquad z^2 = r^2\tan^2\alpha \; and \; dz = r\sec^2\alpha\, d\alpha$

For $z = z_1$, $z_1 = r\tan\alpha_1$ and $\alpha_1 = tan^{-1}\left(\dfrac{z_1}{r}\right)$

For $z = z_2$, $z_2 = r\tan\alpha_2$ and $\alpha_2 = tan^{-1}\left(\dfrac{z_2}{r}\right)$

$$\vec{H} = \int_{\alpha_1}^{\alpha_2} \frac{Ir.r\sec^2\alpha\, d\alpha\,\vec{a}_\varphi}{4\pi[r^2 + r^2\tan^2\alpha]^{3/2}}$$

$$= \int_{\alpha_1}^{\alpha_2} \frac{Ir.r\sec^2\alpha\, d\alpha\,\vec{a}_\varphi}{4\pi r^3\sec^3\alpha}$$

$$= \int_{\alpha_1}^{\alpha_2} \frac{Id\alpha}{4\pi r\sec\alpha}\vec{a}_\varphi$$

$$= \frac{I}{4\pi r}\int_{\alpha_1}^{\alpha_2} \cos\alpha\, d\alpha\,\vec{a}_\varphi$$

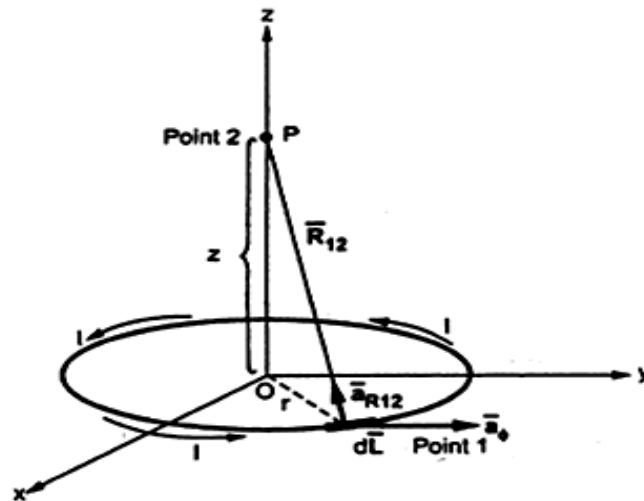$$= \frac{I}{4\pi r}[sin\alpha]_{\alpha_1}^{\alpha_2}\vec{a}_\varphi$$

$$\vec{H} = \frac{I}{4\pi r}[sin\alpha_2 - sin\alpha_1]\vec{a}_\varphi A/m$$

$$\boxed{\vec{B} = \mu\vec{H} = \frac{\mu I}{4\pi r}[sin\alpha_2 - sin\alpha_1]\,\vec{a}_\varphi\ Wb/m^2}$$
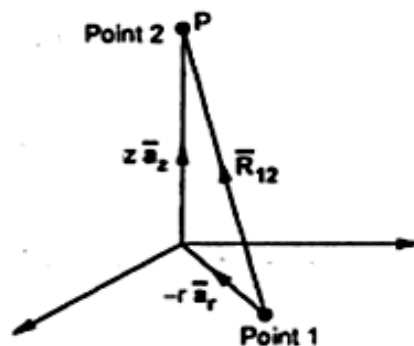
4. Obtain the expression for magnetic field intensity on the axis of a circular ring.

(Or)

$\vec{H}$ **on the axis of a circular loop.[CO3-H2-Nov/Dec   2013, Nov/Dec 2010 & April/May 2015]**

Consider a circular loop carrying a direct current I, placed in 'xy' plane. The magnetic field intensity $\vec{H}$ at point P is to be obtained. The point P is at a distance 'z' from the plane of the circular loop along its axis.

In cylindrical co-ordinate system,

$$d\vec{L} = dr\vec{a}_r + rd\vec{a}_\varphi + dz\vec{a}_z$$

The Id$\vec{L}$ is tangential at a point in $\vec{a}_\varphi$ direction.

$$Id\vec{L} = I\,rd\varphi\vec{a}_\varphi$$

The distance vector is

$$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$$
$$= (z\vec{a}_z) + (-r\vec{a}_r)$$
$$= -r\vec{a}_r + z\vec{a}_z$$

The magnitude is given by,

$$\left|\vec{R}_{12}\right| = \sqrt{(-r)^2 + (z)^2}$$

The unit vector is given by,

$$\vec{a}_{R12} = \frac{\vec{R}_{12}}{\left|\vec{R}_{12}\right|}$$

$$\vec{a}_{R12} = \frac{-r\vec{a}_r + z\vec{a}_z}{\sqrt{r^2 + z^2}}$$

$$d\vec{L} \times \vec{a}_{R12} = \begin{vmatrix} \vec{a}_r & \vec{a}_\varphi & \vec{a}_z \\ 0 & rd\varphi & 0 \\ \frac{-r}{\sqrt{r^2+z^2}} & 0 & \frac{z}{\sqrt{r^2+z^2}} \end{vmatrix}$$

$$= \vec{a}_r \left[r\, d\varphi\, \frac{z}{\sqrt{r^2+z^2}} - 0 \right] - \vec{a}_\varphi [0 - 0] + \vec{a}_z \left[0 + \frac{r^2 d\varphi}{\sqrt{r^2+z^2}} \right]$$

$$= \frac{zr\, d\varphi\, \vec{a}_r}{\sqrt{r^2+z^2}} + \frac{r^2 d\varphi\, \vec{a}_z}{\sqrt{r^2+z^2}}$$

$$= \frac{1}{\sqrt{r^2+z^2}}\left[zrd\varphi\vec{a}_r + r^2 d\varphi\vec{a}_z \right]$$

According to Biot-savart Law

$$d\vec{H} = \frac{Id\vec{L} \times \vec{a}_{R12}}{4\pi R_{12}^2}$$

$$= \frac{I\left[zrd\,\varphi\vec{a}_r + r^2 d\varphi\vec{a}_z \right]}{4\pi\left(\sqrt{r^2+z^2}\right)^2 \sqrt{r^2+z^2}}$$

The total $\vec{H}$ is to be obtained by integrating $d\vec{H}$ over the circular loop with $\varphi = 0$ to 2π

$$\vec{H} = \int_0^{2\pi} \frac{I[zr\,\vec{a}_r + r^2\vec{a}_z]d\varphi}{4\pi(r^2+z^2)^{3/2}}$$

$$= \frac{I}{4\pi}\left\{ \int_0^{2\pi} \frac{zrd\,\varphi}{(r^2+z^2)^{3/2}}\vec{a}_r + \int_0^{2\pi} \frac{r^2\,d\varphi}{(r^2+z^2)^{3/2}}\vec{a}_z \right\}$$

Considering the first integral and expressing the unit vector $\vec{a}_r$ in rectangular co-ordinate system,

$$\int_0^{2\pi} \frac{zrd\varphi}{(r^2+z^2)^{\frac{3}{2}}}\vec{a}_r = \int_0^{2\pi} \frac{zrd\varphi}{(r^2+z^2)^{3/2}}[cos\varphi\vec{a}_x + sin\varphi\vec{a}_y$$

$$\int_0^{2\pi} cos\varphi d\varphi = [sin\varphi]_0^{2\pi} = sin2\pi - sin0 = 0$$

$$\int_0^{2\pi} sin\varphi d\varphi = [-cos\varphi]_0^{2\pi} = -cos2\pi[-cos0] = 0$$

$$\int_0^{2\pi} \frac{zrd\varphi}{(r^2+z^2)^{3/2}} \vec{a}_r = 0$$

This proves that $\vec{H}$ at P cannot have any radial component.

$$\therefore H = \frac{I}{4\pi} \int_0^{2\pi} \frac{r^2 \, d\varphi}{(r^2 + z^2)^{\frac{3}{2}}} \vec{a}_z$$

$$= \frac{Ir^2\vec{a}_z}{4\pi(r^2 + z^2)^{3/2}} \int_0^{2\pi} d\varphi$$

$$= \frac{Ir^2\vec{a}_z}{4\pi(r^2+z^2)^{\frac{3}{2}}} \cdot [\varphi]_0^{2\pi}$$

$$= \frac{Ir^2\vec{a}_z[2\pi]}{4\pi(r^2 + z^2)^{3/2}}$$

$$\boxed{\vec{H} = \frac{Ir^2}{2(r^2+z^2)^{3/2}}\vec{a}_z \text{ A/m}}$$

Where r = radius of the circular loop

Z = distance of point P along the x axis

If the point P is shifted to the center of the circular loop, then height z = 0

$$\vec{H} = \frac{Ir^2}{2(r^2)3/2}\vec{a}_z$$

$$= \frac{Ir^2}{2r^3}\vec{a}_z$$

$$\boxed{\vec{H} = \frac{I}{2r}\vec{a}_z\text{A/m}}$$

W.K.T., $\vec{B} = \mu\vec{H}$

Therefore,

$$\boxed{\vec{B} = \frac{\mu I}{2r}\vec{a}_z \quad Wb/m^2}$$

5. **State and prove Ampere's circuital law.[CO3-L1-Nov / Dec 2006]**

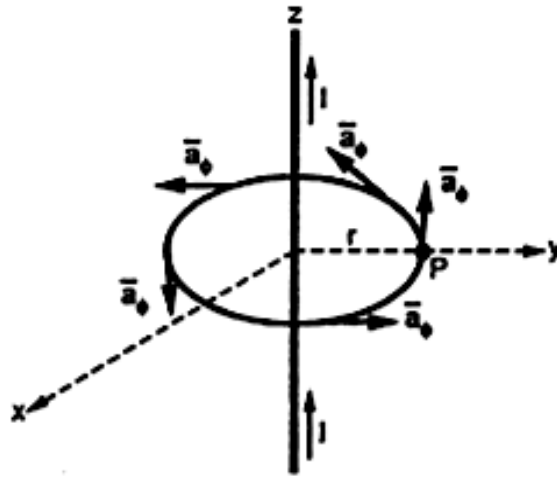   **Ampere's circuital law:**

   Ampere's circuital law is used in the case of <u>magnetic field intensity</u>.

Definition:

   It states that the line integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by the path.

$$\oint \vec{H}.d\vec{L} = I$$

Proof of Ampere's circuital law:

       Consider a long straight conductor carrying direct current I placed along z axis as show in figure.

       Consider a closed circular path of radius 'r' which encloses the straight conductor carrying direct current I. The point P is at perpendicular distance 'r' from the conductor. Consider differential element $d\vec{L}$ at point P which is in $\vec{a}_\varphi$ direction tangential to circular path at point P.

$$\vec{dL} = rd\varphi\vec{a}_\varphi \text{------------------------(1)}$$

While $\vec{H}$ obtained at point P, from Biot-savart law due to infinitely long conductor is

$$\vec{H} = \frac{I}{2\pi r}\vec{a}_\varphi \text{------------(2)}$$

Taking dot product of equation (1) and (2), we get

$$\vec{H}.d\vec{L} = \frac{I}{2\pi r}\vec{a}_\varphi.\ rd\varphi.\ \vec{a}_\varphi$$
$$= \frac{I}{2\pi r}rd\varphi.$$
$$= \frac{I}{2\pi}d\varphi$$

This proves that the integral $\vec{H}.d\vec{L}$ along the closed path gives the direct current enclosed by the closed path.

$$\oint \vec{H}.\vec{dl} = \int_{\varphi=0}^{2\pi} \frac{I}{2\pi}\ d\varphi$$

$$\oint \vec{H}.\vec{dl} = \frac{I}{2\pi}\int_{\varphi=0}^{2\pi}d\varphi$$

$$= \frac{I}{2\pi}[\varphi]_0^{2\pi}$$

$$= \frac{I[2\pi]}{2\pi} \quad = I$$

$$\boxed{\oint \vec{H}.d\vec{L} = I}$$

Hence the ampere's circuital law is proved.

<u>Applications</u>

    i)Due to infinitely long straight conductor along z- axis carrying I.

    (ii) Due to conductor of finite length.

    (iii) At the centre of circular conductor.

    (iv) On the axis of a circular loop, at a distance x.

6. Determine $\vec{H}$ due to infinitely long straight conductor using Ampere's Law. [CO3-H1-Apr/May 2010]
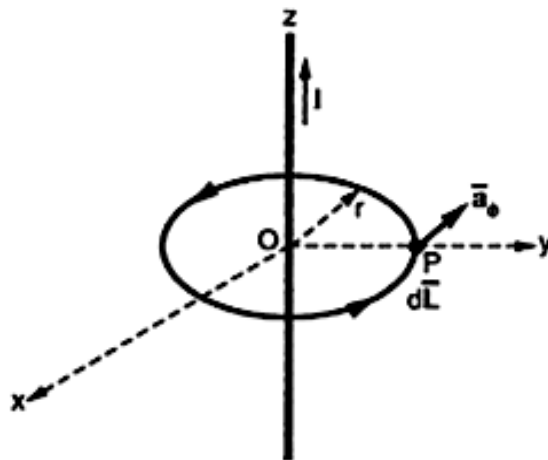
Consider an infinite long straight conductor placed along 'z' axis carrying a direct current I as shown in figure. Consider point 'P' on the closed path at which $\vec{H}$ is to be obtained.

The magnitude of $\vec{H}$ depends on 'r' and the direction is always tangential to the closed path and it has the only component $\vec{a}_\varphi$.

$$\vec{H} = H_\varphi \vec{a}_\varphi,$$

$d\vec{L}$ is given as,

$$d\vec{L} = r\, d\varphi\, \vec{a}_\varphi$$

Now, $\vec{H}.d\vec{L} = H_\varphi \vec{a}_\varphi . r\, d\varphi \vec{a}_\varphi$

$$= H_\varphi\, r\, d\varphi \qquad \text{Since}[\vec{a}_\varphi . \vec{a}_\varphi = 1]$$

According to Ampere's circuital law,

$$\oint \vec{H}.d\vec{L} = I$$

$$\int_{\emptyset=0}^{2\pi} H_\varphi r d\varphi = I$$

$$H_\emptyset r \int_{\varphi=0}^{2\pi} d\varphi = I$$

$$H_\varphi r(2\pi) = I$$
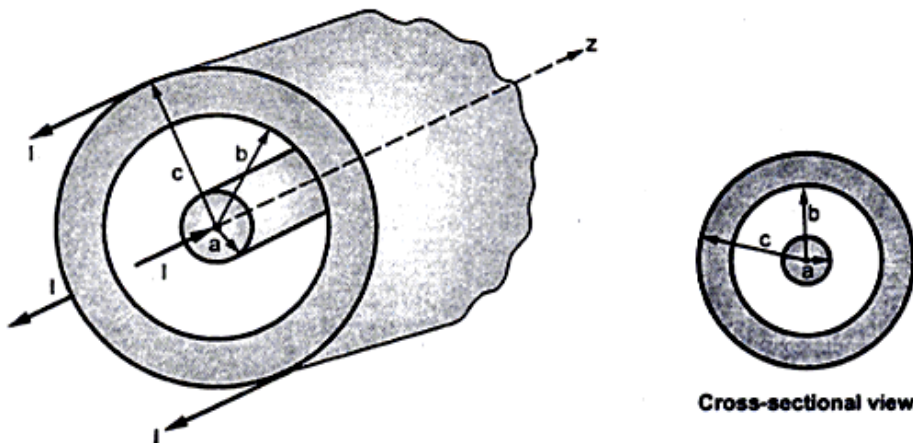
$$H_\varphi = \frac{I}{2\pi r}$$

Sub.H_φ in $\vec{H}$

$$\vec{H} = H_\varphi \vec{a}_\varphi$$

$$\boxed{\vec{H} = \frac{I}{2\pi r}\vec{a}_\varphi}$$

Hence $\vec{H}$ due to infinitely long straight conductor is obtained using ampere's circuital law.

7. Derive $\vec{H}$ due to a Co-axial Cable.(Application of ampere's circuital law)[CO3-H2]

Cross-sectional view

- Consider a co-axial cable as shown in figure. Its inner conductor is solid with radius a, carrying direct current I.

- The outer conductor is in the form of concentric cylinder whose inner radius is b and outer radius is c. This cable is placed along z axis. The current I is uniformly distributed in the inner conductor. While – I is uniformly distributed in outer conductor.

- The space between inner and outer conductor is filled with dielectric say air. The calculation of $\vec{H}$ is divided corresponding to various regions of the cable.

Region 1: Within the inner conductor, $r < a$

- Consider a closed path having radius $r < a$. As in the figure it encloses only part of the conductor. The area of cross section enclosed is $\pi r^2 m^2$. The total current flowing is I through the area πa^2. Hence the current enclosed by closed path is

$$I' = \frac{\pi r^2}{\pi a^2} I = \frac{r^2}{a^2} I$$

The $\vec{H}$ is again only in $\vec{a}_\varphi$ direction and depends only on 'r'.

$$\vec{H} = H_\varphi \vec{a}_\varphi$$

So consider $d\vec{L}$ in the $\vec{a}_\varphi$ direction which is r $d\varphi$

$$d\vec{L} = \text{r}d\varphi\vec{a}_\varphi$$

$$\vec{H}.d\vec{L} = H_\varphi \vec{a}_\varphi . rd\varphi\vec{a}_\varphi = H_\varphi rd\varphi$$

According to Amperes circuital law,

$$\oint \vec{H}.d\vec{L} = I'$$
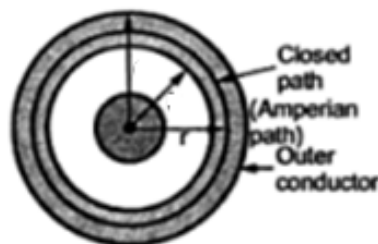$$\oint H_\varphi \, rd\varphi = \frac{r^2}{a^2}I$$
$$\int_{\varphi=0}^{2\pi} H_\varphi \, rd\varphi = \frac{r^2}{a^2}I$$
$$H_\varphi r[2\pi] = \frac{r^2}{a^2}I$$
$$H_\varphi = \frac{r^2}{2\pi r a^2}I$$
$$\boldsymbol{\vec{H} = \frac{Ir}{2\pi a^2}\vec{a}_\varphi \text{ A/m}}$$
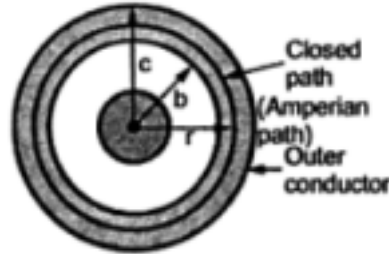
Region 2: Within $a < r < b$

- Consider a circular path which encloses the inner conductor carrying direct current I. this case of infinitely long conductor along z axis. Hence $\vec{H}$ in this region is

$$\vec{H} = \frac{I}{2\pi r}\vec{a}_\varphi \text{ A/m}$$

Region 3: Within outer conductor $b < r < c$

- Consider the closed path .the current enclosed by the closed path is only part of the current −I, in the outer conductor. The total current −I is flowing through the cross section $\pi(c^2 - b^2)$ while the closed path encloses the cross section $\pi(r^2 - b^2)$.
- Hence the current enclosed by the closed path of outer conductor is

$$I'' = \frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)}(-I) = -\frac{(r^2 - b^2)}{(c^2 - b^2)}I$$

Where, I'' = Current in the inner conductor enclosed
- Total current enclosed by the closed path is,

$$I_{enc} = I' + I'' = -\frac{(r^2 - b^2)}{(c^2 - b^2)}I + I$$

$$= I\left[1 - \frac{(r^2 - b^2)}{(c^2 - b^2)}\right] = I\left[\frac{c^2 - r^2}{c^2 - b^2}\right]$$

According to Amperes circuital law,

$$\oint H.dL = I_{enc}$$

Now $\vec{H}$ is again in $\vec{a}_\varphi$ direction only and is a function of 'r' only

$$\vec{H}.d\vec{L} = H_\varphi \vec{a}_\varphi . \text{r}d\varphi\vec{a}_\varphi = H_\varphi \text{ r } d\varphi$$

$$\int_{\varphi=0}^{2\pi} H_\varphi \, r d\varphi = I_{enc}$$

$$H_\varphi r\lfloor 2\pi \rfloor = I\left[\frac{c^2 - r^2}{c^2 - b^2}\right]$$

$$H_\varphi = \frac{I}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right]$$

$$\vec{H} = H_\varphi \vec{a}_\varphi = \frac{I}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right]\vec{a}_\varphi \, ^A/_m$$

Region 4: Outside the cable r > c
- Consider the closed path r > c such that it encloses both the conductors.
- The total current enclosed is

$$I_{enc} = +I - I = 0A$$

$$\oint H.dL = 0$$
$$\vec{H} = 0 \quad A/m$$

Thus magnetic field does not exist outside the cable.

Variation of $\overline{H}$ against r in co-axial cable

8. **For an infinite current sheet of uniform current density 'K' A/m, derive the expression for the magnetic field intensity. (Application of ampere's circuital law) [CO3-H2]**

- Consider an infinite sheet of charge is placed in xy plane and the coil is marked as 1,2,3,4
- The total current of the sheet is

$$I = k_y b$$

For loop 2-3:

$$\vec{H} = -H_x \vec{a}_x$$
$$d\vec{L} = dx\vec{a}_x$$

According to ampere's circuit law,

$$\int_2^3 \vec{H}\, d\vec{L} = \int_2^3 (-H_x \overrightarrow{a_x}).(dx\overrightarrow{a_x})$$

$$= -H_x \int_2^3 dx$$

$$= -H_x\, b$$

For loop 1-4:

$$\vec{H} = H_x\, \vec{a}_x$$

$$d\vec{L} = dx\overrightarrow{a_x}$$

According to ampere's circuit law,

$$\int_4^1 \vec{H}.d\vec{L} = \int_4^1 (H_x a_x).(dx\overrightarrow{a_x})$$

$$= H_x \int_4^1 dx$$

$$= H_x b$$

Total magnetic field is given as,

Total Magnetic Field = loop 2-3 + loop 1-4

$$= H_x b + H_x b$$

Total Magnetic Field = $2\, H_x b$

According to ampere's circuital law,

$$\oint \vec{H}.\, d\vec{L} = I$$

$$2\, H_x b = k_y\, b$$

$$H_x = \frac{1}{2} k_y$$

Therefore,

$$\boxed{\vec{H} = \frac{1}{2} k_y \vec{a}_x}$$

9.What is stokes theorem? Derive and prove it using a closed surface S.[CO3-L1]
STOKE'S THEOREM:
The stoke's theorem relates the line integral to a surface integral. It states that The line integral of vector H around a closed path L is equal to the integral of curl of vector H over the open surface S enclosed by the closed path L. Mathematically it is expressed as,

$$\oint_L \vec{H}.d\vec{L} = \iint_S (\nabla \times \vec{H}).\, d\vec{S}$$

PROOF OF STOKE'S THEOREM:
- Consider a surface S which is splitted into number of incremental surfaces
- Each incremental surface is having area Δs as shown in the figure.

(a)   (b)

- Applying definition of the curl to any of these incremental surfaces we can write,

$$(\nabla \times \overline{H})_N = \frac{\oint \overline{H} \cdot d\overline{L}_{\Delta S}}{\Delta S}$$

N = Normal to Δ S according to right hand rule

dL$_{\Delta S}$ = Perimeter of the incremental surface Δ S

- Now the curl of H in the normal direction is the dot product of curl of H with $\overline{a}_N$ normal to the surface Δs, according to right hand rule

$$\therefore \quad (\nabla \times \overline{H})_N = (\nabla \times \overline{H}) \cdot \overline{a}_N$$
$$\therefore \quad \oint \overline{H} \cdot dL_{\Delta S} = (\nabla \times \overline{H}) \cdot \overline{a}_N \, \Delta S$$
$$\therefore \quad \oint \overline{H} \cdot dL_{\Delta S} = (\nabla \times \overline{H}) \cdot \Delta \overline{S}$$

- To obtain total curl of every incremental surface add the closed line integrals for each Δs.
- From fig B it can be seen that at a common boundary between the two incremental surfaces the line integral is getting cancelled as the boundary is getting traced in two opposite directions.
- The summation of all closed line integrals for each and every Δs ends up in a single closed line integral for the outer boundary of total surface S.

$$\oint_L \vec{H} \cdot d\vec{L} = \iint_S (\nabla \times \vec{H}) \cdot dS$$

10. Explain Magnetic flux and Magnetic flux density. [CO3-L1]

Magnetic flux

The total number of lines of force existing in a particular magnetic field is called magnetic flux which is denoted as ϕ.

Magnetic flux density

The magnetic flux in weber passing through unit area in a plane at right angles to the direction of flux is called magnetic flux density.

It is denoted as $\vec{B}$ is a vector quantity.

$$\vec{B} = \frac{\varphi}{S} \quad \text{Wb/m}^2 \text{ (or) tesla (T)}$$

Relation between Magnetic Flux Density and Magnetic Field Intensity?

The magnetic flux density B is related to the magnetic field intensity H through the permeability of the medium (μ).

$$\vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H}$$

Where, μ_0 = permeability of free space = $4\pi \times 10^{-7}$, μ_r = Relative permeability.

If the

If the flux passing through the unit area is not exactly at right angles to the plane consisting the area but making some angle with the plane then the flux ϕ crossing the area is given by,

$$\phi = \int_S \overline{B} \cdot d\overline{S} \quad \text{webers (Wb)}$$

where
ϕ = Magnetic flux in webers

$\overline{B}$ = Magnetic flux density in Wb/m² or Tesla (T)

$d\overline{S}$ = Open surface through which flux is passing.

The integral of $\overline{B}\,\overline{ds}$ evaluated over a closed surface is always zero.

$$\oint_S \overline{B} \cdot d\overline{S} = 0$$

This is called law of conservation of magnetic flux or Gauss,s law in integral form for magnetic fields.

Applying divergence theorem

$$\oint_S \overline{B} \cdot d\overline{S} = \int_{vol} \nabla \cdot \overline{B}\, dv = 0$$

Where dv = Volume enclosed by the closed surface

But as dv is not aero , we can write ,

$$\nabla \cdot \overline{B} = 0$$

11.What is scalar magnetic potential? Also derive an expression for vector magnetic potential. [CO3-L1-Nov/Dec 2010]

Magnetic scalar and vector potentials:

In electrostatics it is seen that there exist a scalar electric potential v which is related to the electric field intensity $\vec{E}$ as $\vec{E} = -\Delta V$

In case of magnetic fields there are two types of potential which can be defined as
> 1. The scalar magnetic potential denoted as V_m
> 2. The vector potential denoted as $\bar{A}$

To define scalar and vector potential, let us consider two vector identities which are listed as properties of curl

$$\nabla \times \nabla V = 0, V = \text{Scalar}\text{-----------}(1)$$

$$\nabla . \left(\nabla \times \vec{A} \right) = 0, \overrightarrow{A} = \text{vector}\text{-------------}(2)$$

Scalar magnetic potential:

The scalar magnetic potential can be written as,

$$\vec{H} = -\nabla V_m \quad for\ J = 0$$

Where, $\vec{H} = \text{Magnetic field}$

$$V_m = \text{Magnetic scalar potential}$$

$$J = \text{Current density}$$

Proof:

If V_m is the scalar magnetic potential, then it must satisfy

$$\nabla \times \nabla V_m = 0 \text{ -----------}(3)$$

But scalar magnetic potential is related to the magnetic field intensity $\bar{H}$

$$\vec{H} = -\nabla V_m \text{-----------}(4)$$

Sub. (4) in (3)

$$\nabla \times \left(-\vec{H} \right) = 0\text{-----------}(5)$$

By ampere's circuital law in differential form,

$$\nabla \times \vec{H} = \vec{J}\text{-----------}(6)$$

Compare (5) & (6),we get $J = 0$

From this we can conclude that $\vec{H} = -\nabla V_m$ is possible only for $J = 0$

Laplace's equation for scalar magnetic potential

From the gauss law in differential form,

$$(\nabla . \vec{B}) = 0 \text{ ------------}(7)$$

W.K.T., $\vec{B} = \mu_0 \vec{H}$ -----------(8)

Sub. (8) in (7),

$$\nabla . \left(\mu_0 \vec{H} \right) = 0 \qquad \text{Here } \mu_0 \neq 0 (magnetic\ permitivity)$$

$$(\nabla . \vec{H}) = 0$$

According to scalar magnetic potential,

$$\nabla. -(\nabla V_m) = 0$$

$$-(\nabla.\nabla V_m) = 0 \qquad\qquad Since \ [\nabla.\nabla = \nabla^2]$$

$$\boxed{\nabla^2 V_m = 0}$$

This is the expression for magnetic scalar magnetic potential in Laplace form.

Vector Magnetic Potential

The vector magnetic potential is denoted as $\vec{A}$ and measured in Wb/m.

If $\vec{A}$ is a vector, then it should satisfy

$$\nabla. (\nabla \times \vec{A}) = 0\text{--------------------(1)}$$

From Gauss law in differential form,

$$(\nabla.\vec{B}) = 0\text{--------------------(2)}$$

Comparing (1) & (2) We get,

$$\vec{B} = \nabla \times \vec{A}$$

According to ampere's circuital law in differential form,

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \times \frac{\vec{B}}{\mu_0} = \vec{J} \qquad\qquad Since, \vec{H} = \frac{\vec{B}}{\mu_0}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \times \nabla \times \vec{A} = \mu_0 \vec{J}\text{--------------------(3)}$$

According to vector triple product formula,

$$\nabla \times \nabla \times \vec{A} = \nabla(\nabla.\vec{A}) - \nabla^2 \vec{A}\text{--------------------(4)}$$

Sub (4) in (3)

$$\nabla(\nabla.\vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J}$$

$$\vec{J} = \frac{1}{\mu_0}[\nabla \times \nabla \times \vec{A}] = \frac{1}{\mu_0}[\nabla(\nabla.\vec{A}) - \nabla^2 \vec{A}]$$

The above equation is called as vector magnetic potential. It is also used to calculate current density J.

Poisson's equation for magnetic field:

In vector algebra, a vector can be fully defined if its curl and divergence are defined.

For a vector magnetic potential $\vec{A}$ is curl defined $\nabla \times \vec{H} = \vec{B}$ which is known.

But to completely define $\vec{A}$ its divergence must be known assume that $\nabla \times A,$ the divergence of $\vec{A}$ is zero.

$$\therefore \vec{J} = \frac{1}{\mu_0}\left[-\nabla^2\vec{A}\right]$$

$$\nabla^2\vec{A} = -\mu_0\vec{J}$$

12.If the vector magnetic potential is given by $= \frac{10}{x^2+y^2+z^2}a_x$ **, obtain the magnetic flux density in vector form. [CO3-H2]**

The magnetic flux density is obtained as,

$$= \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \dfrac{10}{x^2+y^2+z^2} & 0 & 0 \end{vmatrix}$$

$$= \vec{a}_x(0-0) - \vec{a}_y\left[0 - \frac{\partial}{\partial z}\left(\frac{10}{x^2+y^2+z^2}\right)\right] + \vec{a}_z\left[0 - \frac{\partial}{\partial y}\left(\frac{10}{x^2+y^2+z^2}\right)\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{10}{x^2+y^2+z^2}\right]\vec{a}_y - \frac{\partial}{\partial y}\left[\frac{10}{x^2+y^2+z^2}\right]\vec{a}_z$$

$$= 10\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1}\vec{a}_y - 10\frac{\partial}{\partial y}(x^2+y^2+z^2)^{-1}\vec{a}_z$$

$$= 10(-1)(x^2+y^2+z^2)^{-2}(2z)\vec{a}_y - 10(-1)(x^2+y^2+z^2)^{-2}(2y)\vec{a}_z$$

$$= \frac{-20z}{(x^2+y^2+z^2)^2}\vec{a}_y + \frac{20y}{(x^2+y^2+z^2)^2}\vec{a}_z$$

Unit – IV

Magnetic forces and materials

Part A

1.What is meant by electric force? Write expression of it.[CO4-L1]

Electric force $(\bar{F}_e)$
The electric force exerts on static or a moving charge.
It is given as,

$$\vec{F}_e = Q\vec{E}$$

2.Define magnetic force. [CO4-L1]

 <u>**Magnetic force**</u> $(\bar{F}_m)$

 The magnetic force exerts only moving charge with a velocity $\bar{V}$ in steady magnetic field $\bar{B}$.

 It is given as,

 $\vec{F}_m = Q\vec{v} \times \vec{B}$

3. Write Lorentz force equation and mention its applications.[CO4-L1-May/June 2007,Nov/Dec 2007 & May/June 2013]

 The Lorentz force equation is given by,

$$\vec{F} = m\vec{a} = Q[\vec{E} + \vec{V} \times \vec{B}]$$

 This equation relates the unidirectional force with electrical force. The Lorentz force equation is generally used to determine proton paths in cyclotron. It is extensively used in determining plasma characteristics in a magnetic hydrodynamic generator (MHD generator).

 In general, a motion of charged particle in a combined electric and magnetic fields can be determined using Lorentz force equation.

4.Difference between electric and magnetic force. [CO4-H1]

Electric force	Magnetic force
It can exert on either static or moving charge	It exerts only on a charge moving with certain velocity
It is always in the direction of an electric field	It is always perpendicular to the magnetic field
It performs work while displacing a point charge.	It cannot perform any work while displacing a point charge.

5.Define Torque and write its expression. [CO4-L1-April/May 2015]

 The moment of force (or) torque about a specified point is defined as the vector product of the moment arm $\vec{R}$ and the vector $\vec{F}$. It is measured in newton meter.

 It is represented mathematically as,

$$\vec{T} = \vec{R} \times \vec{F} \text{ (Nm)}$$

6. What do you mean by magnetization? [CO4-L1]

 The field produced due to movement of bound charger is called magnetization.

 it is represented by M

7. Classify the magnetic material? [CO4-L1-NOV/DEC2008]

The magnetic material are
(i)diamagnetic   (vi)anti-feromagnetic
(ii)ferromangetic   (v)supermagnetic
(iii)paramagnetic

8. Define diamagnetic materials. [CO4-L1]

- The magnetic material in which the orbital magnetic moment and electron spin moment cancel each other making net magnet power zero this is called Diamagnetic material.
    Examples: Bismuth,lead,copper.

9. Define paramagnetic material. [CO4-L1]

- The magnetic material in which the orbital magnetic moment and electron spin moment do not cancel each other resulting in a net magnetic moment is called paramagnetic material.
    Examples: Potassium and Tungsten

10. Define Ferromagnetic material. [CO4-L1]

- The magnetic material in which a large dipole moment is produced due to electron spin is known as ferromagnetic material.
- The atoms line up their magnetic dipoles in parallel fashion in a lattice manner.
- The region in which the large dipoles line up in parallel is called as domain.
- The region in which the small moments is present is called residual field and this process is called hysteresis.
        Examples:Iron ,Nickel and cobalt.

11. Define Antiferromagnetic materials. [CO4-L1]

- The materials in which the dipoles moment of adjacent atoms line up in antiparallel fashion and net magnetic moment is zero is called anti ferromagnetic material.
            Examples: Oxides and chlorides

12. Define Ferrimagnetic materials. [CO4-L1]

- The materials in which the dipoles moment of adjacent atoms line up in antiparallel fashion and net magnetic moment is non zero is called ferrimagnetic material.
            Examples: Nickel ferrite

13. Define Supermagnetic materials. [CO4-L1]

- It has large magnetic domains and have high magnetic material.

Examples: Magnetic tapes for audio and video.

14.Define dielectric strength of material given its unit. [CO4-L1-Arp/May2010]

The minimum value of the applied electric field at which the dielectric breakdown is called dielectric strength of the material

its unit is V/M

15.Define inductance. [CO4-L1-Nov/Dec 2012]

The ratio of total flux linkage to the current producing that flux is called inductance. It is measured in Henry.

16. Define self inductance& mutual inductance. [CO4-L1]

Self inductance:

The ratio of total flux linkage to the current producing that flux is called inductance .

it is denoted by L,

it is measured in henry(H),

Mutual inductance:

With respect to the flux linkage ,the mutual inductance between the two coils is defined as the ratio of the flux linkage of one coil to the current in other coil

For linear medium   M12=M21=M

17.What do you mean magnetic moment? [CO4-L1-May/June 2013]

The magnetic dipole moment of a current loop is defined as the product of current through the loop and the area of the loop, directed normal to the current loop.
It is given by,

$$\vec{m} = (IS)\vec{a_n} \ (A.m^2)$$

Using definition for magnetic dipole moment, the torque can be expressed as,

$$\vec{T} = \vec{m} \times \vec{B} \ (N.m)$$

18.Find maximum torque on an 85 turn rectangular coil 0.2 m by 0.3 m carrying current of 2A in a field of B = 0.5 tesla. [CO4-H1]

Given:

n= 85

I = 2A

B = 0.5 tesla

To find:

Maximum torque, $T_{max} = mB$ = ?

Solution:

For a single turn of a rectangular coil, the total area is given by,

$$S' = (0.2)(0.3) = 0.06\ m^2$$

Therefore for 85 turn rectangular coil, the total area is given by,

$$S = nS'$$

$$S = 85\ (0.06)$$

$$S = 5.1\ m^2$$

The magnitude of maximum torque is given by,

$$T_{max} = mB = 10.2\ \times 0.5 = 5.1\ Nm$$

19.Mention the importance of Lorentz force equation. [CO4-L1-May/June 2007 & Nov/Dec 2007]

Whenever a charged particle passes through a region having electric field and magnetic field acting simultaneously, the particle will feel two forces acting on it.

- Force due to **electric field**→it is a velocity independent force and is given by,

$$\vec{F}_e = q\vec{E}$$

It acts in the direction of the field.

- Force due to **magneticfield**→It is the velocity dependent force and is given by

$$\vec{F}_m = q(\vec{V} \times \vec{B})$$

It acts in the direction perpendicular to both $\vec{V} \& \vec{B}$.

20. A long conductor with the current 5A is in coincidence with positive 'z' direction. If $\vec{B} = 4i + 5j$. Find the force per unit length. [CO4-H1-Apr/may 2008]
Solution:

$$F = BIl\ sin\theta$$

$$\frac{F}{l} = BI\ sin\theta$$

$$F_2/_l = \vec{I} \times \vec{B}$$

$$= 5\vec{k} \times (4\vec{i} + 5\vec{j})$$

$$= 20\vec{j} + 25\vec{i})\ \text{N/m}$$

21. What is solenoid? [CO4-L1]

Solenoid is cylindrical in shape which consists of a large number of closely spaced turns of insulated wire wound usually on a non-magnetic frame.

22. What is the relationship between magnetic flux density , magnetic field intensity and magnetization? [CO4-L1-April/May 2015]

The relationship between magnetic flux density $\vec{B}$, magnetic field intensity $\vec{H}$ and magnetization $\vec{M}$ is given by,

$$\vec{B} = \mu(\vec{H} + \vec{M})$$

23. State the boundary condition of magnetic media. [CO4-L1]

The normal component of magnetic flux density $\vec{B}$ is continous at boundary between two magnetic media

$$\therefore \qquad \frac{B_{tan1}}{\mu_1} = \frac{B_{tan2}}{\mu_2}$$

The tangential component of magnetic field intensity $\vec{H}$ is continous at boundary between two magnetic media

$$\therefore \qquad \boxed{\begin{aligned} \mu_1 H_{N_1} &= \mu_2 H_{N_2} \\ \frac{H_{N_1}}{H_{N_2}} &= \frac{\mu_2}{\mu_1} = \frac{\mu_{r2}}{\mu_{r1}} \end{aligned}} \qquad \text{... (8)}$$

Part – B

1. Explain in detail about force on a moving point charge. [CO4-L1]

<u>Force on a moving point charge:</u>

There are two types of force exerted on a charge

i)   Electric force $(\vec{F}_e)$

ii)   Magnetic force $(\vec{F}_m)$

<u>Electric force $(\vec{F}_e)$</u>

The electric force exerts on static or a moving charge.
It is given as,

$$\vec{F}_e = q\vec{E} \text{----- (1)}$$

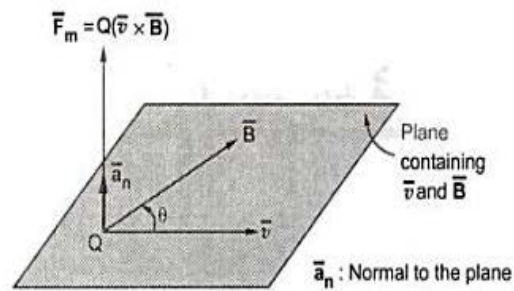**<u>Magnetic force $(\vec{F}_m)$</u>**

The magnetic force exerts only moving charge with a velocity $\vec{V}$ in steady magnetic field $\vec{B}$.

It is given as,

$$\vec{F}_m = q(\vec{V} \times \vec{B}) \text{------ (2)}$$

The magnitude of the magnetic force is directly propotional to the magnitudes of Q, $\bar{v}$ and $\bar{B}$ and also the sine of the angle between $\bar{v}$ and $\bar{B}$.

$$\vec{F}_m = Q(\bar{v} \times \overline{B})$$

B

$\bar{a}_n$

θ

Q

$\bar{v}$

Plane containing $\bar{v}$ and $\overline{B}$

$\bar{a}_n$: Normal to the plane

Total force

The total force on the charge is given by,

$$\vec{F} = \vec{F}_e + \vec{F}_m$$
$$= q \, (\vec{E} + \vec{v} \times \vec{B})$$

2. Explain in detail about Force on a differential current element. [CO4-L1]
Force on a differential current element:

The force exerted on differential element dQ moving in a steady magnetic field is,

$$d\overline{F} = dQ \, \bar{v} \times \overline{B} \quad N \text{ -------(1)}$$

The differential element of the charge can be expressed in

$$dQ = \rho_v \, dv \quad \text{------ (2)}$$

We can write the above equation as ($\rho_v = \dfrac{dQ}{dV}$)

Sub equation 2 in 1 , we get

$$d\overline{F} = \rho_v \, dv \, \bar{v} \times \overline{B}$$

The current density $\bar{J}$ can be expressed interms of velocity of a volume charge density as,

$$\overline{J} = \rho_v \, \bar{v}$$

Expressing $d\overline{F}$ interms of $\overline{J}$ using equation   we can write,

$$d\overline{F} = \overline{J} \times \overline{B} \, dv$$

Integral of the above equation is,

$$\overline{F} = \int\limits_{vol} \overline{J} \times \overline{B} \, dv$$

Surface form

The force exerted on surface current density is given by,

$$d\overline{F} = \overline{K} \times \overline{B} \, dS$$

Integral of the above equation is,

$$\overline{F} = \int\limits_{S} \overline{K} \times \overline{B} \, dS$$

Similarly the force exerted on a differential current element is given by,

$$d\bar{F} = (I\,d\bar{L} \times \bar{B})$$

Integral of the above equation is,

$$\bar{F} = \oint I\,d\bar{L} \times \bar{B}$$

The magnitude of the force is given by,

$$F = I\,L\,B\,\sin\theta$$

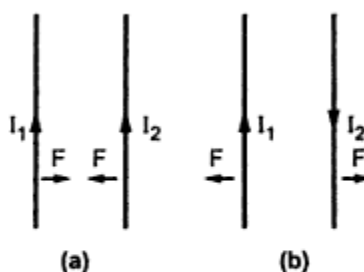## Force between Differential Current Elements

Fig.   Force between two parallel current carrying conductors

Consider two current carrying conductor are placed parallel to each other and separated by the distance R.

Each produce its own flux.

Two conductors are placed closed to each other, there exists a force due to the interaction of two fluxes.

If the directions of both the currents are same , then the conductors experience a force of attraction.

If the directions of both the currents are opposite , then the conductors experience a force of repulsion.

The point P produce flux density which is given as,

$$B = \frac{\mu_o I_1}{2\pi d} \quad \text{---------- (1)}$$

The force on Q is given as,

$$F = B\,I_2\,l \quad \text{-------- (2)}$$

Where l= length of the conductor

Sub eqn 2 in 1, we get,

$$F = \frac{\mu I_1\,I_2\,l}{2\pi d}$$

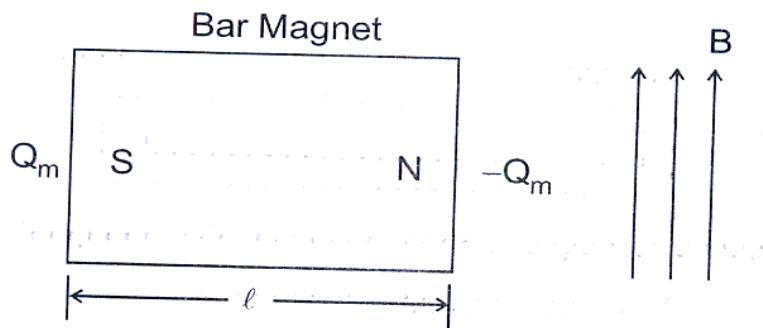This is the force of attractive conductor

if the current of the conductor is in opposite direction, the value of the force is same but repulsive.

$$\bar{F}_2 = -\bar{F}_1$$

The force of repulsive conductor,

$$F = -\frac{\mu I_1 \, I_2 l}{2\pi d}$$

3. Find the expression for torque acting on a square loop carrying a current I. [CO4-H1-Nov/Dec 2013]

Consider a loop of length l and breadth b carrying a current I in a uniform magnetic flux density B.

The force acting on the loop is

$$F = BIl \, sin\theta$$

The torque T is given as

$$T = 2 \times \text{torque on each side}$$

$$= 2 \times force \times radial \; distance$$

$$T = 2 \times BIl \, sin\theta \times \frac{b}{2}$$

$$= BIlb \, sin\theta$$

Here,

$$lb = A(Area)$$

$$T = BIA \, sin\theta$$

$$T = Bm \, sin\theta$$

Since the torque acts perpendicular to the loop,

$$T = \; Bm \; sin\theta \times \vec{n}$$

The torque can be written in vector form as,

$$\vec{T} = m \times \vec{B}$$

4. Explain various types of magnetic materials. [CO4-L1]
DIAMAGNETIC MATERIALS:
- The magnetic material in which the orbital magnetic moment and electron spin moment cancel each other making net magnet power zero this is called Diamagnetic material.

  Examples: Bismuth,lead,copper.

PARAMAGNETIC MATERIAL:
- The magnetic material in which the orbital magnetic moment and electron spin moment do not cancel each other resulting in a net magnetic moment is called paramagnetic material.

  Examples: Potassium and Tungsten

FERROMAGNETIC MATERIAL:
- The magnetic material in which a large dipole moment is produced due to electron spin is known as ferromagnetic material.
- The atoms line up their magnetic dipoles in parallel fashion in a lattice manner.
- The region in which the large dipoles line up in parallel is called as domain.
- The region in which the small moments is present is called residual field and this process is called hysteresis.

  Examples:Iron ,Nickel and cobalt.

ANTIFERROMAGNETIC MATERIALS:
- The materials in which the dipoles moment of adjacent atoms line up in antiparallel fashion and net magnetic moment is zero is called anti ferromagnetic material.

  Examples: Oxides and chlorides

FERRIMAGNETIC MATERIALS:
- The materials in which the dipoles moment of adjacent atoms line up in antiparallel fashion and net magnetic moment is non zero is called ferrimagnetic material.
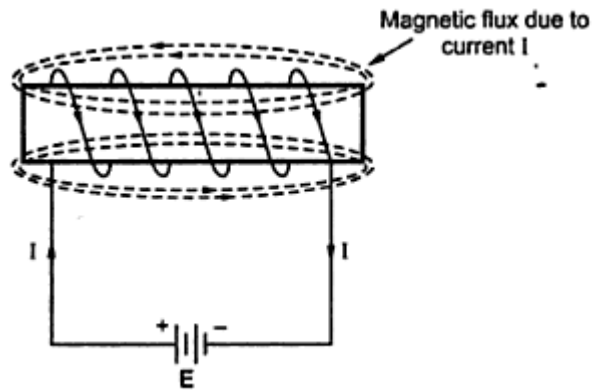
  Examples: Nickel ferrite

SUPERMAGNETIC MATERIALS:
- It has large magnetic domains and have high magnetic material.

Examples: Magnetic tapes for audio and video.

5.Write in detail about inductance. [CO4-L1]

Inductance:

- Let us take a N turns of coil.
- Current I is flowing through it.
- The flux of each turn of coil is Φ.
- The flux of total no of coil is NΦ.

Definition:

The ratio of total flux linkage to the current producing that flux is called inductance. It is measured in Henry.

$$L = \frac{N\varphi}{I} H$$

6..Derive an expression for the inductance of solenoid. [CO4-H2-Nov/Dec 2013]

INDUCTANCE OF A SOLENOID

- Consider a solenoid of N turns as shown in the figure above
- Let the current flow be I and the length of the solenoid is L and cross section is A.

   The magnetic field intensity of solenoid is

$$H = \frac{NI}{l} \, A/m \qquad\qquad \dots (2)$$

   The total flux linkage is given by

$$N\phi = N\,(B)\,(A)$$

$$= N\,(\mu H)\,(A)$$

$$= \mu\,NHA$$

$$= \mu N\left[\frac{NI}{l}\right]A$$

$$= \frac{\mu N^2 IA}{l} \qquad\qquad \text{... (3)}$$
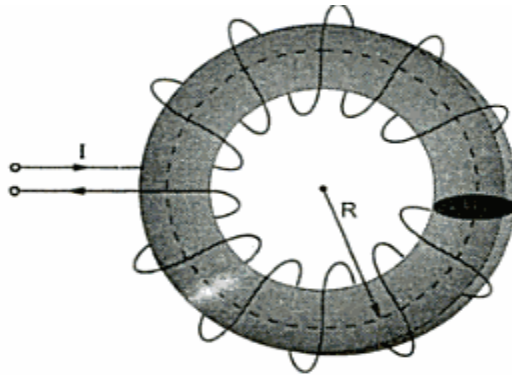
The inductance of solenoid is given by

$$L = \frac{\text{Total flux linkage}}{\text{Total current}}$$

$$\therefore \qquad L = \frac{\mu N^2 IA}{l(I)}$$

$$\therefore \qquad L = \frac{\mu N^2 A}{l}\,H \qquad\qquad \text{... (4)}$$

7. Derive the expression for inductance of a toroidal coil carrying current. [CO4-H2-Nov/Dec 2012 & April/May 2015]

INDUCTANCE OF A TOROID

The magnetic flux density of toroid is

$$B = \frac{\mu NI}{2\pi R} \qquad\qquad \text{... (5)}$$

The total flux linkage is

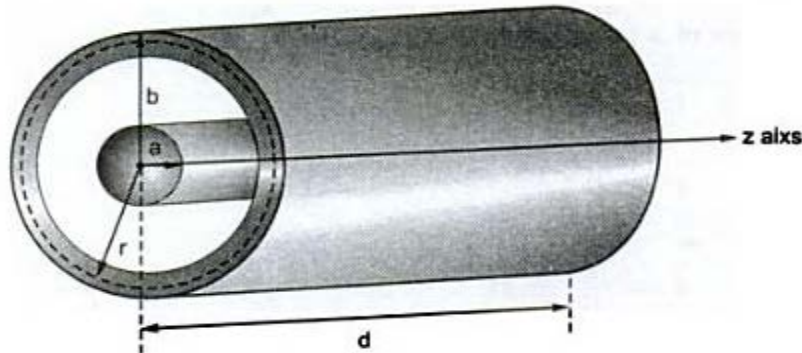$$N\,(B)\,(A) = N\left(\frac{\mu NI}{2\pi R}\right)(A) \qquad\qquad = \frac{\mu N^2 IA}{(2\pi R)}$$

The inductance of toroid is

$$L = \frac{\text{Total flux linkage}}{\text{Total current}}$$

$$\therefore \qquad L = \frac{\mu N^2 IA}{(2\pi R)(I)}$$

$$\therefore \qquad L = \frac{\mu N^2 A}{(2\pi R)}\,H \qquad\qquad \text{... (6)}$$

The inductance of toroid with 2 radius is given as

$$L = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{r_2}{r_1}\right) \qquad \qquad \text{... (7)}$$

8.Explain inductance of a coaxial cable. [CO4-L1]
INDUCTANCE OF A COAXIAL CABLE

- Consider a coaxial cable with inner radius a and outer radius b as shown in the figure.
- I is the current in coaxial cable

The field intensity of coaxial cable is

$$H = \frac{I}{2\pi r} \quad \text{where} \quad a < r < b \qquad \qquad \text{... (8)}$$

The magnetic flux is

$$B = \mu H = \frac{\mu I}{2\pi r}$$

The cable is on Z axis .The radius varies from r=a to r=b and z=0 andz=d
Since the unit vector of coaxial cable is in Φ direction

$$\overline{B} = \frac{\mu I}{2\pi r} \overline{a}_\phi \ T \qquad \qquad \text{... (9)}$$

The total magnetic flux is given by

$$\phi = \int_S \overline{B} \cdot d\overline{S}$$

Now $\qquad d\overline{S} = dr\ dz\ \overline{a}_\phi \qquad \qquad \text{... (10)}$

$$\therefore \qquad \phi = \int_{z=0}^{z=d} \int_{r=a}^{r=b} \left(\frac{\mu I}{2\pi r} \overline{a}_\phi\right) \cdot (dr\ dz \overline{a}_\phi)$$

$$\therefore \qquad \phi = \frac{\mu I}{2\pi} [z]_0^d\ [\ln r]_a^b$$

$$\therefore \qquad \phi = \frac{\mu I d}{2\pi} \ln\left(\frac{b}{a}\right) \qquad \qquad \text{... (11)}$$

The inductance of a coaxial cable is given by

$$L = \frac{\text{Total flux linkage}}{\text{Total current}}$$

$$\therefore \qquad L = \frac{\dfrac{\mu I\,d}{2\pi}\,ln\left(\dfrac{b}{a}\right)}{I}$$

$$\therefore \qquad L = \frac{\mu\,d}{2\pi}\,ln\left(\frac{b}{a}\right)\ H \qquad\qquad \dots (12)$$

The inductance of a coaxial cable per unit length is

$$\frac{L}{d} = \frac{\mu}{2\pi}\,ln\left(\frac{b}{a}\right)\ H/m \qquad\qquad \dots (13)$$

9. Explain mutual inductance. [CO4-L1]

MUTUAL INDUCTANCE

- Consider two circuits carrying two different currents of coils
- The coil 1 have N_1 turns , inductance L_1, current I_1 and flux Φ_{11}.
- The coil 2 have N_2 turns , inductance L_2, current I_2 and flux Φ_{22}.
- If the circuits are brought together , the flux from coil 1 links with coil 2 which
  produces Φ_{12}.

The flux from coil 2 links with coil 1 which produces Φ_{21}.

Circuit 1   Circuit 2

- The total flux of second coil is $N_2\Phi_{12}$.
- The total flux of first coil is $N_1\Phi_{21}$.
- Mutual inductance is given by M.
  It is given by

$$M_{12} = \frac{N_2\,\phi_{12}}{I_1}\ H \qquad\qquad \dots (14)$$

$$M_{21} = \frac{N_1\,\phi_{21}}{I_2}\ H \qquad\qquad \dots (15)$$

Definition:

The mutual inductance between two coil is defined as the ratio of flux linkage of one coil to
the current in other coil.

10.Explain energy density in magnetic field. [CO4-L1]

    ENERGY DENSITY IN MAGNETIC FIELD

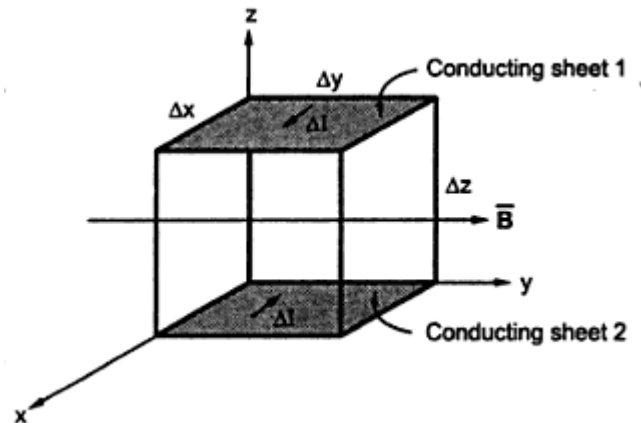   Inductor is also a capacitor in storing energy, capacitor stores energy in electric field where inductor stores energy in magnetic field.

    The energy stored in inductor is

$$W_m = \frac{1}{2} L I^2 \qquad \text{... (1)}$$

    The inductance ΔL of a differential volume is given by

$$\Delta L = \frac{\Delta\phi}{\Delta I} = \frac{B\,\Delta S}{\Delta I} \qquad \text{...} \phi = \int_S \overline{B}\cdot d\overline{S}$$

    ΔS can be written as

$$\Delta L = \frac{B(\Delta x\,\Delta z)}{\Delta I}$$

$$B = \mu H$$

$$\Delta L = \frac{\mu H \Delta x\,\Delta z}{\Delta I} \qquad \text{... (2)}$$

    The decremented current can be expressed as

$$\Delta I = (H)(\Delta y) \qquad \text{... (3)}$$

    The energy stored in the inductance of a differential volume is

$$\Delta W_m = \frac{1}{2} \Delta L\,\Delta I^2$$

    Sub equations of ΔL and ΔI

$$\Delta W_m = \frac{1}{2}\left[\frac{\mu H \Delta x \Delta z}{H \Delta y}\right][H\Delta y]^2$$

$$\therefore \qquad \Delta W_m = \frac{1}{2} \mu H^2 (\Delta x\,\Delta y\,\Delta z)$$

    The differential volume can be written as

$$\Delta \dot{v} = \Delta x \, \Delta y \, \Delta z$$

The energy stored in a inductor of differential volume is given by

$$\Delta W_m = \frac{1}{2} \mu H^2 \, \Delta v \qquad\qquad\qquad \text{... (4)}$$

11. Explain about Magnetization. [CO4-L1]

Magnetization :

A small bar magnet with pole strength Q_m and length 'l' is defined as magnetic dipole whose magnetic moment is $Q_m l$.

A magnetic dipole is considered as equivalent to current loop with area A and current I with magnetic moment m= I_A i.e., $Q_m l = I_A$.

Magnetic Dipole

Current Loop

Consider a bar magnet as shown in the figure .

Without the external B field applied to this bar magnet , the sum of magnetic moment is zero due to random orientation.

S   N

When an external field is applied to the bar magnet , the magnetic moments of the electronic more or less align themselves with B so that the net magnetic moment is not zero.

Bar magnet

Magnetization 'M' in A/m is the magnetic dipole moment per unit volume.

$$M = \frac{\text{dipole moment}}{\text{volume}} \quad ; \text{volume} = Al$$

$$= \frac{Q_m l}{\text{volume}} = \frac{Q_m l}{Al}$$

$$M = \frac{Q_m}{A} \text{ A/m}$$

Or

$$M = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} mk}{\Delta v}$$

Where m = magnetic moment

if M 0 then tne medium is said to be magnetized everywhere.

Magnetic susceptibility (ψ_m)

It is defined as the ratio of magnetization to the magnetic field intensity.

$$\psi_m = \frac{M}{H}$$

$$B = \mu_0 \, H$$

For magnet , the flux density

$$B = \mu_0 \, (H + M)$$

$$= \mu_0 \, H \left(1 + \frac{M}{H}\right)$$

$$B = \mu_0 \, H \, (1 + \psi_m) \text{ ------ (1)}$$

$$B = \mu \, H$$

$B = \mu_0 \, \mu_r H$ --------(2)

Equating 1 and 2,, $\mu_r = 1 + \psi_m$

12. Derive magnetic boundary condition and give its equation. [CO4-H2-Nov/Dec 2013 & April/May 2015]

MAGNETIC BOUNDARY CONDITIONS:

- The conditions of magnetic field existing at the boundary when the magnetic field passes from one medium to another medium.
- It is resolved into two components.1.Tangential component 2.Normal component.

BOUNDARY CONDITION FOR NORMAL COMPONENT:

According to Guass law

$$\oint_S \overline{B} \cdot d\overline{S} = 0 \qquad \qquad \dots (1)$$

$$\therefore \oint_{top} \overline{B} \cdot d\overline{S} + \oint_{bottom} \overline{B} \cdot d\overline{S} + \oint_{lateral} \overline{B} \cdot d\overline{S} = 0 \qquad \qquad \dots (2)$$

$$\oint_{Top} \overline{B} \cdot d\overline{S} = B_{N_1} \oint_{Top} d\overline{S} = B_{N_1} \Delta S \qquad \qquad \dots (3)$$

$$\oint_{Bottom} \overline{B} \cdot d\overline{S} = B_{N_2} \oint_{Bottom} d\overline{S} = B_{N_2} \Delta S \qquad \qquad \dots (4)$$

$$\oint_{Lateral} \overline{B} \cdot d\overline{S} = 0 \qquad \qquad \dots (5)$$

$$B_{N_1} \Delta S - B_{N_2} \Delta S = 0 \qquad \qquad \dots (6)$$

$$B_{N_1} \Delta S = B_{N_2} \Delta S$$

i.e. $\boxed{B_{N_1} = B_{N_2}} \qquad \qquad \dots (7)$

$$\overline{B} = \mu \overline{H}$$

$$\therefore \boxed{\begin{matrix} \mu_1 H_{N_1} = \mu_2 H_{N_2} \\ \dfrac{H_{N_1}}{H_{N_2}} = \dfrac{\mu_2}{\mu_1} = \dfrac{\mu_{r2}}{\mu_{r1}} \end{matrix}} \qquad \qquad \dots (8)$$

BOUNDARY CONDITION FOR TANGENTIAL COMPONENT:

$$\oint \overline{H} \cdot d\overline{L} = I \qquad \qquad \dots (9)$$

$$\oint \overline{H} \cdot d\overline{L} = \int_a^b \overline{H} \cdot d\overline{L} + \int_b^1 \overline{H} \cdot d\overline{L} + \int_1^c \overline{H} \cdot d\overline{L} + \int_c^d \overline{H} \cdot d\overline{L} + \int_d^2 \overline{H} \cdot d\overline{L} + \int_2^a \overline{H} \cdot d\overline{L} = I \qquad \dots (10)$$

$$K \cdot dw = H_{tan1}(\Delta w) + H_{N1}\left(\frac{\Delta h}{2}\right) + H_{N2}\left(\frac{\Delta h}{2}\right) - H_{tan2}(\Delta w)$$

$$- H_{N2}\left(\frac{\Delta h}{2}\right) - H_{N1}\left(\frac{\Delta h}{2}\right) \qquad \qquad \dots (11)$$

$$\boxed{\overline{H}_{tan1} - \overline{H}_{tan2} = \overline{a}_{N12} \times \overline{K}} \qquad \qquad \dots (13)$$

$$\therefore \quad \frac{B_{tan1}}{\mu_1} - \frac{B_{tan2}}{\mu_2} = K \quad\quad\quad \text{... (14)}$$

$$H_{tan1} - H_{tan2} = 0$$

$$\text{or} \quad\quad\quad H_{tan1} = H_{tan2} \quad\quad\quad \text{... (15)}$$

$$\therefore \quad \frac{B_{tan1}}{\mu_1} = \frac{B_{tan2}}{\mu_2}$$

13. Find the maximum torque on an 85 turns, rectangular coil with dimension (0.2 × 0.3) m, carrying current of 5 Amps in a field B = 6.5T.   [CO4-L1-Apr/May 2010]

Given:

$$N = 85,$$
$$A = 0.2 \times 0.3 = 0.06 \text{ m}^2$$
$$I = 5A$$
$$B = 6.5 \text{ T}$$

Formula:

$$T = NBIA$$
$$= 85 \times 6.5 \times 5 \times 0.06$$
$$= 165.75 \text{ N.m}$$

14. Explain magnetic circuits. [CO4-L1]

In magnetic circuits , we determine magnetic fluxes and magnetic field intensities in various part of the circuits

Consider an example of electromagnet made of iron of constant relative permeability.

The common examples are transformer, toroid motors, generator , relays.

The magnetomotive force (m.m.f) is defined as,

$$e_m = NI = \oint \overline{H} \bullet d\overline{L}$$

The SI unit of m.m.f is amperes(A)

In electric circuits , the resistance is defined as ratio of voltage to current given by

$$R = \frac{V}{I}$$

Reluctance is defined as magnetomotive force to total flux.

$$\Re = \frac{e_m}{\phi}$$

It is measured in (ampere. turn / weber)

The resistance in electric circuit can be expressed interms of conductivity is

$$R = \frac{l}{\sigma S}$$

Where   l = lenth in m

     S = Cross-sectional area in m²

     σ = Conductivity of the linear isotropic homogeneous material

In case of magnetic field the reluctance is

$$\Re = \frac{l}{\mu S}$$

μ = Permeability of the isotropic, linear homogeneous material

For electric circuits , the ohms law is

$$\bar{J} = \sigma \bar{E}$$

The magnetic flux density is

$$\bar{B} = \mu \bar{H}$$

The basic equations are

$$\nabla \cdot \bar{B} = 0 , \text{ and}$$

$$\nabla \times \bar{H} = \bar{J}$$

The total current in the magnetic circuit is given by,

$$I = \int_S \bar{J} \cdot d\bar{S}$$

The total magnetic flux density is

$$\phi = \int_S \bar{B} \cdot d\bar{S}$$

In electric circuits , the reciprocal of the resistance is called conductance.

In magnetic circuits , the reciprocal of the reluctance is called permeance denoted by $\mathcal{P}$

$$P = \frac{\mu S}{l}$$

(a) Electric circuit   **(b) Magnetic circuit**

(c) Coil of N turns carrying current

15. Explain about magnetic force in the closed circuits. [CO4-L1]

The magnetic force is the force that a magnetic field exist on a piece of magnetic material in the field.

The coil has N turns and carrying current I. fringing effects is assumed to be null.

The magnetic field in the air gap is same as that in iron. To find the force between the two pieces of iron, we calculate the change in the total energy that would result were the two pieces of the magnetic circuit separated by the differential displacement dl.

The work required to effect the displacement is equal to the change in stored energy in air gap

$$-Fdl = d\omega_m = 2\left[\frac{1}{2}\frac{B^2}{\mu_0}Sdi\right]$$

Whers S is the cross sectional air of the gap.

Factor 2 accounts for 2 air gaps.

Negative sign indicates that the force acts to reduce the air gap

$$F = -2\frac{B^2S}{2\mu_0}$$

The attractive force across a single gap can be obtained from,

$$F = -\frac{B^2S}{2\mu_0}$$

The attractive pressure N/m2 in a magnetized surface is,

$$P = \frac{F}{S} = \frac{B^2}{2\mu_0} = \frac{1}{2}BH$$

Which is same as the energy density

: *A conductor 6m long, lies along z-direction with a current of 2A in $\bar{a}_z$ direction. Find the force experienced by conductor if $\bar{B} = 0.08\,\bar{a}_x$ (T).*

' : A force exerted on current carrying conductor in a magnetic field is given by

$$\bar{F} = I\,d\bar{L}\times\bar{B}$$

$$\bar{F} = 2(6\bar{a}_z)\times(0.08\bar{a}_x)$$

$$\bar{F} = 12\,\bar{a}_z\times0.08\,\bar{a}_x$$

$$\bar{F} = 0.96\,\bar{a}_y\ N \qquad \ldots\bar{a}_z\times\bar{a}_x = \bar{a}_y$$

A solenoid with $N_1 = 2000$, $r_1 = 2$ cm and $l_1 = 100$ cm is concentric within a second coil of $N_2 = 4000$, $r_2 = 4$ cm and $l_2 = 100$ cm. Find mutual inductance assuming free space conditions.

$$H_1 = \frac{N_1 I_1}{l_1} = \frac{(2000)I_1}{100\times10^{-2}} = (2000)I_1\ A/m$$

$$B_1 = \mu\,H_1 = \mu_0\mu_r\,H_1 = (4\times\pi\times10^{-7})(2000I_1) = (2.5132\times10^{-3})\,I_1\quad Wb/m^2$$

Total flux is produced is

$$\phi_1 = (B_1)(A_1) = (2.5132 \times 10^{-3} \times I_1) \left[\pi (2 \times 10^{-2})^2 \right]$$

$$\phi_1 = (3.1582 \times 10^{-6}) I_1 \quad Wb$$

The mutual inductance between 2 coils is

$$M_{12} = \frac{N_2 \phi_1}{I_1} = \frac{(4000)(3.1582 \times 10^{-6} I_1)}{I_1}$$

$$\therefore \qquad M_{12} = 12.633 \text{ mH}$$

8   *Calculate the inductance of a solenoid of 200 turns wound tightly on a cylindrical tube of 6 cm diameter. The length of the tube is 60 cm and the solenoid is in air.*

                                                      [AU Oct. - 2002]

Sol. :   For a given solenoid in air,

$$\mu = \mu_0 = 4\pi \times 10^{-7} \text{ Wb/A.m}$$

$$N = 200$$

$$d = 6 \text{ cm} = 6 \times 10^{-2} m \quad \text{hence } r = \frac{d}{2} = 3 \times 10^{-2} \text{ m}$$

$$l = 60 \text{ cm} = 60 \times 10^{-2} \text{ m}$$

The inductance of a solenoid is given by,

$$L = \frac{\mu N^2 A}{l}$$

$$= \frac{\mu_0 N^2 (\pi r^2)}{l}$$

$$= \frac{4 \times \pi \times 10^{-7} \times (200)^2 \times \pi \times (3 \times 10^{-2})^2}{60 \times 10^{-2}}$$

$$= 2.3687 \times 10^{-4} \text{ H}$$

$$= 0.2368 \text{ mH}$$

Calculate the inductance of a 10 m length of co-axial cable filled with a material for which $\mu_r = 80$ and radii of inner and outer conductors are 1mm and 4mm respectively.

$$d = 10 \text{ m}$$

$$\mu_r = 80,$$

$$a = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

$$b = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$$

The inductance of coaxial cable is

$$L = \frac{\mu d}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{(\mu_0 \mu_r)(d)}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$= \frac{4 \times \pi \times 10^{-7} \times 80 \times 10}{2\pi} \ln\left(\frac{4 \times 10^{-3}}{1 \times 10^{-3}}\right)$$

$$= 2.218 \times 10^{-4} \text{ H} = 221.8 \times 10^{-6} \text{ H}$$

$$L = 221.8 \ \mu\text{H}$$

Unit V

Time varying fields and maxwell's equations

Part – A

1. **Write down the integral form of Maxwell's equation. [CO5-L1-Nov 2007, May 2006 & Nov 2011]**

   (i)   Maxwell's I equation (From Ampere's law)

   $$\oint \vec{H}.\overrightarrow{dL} = \int_s (\vec{J} + \frac{\partial \vec{D}}{\partial t})\overrightarrow{ds}$$

   (ii)   Maxwell's II equation (From faraday's law)

   $$\oint \vec{E}.\overrightarrow{dl} = -\int_s \frac{\overrightarrow{\partial B}}{\partial t}.\overrightarrow{ds}$$

   (iii)   Maxwell's III equation (From Gauss's law for electric field)

   $$\oint \vec{D}.\overrightarrow{ds} = \int_v \rho_v d_v$$

   (iv)   Maxwell's IVth equation (From Gauss's law for magnetic field)

   $$\int_s \vec{B}.\overrightarrow{ds} = 0$$

2. **Write down the differential form of Maxwell's equations (or) Write point from of Maxwell's equations. . [CO5-L1-Nov/Dec 2008]**

   (i)   Maxwell's I st equation (From Ampere's law)

   $$\nabla \times \vec{H} = \vec{J} + \frac{\overrightarrow{\partial D}}{\partial t}$$

   (ii)   Maxwell's II nd equation (From faraday's law)

   $$\nabla \times \vec{E} = -\frac{\overrightarrow{\partial B}}{\partial t}$$

   (iii)   Maxwell's III equation (From Gauss's law for electric field)
   $$\nabla.\vec{D} = \rho v$$

   (iv)   Maxwell's IV equation (From Gauss's law for magnetic field)
   $$\nabla.\vec{B} = 0$$

3. **Give the Maxwell's equation for free space in point form. [CO5-L1]**

Maxwell's I equation (From Ampere's law)

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

Maxwell's II equation (From faraday's law)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Maxwell's III equation (From Gauss's law for electric field)

$$\nabla . \vec{D} = \rho v$$

Maxwell's IV equation (From Gauss's law for magnetic field)

$$\nabla . \vec{B} = 0$$

4.Write down the Maxwell's equations derived from faraday's law.[CO5-L1-Nov/dec 2010]

INTEGRAL FORM

$$\Rightarrow \oint \vec{E}.\overrightarrow{dL} = \int_S \frac{\partial}{-\partial t}(\vec{B}.\overrightarrow{ds})$$

DIFFERENTIAL FORM

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

5.Write down the Maxwell's equations derived from Ampere's circuital law.[CO5-L1]

INTEGRAL FORM

$$\oint \vec{H}.\overrightarrow{dL} = \int_s (\vec{J} + \frac{\partial \vec{D}}{\partial t})\overrightarrow{ds}$$

DIFFERENTIAL FORM

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

6. Brief about Ampere's circuital law in integral form. [CO5-L1-May/ June 2007]

According to Ampere's circuital law states that, the line integral of magnetic field intensity vector around any closed path is equal to the direct current enclosed by the path.

$$\oint \overrightarrow{H}.\overrightarrow{dL} = I_{enclosed}$$

INTEGRAL FORM:

Replace the current in above equation, the surface integral of conduction current density is $\vec{J}$.

$$\oint \vec{H}.\overrightarrow{dL} = \oint_s \vec{J}.\overrightarrow{ds}$$

Add displacement current density to the conduction current density ∂

$$\oint \vec{H}.\overrightarrow{dL} = \int_S (\vec{J} + \frac{\partial \vec{D}}{\partial t})\overrightarrow{ds}$$

The above equation is Maxwell's equation derived from Ampere's circuital law in Integral form.

7. State Faraday's law of electromagnetic induction. [CO5-L1-May 2011, Dec 2010, Dec 2013 & April/May 2015]

Faraday's law states that, in time varying magnetic fields, the induced emf is equal to the time rate of change of magnetic flux linking with the closed circuit.

$$e = \frac{-Nd\varphi}{dt} \; volts$$

Where N = number of turns in the circuit
e = induced emf
Φ= flux linked with the circuit

8. Define Poynting vector, what is the SI unit for this vector? [CO5-L1-April/May 2008]

The Poynting vector is defined as the rate of flow of energy of a wave as it propagates. It is the vector product of electric field and magnetic field

$$\vec{P} = \vec{E} \times \vec{H}$$

Power = power density x area

Power density is measured in watt/m^2

9. Brief about complex Poynting vector? [CO5-L1-June 2006 & May 2011]

The complex vector is given by

$$p = \frac{1}{2} \vec{E} \times \vec{H}$$

The product of E and H is a vector product mutually perpendicular components of E and H contribute to the power flow. This power flow is directed along the normal to the plane containing E and H

10. State Poynting theorem. [CO5-L1-April/May 2015]

- Poynting vector is based on law of conservation of energy.

- It states that ,'the net power flowing out o a given volume V is equal to the time rate of decrease in the energy stored within volume V minus the ohmic power dissipated.

11. **What are displacement current density and its significance? [CO5-L1-May 2011, May 2010, Dec 2012, Nov 2007, Dec 2012 & June 2012]**

It is defined as the displacement current per unit surface area.

$$J_D = \frac{I_D}{A}$$

Where,

J_D = displacement current density
I_D = displacement current
A = surface area

12. **What is motional emf? (Or) Write down the expression for the emf induced in the moving loop in static   B   field. [CO5-L1]**

The induced emf obtained by a stationary fields and a moving circuit is called motional emf or generator emf. The above action is similar to generator action.

$$\oint \vec{E}_m . \vec{dl} = \int (\vec{v} \times \vec{B}). \vec{dl}$$
$$\text{(or)}$$
$$e = BLv \sin 90°$$
$$e = BLv$$

13. **What is transformer emf? [CO5-L1]**

The induced emf obtained by keeping the circuit stationary and field carrying sinusoidally is called transformer emf. The above action is similar to transformer action.

$$\oint \vec{E} . \vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t}. \vec{ds}$$

14. **Explain in detail about the instantaneous average and complex poynting vector. [CO5-L1-Nov/dec 2010]**

The instantaneous poynting vector is given by,
$$\vec{P} = \vec{E} \times \vec{H}$$
The complex poynting vector is given as

$$\vec{P} = \frac{1}{2}(\vec{E} \times \vec{H}^*)$$

The average poynting vector is given as,
$$\vec{P}_{avg} = \frac{1}{2} Re[\vec{E} \times \vec{H}^*]$$

15. **State Lenz's law. [CO5-L1]**

It states that, "The emf induced in an electric circuit always act in such a direction that the current it drives around the circuit opposes the change in magnetic flux which produces the emf "

16. Give the situations, when the rate of change of flux results in a non- zero value. [CO5-L1-May 2011]

In the following situation, the rate of change of flux results in a non- zero value.

      i)   There exists a relative motion between conductor and flux.

      ii)   Plane of flux and motion of conductor should not be parallel.

      iii)   An alternating flux linking with coils of N turns.

17. State faraday's law for a moving charge in a constant magnetic field? [CO5-L1]

When the charge Q is moving in a constant magnetic field $\vec{B}$ and if the velocity with which it moves is $\vec{V}$, then force on a charge is given by,

$$\vec{F} = Q\vec{V} \times \vec{B}^*$$

18. Discuss the condition under which conduction current is equal to displacement current? [CO5-L1]

$$\frac{\sigma}{\omega \in} = 1$$

Under this condition only the conduction current is equal to the displacement current.

Where,

$\sigma = $ Conductivity

$\omega = $ Angular frequency   , $\in = $ Permeability

19. The magnetic field intensity in free space is given as $\vec{H} = H_0 sin\theta \vec{a_y}$ A/m, where $\theta = (\omega t - \beta z)$ and β is a constant quantity. Determine the displacement current density. [CO5-H1]

Given:

$\vec{H} = H_0 sin\theta \vec{a_y}$

$\theta = (\omega t - \beta z)$

To Find:

Displacement current density

Solution:

$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

For free space $\vec{J} = 0$

$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_D$

Where,

$$\frac{\partial \vec{D}}{\partial t} = 0$$

$$\nabla \times \vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & H_0 sin\theta & 0 \end{vmatrix}$$

$$= \vec{a}_x \left[\frac{\partial}{\partial z} H_0 sin\theta \right] + \vec{a}_z \left[\frac{\partial}{\partial x} H_0 sin\theta \right]$$

$$\theta = (\omega t - \beta z)$$

$$= -\frac{\partial}{\partial z} H_0 sin(\omega t - \beta z)\vec{a}_x + \frac{\partial}{\partial x} H_0 sin(\omega t - \beta z)\vec{a}_z$$

$$\nabla \times \vec{H} = \beta H_0 (\omega t - \beta z)\vec{a}_x$$

$$\vec{J}_D = \beta H_0 (\omega t - \beta z)\vec{a}_x$$

20. Determine the emf developed about the path $\gamma = 0, z = 0$ & $t = 0$. If B = 0.01 sin377 $\vec{a}_z$. [CO5-H2-April/May 2008]

  Given:

B = 0.01 sin377 $\vec{a}_z$
$\gamma = 0, z = 0$ & $t = 0$

To find:

Emf developed

Solution:

Total flux is given by,

$$\varphi = \int_s \vec{B}.\vec{ds}$$

$$\vec{ds} = (\gamma \, dr \, d\varphi)\vec{a}_z$$

$$\varphi = \int_{\varphi=0}^{2\pi} \int_{\gamma=0}^{0.5} [0.01 \sin 377t \, \vec{a}_z][\gamma \, dr \, d\varphi \, \vec{a}_z]$$

$$= (0.01 \sin 377t) \, [\varphi]_0^\pi \, \left[\frac{\gamma^2}{2}\right]_0^{0.5}$$

$$= 0.01 \sin 377t \, (2\pi) \, [\frac{0.5^2}{2}]$$

$$= 0.01 \sin 377t \, (2\pi) \, [\frac{0.25}{2}]$$

$$\varphi = 7.85 \times 10^{-3} \sin 377t$$

Emf induced,

$$e = -\frac{\partial \varphi}{\partial t} = -\frac{\partial}{\partial t}[7.85 \times 10^{-3} \sin 377t]$$

$$e = -2.960 \cos 377t$$

21. What is the electric field and power flow in co-axial cable. [CO5-L1-Nov/Dec 2011]

The electric field intensity for coaxial cable is given by,

$$E = \frac{q}{2\pi\epsilon r}$$

The total power along the cable is

W= V I

Part- B

1. Explain time varying Electric and Magnetic fields. [CO5-L1]

Michael Faraday proved that a magnetic field could produce current. According to Faraday's experiment, a static magnetic field cannot produce any current flow. But with a time varying field can induce an emf which can cause current flow.

This emf is nothing but a voltage that is produced due to time varying fields.

Statement of Faraday's law:

"The electromotive force induced n a closed path is proportional to the rate of change of magnetic flux enclosed by the closed path".

Faraday's law can be stated as,

$e = -N \frac{d\varphi}{dt}$ volts

Where, N = no. of turns in circuits

e = induced emf

If N =1,

$e = -\frac{d\varphi}{dt}$ volts [-ve sign indicates opposite direction]

According to Lenz law, the induced emf produces opposite flux.

According to Gauss law, the flux is given as,

$$\varphi = \int \overrightarrow{B}.\,\overrightarrow{ds}$$

Therefore, $e = -\frac{d}{dt}\int \overrightarrow{B}.\,\overrightarrow{ds}$

The emf can also be written as,

$$e = \oint \overrightarrow{E}.\,\overrightarrow{dL}$$

Therefore, $\vec{E}.\overrightarrow{dL} = -\frac{d}{dt}\int \vec{B}.\overrightarrow{ds}$

2. Derive and explain Maxwell's equations both in integral and point forms. [CO5-H1-Nov/Dec 2011, 2010,May/June 2013]

Maxwell's equations are a set of four partial differential equation that describe the properties of the electric and magnetic fields and relate them to their sources, charge density and current density.

These equation are used to show that, light is an electromagnetic wave. Basically there are four Maxwell's equations. They are

- Maxwell's equation obtained from Ampere's law.
- Maxwell's equation obtained from Faraday's law.
- Maxwell's equation obtained from Gauss law for Electric field.
- Maxwell's equation obtained from Gauss law for Magnetic field.

(I) Maxwell's equation derived from Ampere's circuital law.

According to Ampere's circuital law states that,
The line integral of magnetic field intensity vector around any closed path is equal to the direct current enclosed by the path.

$$\oint \vec{H}.\overrightarrow{dL} = I_{enclosed}$$

Integral form:

Replace the current in above equation, the surface integral of conduction current density is $\vec{J}$.

$$\oint \vec{H}.\overrightarrow{dL} = \oint_{s} \vec{J}.\overrightarrow{ds}$$

Add displacement current density to the conduction current density.

$$\oint \vec{H}.\overrightarrow{dL} = \int_{s} (\vec{J} + \frac{\partial \vec{D}}{\partial t})\overrightarrow{ds}$$

The above equation is Maxwell's equation derived from Ampere's circuital law in Integral form.

Differential form:

Apply Stoke's theorem, to the LHS of Maxwell equation.

$$\int_{s} (\nabla \times \vec{H})\overrightarrow{ds} = \int_{s} (\vec{J} + \frac{\partial \vec{D}}{\partial t})\overrightarrow{ds}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

The above equation is the <u>point (or) differential form</u> of Maxwell's equation derived from Ampere's circuital law.

(II) Maxwell's equation derived from Faraday's law:

According to Faraday's law, $e = -\frac{dQ}{dt}$

According to Gauss law for magnetic field,

$$\mathbf{Q} = \int \vec{B}.\vec{ds}$$

$$e = \int_S \frac{\partial}{\partial t}(\vec{B}.\vec{ds})$$
$$\Rightarrow \oint \vec{E}.\vec{dL} = \int_S \frac{\partial}{\partial t}(\vec{B}.\vec{ds})$$

The above equation is Maxwell's equation derived from <u>Faraday's law   in Integral form</u>

Differential form:

Using Stokes theorem, Convert the line integral into surface integral

$$\int_S (\nabla \times \vec{E})\,\vec{ds} = \int_S \frac{\partial \vec{B}}{\partial t}\vec{ds}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

The above equation is the <u>point (or) differential form</u> of Maxwell's equation derived from Faraday's law.

(III) Maxwell's equation obtained from Gauss's law for Electric field.

The Gauss's law states that, the electric flux passing through any closed surface is equal to the total charge enclosed by that pat.

According to gauss law, $\oint \vec{D}.\vec{ds} = Q_{enclosed}$

Integral form:

Replace charge by volume integral of volume charge density ρ_v

$$\oint \vec{D}.\vec{ds} = \int_v \rho_v dv$$

The above equation is Maxwell's equation for electric field from gauss law in Integral form

Differential form:

Using divergence theorem, to Convert the surface integral into volume integral

$$\int_v (\nabla . \vec{D})\overrightarrow{ds} = \int_v \rho_v dv$$

The above equation is Maxwell's equation for electric field from <u>gauss law in differential form</u>

(IV) <u>Maxwell's equation obtained from Gauss's law for magnetic field.</u>

The magnetic Gauss's law states that the net magnetic flux of lines passing through any closed surface is zero.

According to Gauss law for magnetic field,

    i.e. $Q = 0$; $\int \vec{B}.\overrightarrow{ds}$ **= 0**

The above equation is Maxwell's equation derived from <u>gauss law for magnetic field   in integral form</u>

<u>Differential form</u>

Applying divergence theorem, to Convert the surface integral into volume integral

$$\int_v (\nabla . \vec{B})\overrightarrow{dv} = 0$$

$$\nabla . \vec{B} = 0$$

The above equation is Maxwell's equation derived from gauss law for magnetic field in differential form.

3. Discuss about displacement current and its circuit analysis.[CO5-L3-Nov/Dec 2011]

<u>Displacement current:</u>

For static electromagnetic field, the Ampere circuital law is

$$\nabla \times \vec{H} = \vec{J}_C + \vec{J}_D \quad \text{---------------------(1)}$$

<u>Proof:</u>

Take divergence on both sides of Ampere's circuital law.

$$\nabla . (\nabla \times \vec{H}) = \nabla . \vec{J} \quad \text{-------------------(2)}$$

According to vector identity, the divergence of the curl of any vector field is zero.

$$\text{Therefore, } \nabla . (\nabla \times \vec{H}) = \nabla . \vec{J} = 0$$

The equation of continuity is given as,

$$\nabla . \vec{J} = \frac{\partial \rho_v}{\partial t} = 0 \quad \text{---------------------(3)}$$

Add some unknown term $\vec{N}$

$$\nabla \times \vec{H} = \vec{J} + \vec{N} \quad \text{-------------------(4)}$$

Take divergence on both sides

$$\nabla.(\nabla \times \vec{H}) = \nabla.(\vec{J} + \vec{N})$$

$$= \nabla.\vec{J} + \nabla.\vec{N}$$

$$\nabla.\vec{J} = -\nabla.\vec{N} \text{ --------------(5)}$$

Sub. in $\nabla.\vec{J}$,

$$-\nabla.\vec{N} = -\frac{\partial \rho_v}{\partial t}$$

$$\nabla.\vec{N} = \frac{\partial \rho_v}{\partial t} \text{ ------------------------(6)}$$

According to Gauss law,

$$\rho_v = \nabla.\vec{D}$$

$$\nabla.\vec{N} = \frac{\partial}{\partial t}(\nabla.\vec{D})$$

$$\vec{N} = \frac{\partial}{\partial t}\vec{D}$$

Sub. $\vec{N}$ in eqn.(4)

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Hence $\vec{J} = \vec{J_C}$

$$\frac{\partial \vec{D}}{\partial t} = \vec{J}_D$$

Therefore $\nabla \times \vec{H} = \vec{J_C} + \vec{J_D}$

Circuit analysis of displacement current:

Let us consider a parallel circuit of resistor and capacitor which is applied by a time varying component V.

From the circuit $i_1 = \frac{V}{R}$ (conduction current)

The conduction current density $\vec{J_C}$ is

$\vec{J_C} = \frac{i_c}{A} = \sigma\vec{E}$ [Therefore $i_1 = i_c$]

The displacement current is given as,

$\vec{J_D} = \frac{i_D}{A}$ [Therefore $i_2 = i_D$]

The current i_2 is given as,

$$i_2 = i_D = C\frac{dv}{dt}$$

W.K.T,

$$C = \frac{\varepsilon A}{d}$$

Therefore,

$$i_D = \frac{\varepsilon A}{d} \cdot \frac{dv}{dt} \quad \text{where, } \vec{E} = V/d \; ; v = d\vec{E}$$

$$i_D = \frac{\varepsilon A}{d} \cdot \frac{d}{dt} \, d\vec{E}$$

$$i_D = \varepsilon A \frac{d\vec{E}}{dt}$$

$$\vec{J}_D = \frac{i_D}{A} = \frac{\overline{dE}}{dt}$$

$$= \frac{d}{dt} (\varepsilon\vec{E})$$

$$\vec{J}_D = \frac{d}{dt} \vec{D}$$

The total current density is given by,

$$\vec{J} = \vec{J}_C + \vec{J}_D$$

$$\vec{J} = \sigma\vec{E} + \frac{d\vec{D}}{dt}$$

The above equation depends on the properties of the medium

4. Discuss about Poynting vector and power flow. [CO5-L3-May 2012, Nov/Dec 2011 & Nov/Dec 2010]

Poynting vector

In general, time varying fields constitute the electromagnetic waves. This waves travel through free space (or) dielectric.
 Eg: radio waves
In case of electromagnetic waves, the power and energy relationships can be explained in terms of amplitude of the electric and magnetic field.
The resultant theorem is poynting theorem.
Poynting vector and flow of power:
　　　The energy stored in an electric field and magnetic field is transmitted at a certain rate of energy flow which can be calculated with the help of Poynting theorem.

$\vec{E}$ is expressed in V/m and $\vec{H}$ is expressed in A/m.

The product of two fields will dimentionally give V.A/m^2

The cross product of $\vec{E}$ and $\vec{H}$ gives power density.

$\vec{P} = \vec{E} \times \vec{H}$

Where, $\vec{P}$ = Poynting vector

Fig. Power balance representation in electromagnetic fields

$\vec{P}$ is the instantaneous power density vector associated with the electromagnetic field at a given point. The direction of $\vec{P}$ indicates instantaneous power flow at a point. To get a net power flowing out of any surface, $\vec{P}$ is integrated over same closed surface. Poynting theorem states that,

"The net power flowing out of a given volume v is equal to the time rate of decrease in the energy stored within volume v minus the ohmic power dissipated". This is illustrated in the above Fig.

Suppose,

$$\vec{E} = E_x \vec{a_x}$$

Then,

$$\vec{H} = H_y \vec{a_y}$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$= E_x \vec{a_x} \times H_y \vec{a_y}$$

$$= E_x H_y \vec{a_z}$$

$$= P_z \vec{a_z}$$

5. Discuss about the power density and average power density. [CO5-L3]

Power density:

The electric field in free space is given as,

$$\vec{E} = [E_m(\cos(\omega t - \beta z))]\vec{a}_x$$

The magnetic field in free space is given as,

$$\vec{H} = [H_m(\cos(\omega t - \beta z))]\vec{a}_y$$

The intrinsic impedance η is given as,

$$\eta = \eta_0 = \frac{E_m}{H_m}$$

$$H_m = \frac{E_m}{\eta_0}$$

The magnetic field is given as,

$$\vec{H} = \left[\frac{E_m}{\eta_0}(\cos(\omega t - \beta z))\right]\vec{a}_y$$

According to pointing theorem,

$$\vec{P} = \vec{E} \times \vec{H}$$

$$= [E_m(\cos(\omega t - \beta z))]\vec{a}_x \times \left[\frac{E_m}{\eta_0}(\cos(\omega t - \beta z))\right]\vec{a}_y$$

$$\vec{P} = \frac{E_m^2}{\eta_0} \cos^2(\omega t - \beta z)\vec{a}_z \qquad W/_{m^2}$$

The above equation is the power density in $W/_{m^2}$.

Power:

The power passing through a particular area is given as,

Power = power density × Area

Average power density:

P_{avg} is obtained by integrating power density in z-direction and divide the period T of one cycle.

$$P_{avg} = \frac{1}{T}\int_0^T \frac{E_m^2}{\eta_0} \cos^2(\omega t - \beta z)\, dt$$

$$= \frac{1}{T}\frac{E_m^2}{\eta_0} \int_0^T \left(\frac{1 + \cos2(\omega t - \beta z)}{2}\right) dt$$

$$= \frac{1}{2T}\frac{E_m^2}{\eta_0} \left[\int_0^T dt + \int_0^T \cos2\,(\omega t - \beta z)dt\right]$$

$$= \frac{1}{2T}\frac{E_m^2}{\eta_0} \left[T + \left(\frac{\sin2(\omega t - \beta z)}{2\omega}\right)_0^T\right]$$

$$= \frac{1}{2T} \frac{{E_m}^2}{\eta_0} \left[T + \frac{(sin2(\omega T - \beta z) - sin2(-\beta z))}{2\omega} \right]$$

$$Sub \ \omega T = 2\pi$$

$$P_{avg} = \frac{{E_m}^2}{2T\eta_0} \left[T + \frac{sin2(4\pi - 2\beta z)}{2\omega} + \frac{sin2\beta z}{2\omega} \right]$$

$$P_{avg} = \frac{{E_m}^2}{2T\eta_0} \left[T - \frac{sin2\beta z}{2\omega} + \frac{sin2\beta z}{2\omega} \right]$$

$$P_{avg} = \frac{{E_m}^2}{2T\eta_0} \qquad [since \ \eta = \eta_0]$$

]*The average power is given by,*

$$P_{avg} = \frac{{E_m}^2}{2T\eta_0} \ W/_{m^2}$$

6.Discuss in detail about the pointing vector in integral and point form. [CO5-L3-Dec 2011, Dec 2013]
Integral and point form of pointing vector:

Consider Maxwell's equation as given below,

$$\nabla \times \overline{\mathbf{E}} = -\frac{\partial \overline{\mathbf{B}}}{\partial t} = -\mu \frac{\partial \overline{\mathbf{H}}}{\partial t}$$

$$\nabla \times \overline{\mathbf{H}} = \overline{\mathbf{J}} + \frac{\partial \overline{\mathbf{D}}}{\partial t} = \sigma \overline{\mathbf{E}} + \varepsilon \frac{\partial \overline{\mathbf{E}}}{\partial t}$$

Take dot product on both the sides of the above equations with $\vec{E}$

$$\overline{\mathbf{E}} \bullet (\nabla \times \overline{\mathbf{H}}) = \overline{\mathbf{E}} \bullet (\sigma \overline{\mathbf{E}}) + \overline{\mathbf{E}} \bullet \left(\varepsilon \frac{\partial \overline{\mathbf{E}}}{\partial t} \right)$$

Let us make use of vector identity as given below,

$$\nabla \bullet (\overline{\mathbf{A}} \times \overline{\mathbf{B}}) = \overline{\mathbf{B}} \bullet (\nabla \times \overline{\mathbf{A}}) - \overline{\mathbf{A}} \bullet (\nabla \times \overline{\mathbf{B}})$$

Applying above identity to equation (3) with $\vec{A} = \vec{E}$ and $\vec{B} = \vec{H}$

$$\overline{\mathbf{H}} \bullet (\nabla \times \overline{\mathbf{E}}) - \nabla \bullet (\overline{\mathbf{E}} \times \overline{\mathbf{H}}) = \overline{\mathbf{E}} \bullet (\sigma \overline{\mathbf{E}}) + \overline{\mathbf{E}} \bullet \left(\varepsilon \frac{\partial \overline{\mathbf{E}}}{\partial t} \right)$$

$$\overline{H} \cdot (\nabla \times \overline{E}) - \nabla \cdot (\overline{E} \times \overline{H}) \;=\; \sigma E^2 + \overline{E} \cdot \left(\varepsilon \frac{\partial \overline{E}}{\partial t} \right)$$

Consider first term on left of equation (4). Putting value $\nabla \times \vec{E}$ from equation (1) we can write,

$$\overline{H} \cdot (\nabla \times \overline{E}) \;=\; \overline{H} \cdot \left(-\mu \frac{\partial \overline{H}}{\partial t} \right) = -\mu \, \overline{H} \cdot \frac{\partial \overline{H}}{\partial t}$$

Now consider term,

$$\frac{\partial}{\partial t} (\overline{H} \cdot \overline{H}) \;=\; \overline{H} \cdot \frac{\partial \overline{H}}{\partial t} + \overline{H} \cdot \frac{\partial \overline{H}}{\partial t}$$

Therefore,

$$\frac{\partial}{\partial t} H^2 \;=\; 2 \overline{H} \cdot \frac{\partial \overline{H}}{\partial t}$$

$$\frac{1}{2} \frac{\partial}{\partial t} (H^2) \;=\; \overline{H} \cdot \frac{\partial \overline{H}}{\partial t}$$

Similarly we can write,

$$\frac{1}{2} \frac{\partial}{\partial t} (E^2) \;=\; \overline{E} \cdot \frac{\partial \overline{E}}{\partial t}$$

Using results obtained in equations (i), (ii), (iii) in equation (4),

$$-\frac{\mu}{2} \frac{\partial}{\partial t} (H^2) - \nabla \cdot (\overline{E} \times \overline{H}) \;=\; \sigma E^2 + \frac{\varepsilon}{2} \frac{\partial}{\partial t} (E^2)$$

$$-\nabla \cdot (\overline{E} \times \overline{H}) \;=\; \sigma E^2 + \frac{1}{2} \frac{\partial}{\partial t} \left[\mu H^2 + \varepsilon E^2 \right]$$

But $\vec{E} \times \vec{H}$ is nothing but Poynting vector, $\vec{P}$, rewriting equation,

$$-\nabla \cdot \overline{P} \;=\; \sigma E^2 + \frac{1}{2} \frac{\partial}{\partial t} \left[\mu H^2 + \varepsilon E^2 \right]$$

Equation (5) represents Poynting theorem in point form.

Integral form

If we integrate this power over a volume, we get energy distribution as,

$$-\int_V \nabla \cdot \overline{P}\, dv \;=\; \int_V \sigma E^2 \, dv + \frac{\partial}{\partial t}\int_V \frac{1}{2}\left[\mu H^2 + \varepsilon E^2\right] dv$$

Applying divergence theorem to left of above equation, we get,

$$-\oint_S \overline{P} \cdot d\overline{S} \;=\; \int_V \sigma E^2 \, dv + \frac{\partial}{\partial t}\int_V \frac{1}{2}\left[\mu H^2 + \varepsilon E^2\right] dv$$

Equation (6) represents Poynting theorem in integral form.

The negative sign indicates power flow inside the surface.

Power flow out of the volume is given as,

$$\oint_S \overline{P} \cdot d\overline{S} \;=\; -\int_V \sigma E^2 \, dv - \frac{\partial}{\partial t}\int_V \frac{1}{2}\left[\mu H^2 + \varepsilon E^2\right] dv$$

The first term on R.H.S gives the total ohmic power loss and second term represents the increase of total energy stored in electric and magnetic field.

The above eqn represents the pointing vector in integral form.

If $\vec{E}$ and $\vec{H}$ are expressed in phasor form, then the average power is given by,

$$\overline{P}_{avg} \;=\; \frac{1}{2}\,\mathrm{Re}\left[\overline{E}\times\overline{H}^*\right] = \frac{1}{2}\,\mathrm{Re}\left[\overline{E}^*\times\overline{H}\right]$$

Where $\overrightarrow{H^*}$ is the complex conjugate of $\vec{H}$ and $\overrightarrow{E^*}$ is the complex conjugate of $\vec{E}$.

7. Explain the power flow in Coaxial cable. [CO5-L1-May 2011, Nov/Dec 2012 & April/May 2015]

Power flow in Coaxial cable:

Consider a coaxial cable in which the power is transferred to the load resistance R.

There are two conductor namely inner and outer conductor.

➤ Radius of the inner conductor is 'a' and the Radius of the outer conductor is 'b'.

Consider a coaxial cable in which the power is transferred to the load resistance R.

There are two conductor namely inner and outer conductor.

➢ Radius of the inner conductor is 'a' and the Radius of the outer conductor is 'b'.
➢ The magneto force is given by,

$$H= \frac{I}{2\pi r} \qquad \text{----------------(1)}$$

Potential difference between the two conductor is

$$v= \frac{q}{2\pi \epsilon} \log\left(\frac{b}{a}\right) \qquad \text{---------------- (2)}$$

The electric field intensity for coaxial cable is given by,

$$E= \frac{q}{2\pi \epsilon r} \qquad \text{----------------(3)}$$

Eqn (2) becomes

$$\frac{q}{2\pi \epsilon} = \frac{V}{\log\left(\frac{b}{a}\right)}$$

$$E= \frac{V}{r\left(\log\left(\frac{b}{a}\right)\right)}$$

According to the poynting vector,

$$\vec{P} = \vec{E} \times \vec{H}$$

The magnitude of the poynting vector is, $P = E \times H$

The total power along the cable is,

$$W = \int_{surface} \vec{P} \, \overrightarrow{da}$$

$$=\int_{surface} (\vec{E} \times \vec{H}) \, \overrightarrow{da}$$

$$= \int_a^b \left(\frac{V}{r\left(\log\left(\frac{b}{a}\right)\right)}\right) \times \frac{I}{2\pi r}\right) \, 2\pi r \, dr \qquad (\overrightarrow{da} = 2\pi r \, dr)$$

$$= \left(\frac{V I}{\left(\log\left(\frac{b}{a}\right)\right)}\right) \int_a^b \frac{dr}{r}$$

$$= \left(\frac{V I}{\left(\log\left(\frac{b}{a}\right)\right)}\right) \, [\log\,]$$

$$= \left(\frac{V I}{\left(\log\left(\frac{b}{a}\right)\right)}\right) [\log b - \log a]$$

$$= \left(\frac{V I}{\left(\log\left(\frac{b}{a}\right)\right)}\right) \log\left(\frac{b}{a}\right)$$

W= V I   The power flow is the product of voltage and current.

8.Explain in detail about the instantaneous average and complex pointing vector.

[CO5-L1-Nov/Dec 2010], Dec 2013]

The relationship between the instantaneous, average and complex pointing vector can be derived as follows.

The pointing vector is given by,

$$\vec{P} = \vec{E} \times \vec{H}$$

The complex pointing vector is given as

$$\vec{P} = \frac{1}{2}(\vec{E} \times \vec{H}^*)$$

The instantaneous pointing vector can be written as,

$$\vec{P}_{inst} = \vec{P}_{avg} + \vec{P}_{react}$$

The average pointing vector is given as,

$$\vec{P}_{avg} = \frac{1}{2} Re[\vec{E} \times \vec{H}^*]$$

The reactive pointing vector is given as,

$$\vec{P}_{react} = \frac{1}{2} Im[\vec{E} \times \vec{H}^*]$$

Therefore,

$$\vec{P}_{inst} = \frac{1}{2} Re[\vec{E} \times \vec{H}^*] + \frac{1}{2} Im[\vec{E} \times \vec{H}^*]$$

9. Find the total power passing through a circular disc of radius 5cm In free space $\vec{H} = 0.2 \, e^{-j\beta x}\vec{a}_z$ [CO5-H2-Dec 2010, May 2010,2012]

Solution:

In free space, the intrinsic impedance is given by,

$$\eta_0 = 120\pi = 377\Omega$$

The magnetic field in phasor form is given by,

$$\vec{H} = 0.2 \, e^{-j\beta x} \, \vec{a}_z \quad \text{A/m}$$

Dielectric field in phasor form is given by,

$$\vec{E} = \eta_0 \vec{H}$$
$$= (120\pi)[0.2e^{-j\beta x} \, \vec{a_y}]$$
$$= 24 \, \pi e^{-j\beta x} \, \vec{a}_y \quad \text{V/m}$$

According to the Poynting theorem, the average power density is given by,

$$\vec{P}_{avg} = \frac{1}{2} Re[\vec{E} \times \vec{H}^*]$$
$$\vec{P}_{avg} = \frac{1}{2} Re\{[24 \, \pi e^{-j\beta x}\vec{a}_y] \times [0.2 \, e^{-j\beta x} \, \vec{a}_z]\}$$
$$= \frac{1}{2} (24\pi)(0.2)[\vec{a}_y \times \vec{a}_z]$$
$$= 7.5398 \, \vec{a}_x \quad \text{watt} / m^2$$

The area of a circular ring is given by,

S = $Area = \pi r^2$

$= \pi(5 \times 10^{-2})^2 = 7.5398 \times 10^{-3} \, m^2$

Hence magnitude of the total power passing through a circular disc is given by,

P = (Power average density) (Area)

$= (7.5398)(7.5398 \times 10^{-3}) = 59.2168$ Mw

10. In a material for which $\sigma = 5.0 \ \frac{s}{m}$ and $\varepsilon_r = 1$ and $\vec{E} = 250 \sin 10^{10} t$ (V/m). Find the conduction and displacement current densities, and the frequency at which both have equal magnitudes. [CO5-H1-May 2012]

Sol:
$$J_c = \sigma E = 5[250 \sin 10^{10} t]$$
$$J_c = 1250 \sin 10^{10} t \ A/m^2$$
$$J_D = \frac{\partial \vec{D}}{\partial t} = \frac{\partial}{\partial t}[\varepsilon \vec{E}] = (\varepsilon_0 \varepsilon_r) \frac{\partial \vec{E}}{\partial t}$$
$$= (8.854 \times 10^{-12})(1) \frac{d}{dt}[250 \sin 10^{10} t]$$
$$= 8.854 \times 10^{-12} \times 250 \times 10^{10} \times \cos 10^{10} t$$
$$J_D = 22.135 \cos 10^{10} t \ A/m^2$$

For the two densities, the conduction for magnitudes to be equal is,
$$\frac{J_c}{J_D} = \frac{\sigma}{\omega \epsilon} = 1$$
$$\omega = \frac{\sigma}{\varepsilon} = \frac{5}{8.854 \times 10^{-12} \times 1} = 5.6471 \times 10^{11}$$
$$\text{But } \omega = 2\pi f$$
$$f = \frac{\omega}{2\pi} = \frac{5.6471 \times 10^{11}}{2\pi} = 89.87 \ GHz$$

11. In free space, $\vec{E} = 50 \cos(\omega t - \beta z) \vec{a}_x$ V/m. find the average power crossing a circular area of radius 2.5m in the plane z = 0. Assume $E_m = H_m \cdot \eta_0$ and η_0 = 120 π Ω. [CO5-H1-Dec 2011]

Sol:

In complex form,
$$\vec{E} = 50 \ e^{j(\omega t - \beta z)} \vec{a}_x \text{ and}$$
$$\vec{H} = \frac{50}{\eta_0} e^{j(\omega t - \beta z)} \vec{a}_y$$
$$\vec{H} = \frac{50}{120\pi} e^{j(\omega t - \beta z)} \vec{a}_y$$

Hence we can write,
$$\vec{H}^* = \frac{50}{120\pi} e^{j(-\omega t + \beta z)} \vec{a}_y$$

Hence,
$$\vec{P}_{avg} = \frac{1}{2} Re[\vec{E} \times \vec{H}^*]$$
$$= \frac{1}{2}(50) \ (\frac{50}{120\pi})[e^{j(\omega t - \beta z)} \vec{a}_x \times e^{j(-\omega t + \beta z)} \vec{a}_y]$$
$$= 3.31572 \vec{a}_z \ W/m^2$$

Area of the circle with radius 2.5m is given by, $Area = \pi r^2 = \pi(2.5)^2 = 19.6349 \ m^2$

Now the average power crossing circular area is given by,
$$P_{avg} = (|\overrightarrow{P_{avg}}|) \ (A)$$
$$= (3.31572)(19.6349) = 65.1 \ W$$

The power flow is normal to the circular area.

12. if electric field intensity in free space is given by $\vec{E} = \frac{50}{\rho}\cos(10^8 t -$

$10z)\vec{a}_\rho\ V/m$**. Find the magnetic field intensity** $\overrightarrow{H}$**. [CO5-H1May 2011]**

Sol:

For time varying fields, using Maxwell's equation we can write,

$$\nabla \times \vec{E} = - \frac{\partial\vec{B}}{\partial t} - - - - - - - - - - (1)$$

Consider L.H.S of above equation. Using cylindrical co-ordinate system, we can write,

$$\nabla \times \vec{E} = \frac{1}{\rho}\begin{vmatrix} \vec{a}_\rho & \vec{a}_\varphi & \vec{a}_z \\ \partial/\partial\rho & \partial/\partial\varphi & \partial/\partial z \\ E_\rho & E_\varphi & E_Z \end{vmatrix}$$

$$= \frac{1}{\rho}\begin{vmatrix} \vec{a}_\rho & \vec{a}_\varphi & \vec{a}_z \\ \partial/\partial\rho & \partial/\partial\varphi & \partial/\partial z \\ \frac{50}{\rho}\cos(10^8\ t - 10z) & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left\{[0 - 0]\vec{a}_\rho - \left[0 - \frac{\partial}{\partial z}\left\{\frac{50}{\rho}\cos(10^8\ t - 10z)\right\}\right]\vec{a}_\varphi + [0 - \frac{\partial}{\partial\varphi}\{\frac{50}{\rho}\cos(10^8\ t - 10z\}]\vec{a}_z\}$$

$$= \frac{1}{\rho}\left\{\frac{50}{\rho}\cos(10^8\ t - 10z)\right\}\vec{a}_\varphi$$

$$= \frac{50}{\rho^2}[-\sin(10^8t - 10z)]\vec{a}_\varphi$$

$$= \frac{500}{\rho^2}[\sin(10^8t - 10z)]\vec{a}_\varphi \quad ---------------------(2)$$

Putting L.H.S value from eqn.(2) in, eqn.(1), we get,

$$\frac{500}{\rho^2}[\sin(10^8t - 10z)]\vec{a}_\varphi = - \frac{\partial\vec{B}}{\partial t}$$

Separating variables,

$$\partial\vec{B} = \left[-\frac{500}{\rho^2}\boldsymbol{sin}\ (10^8t - 10z)\vec{a}_\rho\right]\partial t$$

Integrating both the sides with respect to corresponding variables, we get,

$$\vec{B} = -\frac{500}{\rho^2}\left[-\cos(10^8t - 10z)\left(\frac{1}{10^8}\right)\vec{a}_\rho\right]$$
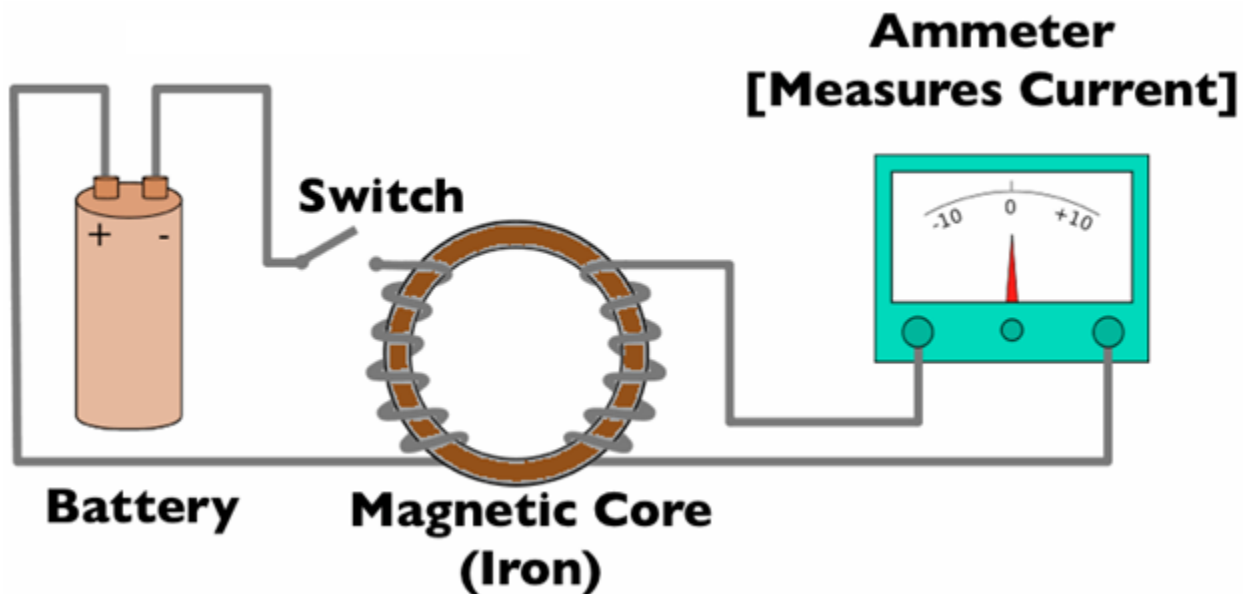
Therefore, $\vec{B} = \frac{5}{\rho^2} cos(10^8 t - 10z)\vec{a}_\rho \quad \mu A/m$

13. State Faraday's law of electromagnetic induction and also derive the Maxwell's equation. [CO5-H2-Nov/Dec 2003, April/May 2004, May 2005, May/June 2009].

Faraday's Law: Faraday"s law states that" the electromotive force (mmf) induced in a circuit is equal to the rate of decrease of the magnetic flux linkage in the magnetic circuit".

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday was a scientist experimenting with circuits and magnetic coils way back in the 1830s. His experiment setup, which led to Farday's Law, is shown in Figure

The experiment itself is somewhat simple. When the battery is disconnected, we have no electric current flowing through the wire. Hence there is no magnetic flux induced within the Iron (Magnetic Core). The Iron is like a highway for Magnetic Fields - they flow very easily through magnetic material. So the purpose of the core is to create a path for the Magnetic Flux to flow.

When the switch is closed, the electric current will flow within the wire attached to the battery. When this current flows, it has an associated magnetic field (or magnetic flux) with it. When the wire wraps around the left side of the magnetic core (as shown in Figure), a magnetic field (magnetic flux) is induced within the core. This flux travels

around the core. So the Magnetic Flux produced by the wired coil on the left exists within the wired coil on the right, which is connected to the ammeter.

Now, a funny thing happens, which Faraday observed. When he closed the switch, then current would begin flowing and the ammeter would spike one way (say measuring +10 Amps on the other side). But this was very brief, and the current on the right coil would go to zero. When the switch was opened, the measured current would spike to the other side (say -10 Amps would be measured), and then the measured current on the right side would again be zero.

Faraday figured out what was happening. When the switch was initially changed from open to closed, the magnetic flux within the magnetic core increased from zero to some maximum number (which was a constant value, versus time). When the flux was increasing, there existed an induced current on the opposite side.

Similarly, when the switch was opened, the magnetic flux in the core would decrease from it's constant value back to zero. Hence, a decreasing flux within the core induced an opposite current on the right side.

Faraday figured out that a changing Magnetic Flux within a circuit (or closed loop of wire) produced an induced EMF, or voltage within the circuit. He wrote this as

$$\text{EMF} = -\text{N}\frac{d\varphi}{dt} \text{ volts}$$

Where   N = Number of turns in the circuit

               Φ = total magnetic flux

       Let us assume N=1

$$\text{EMF} = -\frac{d\varphi}{dt} \text{ volts}$$

The total magnetic flux is the integral of the $\vec{B}$ field over the area enclosed by the wire

$$\varphi = \iint_S \vec{B} \cdot d\vec{S}$$

$$\text{EMF} = -\frac{d}{dt}\iint_S \vec{B} \cdot d\vec{S} \dots \dots \dots \dots .1$$

We know that the induced emf is a scalar quantity measured in volts.

$$\text{EMF} = \oint_L \vec{E} \cdot d\vec{L} \quad \ldots\ldots\ldots 2$$

From equations 1 and 2

$$\oint_L \vec{E} \cdot d\vec{L} = -\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{S}$$

By using stokeses theorem

$$\iint_S (\nabla \times \vec{E}) \cdot d\vec{S} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

From the above equation

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

14. Explain in detail about Electromagnetic spectrum. [CO5-L2]

The electromagnetic spectrum is the distribution of electromagnetic radiation according to energy. The electromagnetic spectrum is the range of all possible frequencies of electromagnetic radiation. The "electromagnetic spectrum" of an object has a different meaning, and is instead the characteristic distribution of electromagnetic radiation emitted or absorbed by that particular object.

Spectrum of Electromagnetic Radiation				
Region	**Wavelength (Angstroms)**	**Wavelength (centimeters)**	**Frequency (Hz)**	**Energy (eV)**
Radio	$> 10^9$	> 10	$< 3 \times 10^9$	$< 10^{-5}$
Microwave	$10^9 - 10^6$	$10 - 0.01$	$3 \times 10^9 - 3 \times 10^{12}$	$10^{-5} - 0.01$
Infrared	$10^6 - 7000$	$0.01 - 7 \times 10^{-5}$	$3 \times 10^{12} - 4.3 \times 10^{14}$	$0.01 - 2$
Visible	$7000 - 4000$	$7 \times 10^{-5} - 4 \times 10^{-5}$	$4.3 \times 10^{14} - 7.5 \times 10^{14}$	$2 - 3$
Ultraviolet	$4000 - 10$	$4 \times 10^{-5} - 10^{-7}$	$7.5 \times 10^{14} - 3 \times 10^{17}$	$3 - 10^3$
X-Rays	$10 - 0.1$	$10^{-7} - 10^{-9}$	$3 \times 10^{17} - 3 \times 10^{19}$	$10^3 - 10^5$
Gamma Rays	< 0.1	$< 10^{-9}$	$> 3 \times 10^{19}$	$> 10^5$

Electromagnetic waves are typically described by any of the following three physical properties: the frequency f, wavelength λ, or photon energy E. Frequencies observed in astronomy range from 2.4×10^{23} Hz (1 GeV gamma rays) down to the local plasma frequency of the ionized interstellar medium (~1 kHz). Wavelength is inversely proportional to the wave frequency, so gamma rays have very short wavelengths that are fractions of the size of atoms, whereas wavelengths on the opposite end of the spectrum can be as long as the universe. Photon energy is directly proportional to the wave frequency, so gamma ray photons have the highest energy (around a billion electron volts), while radio wave photons have very low energy (around a femtoelectronvolt). These relations are illustrated by the following equations:

$$f = \frac{c}{\lambda}, \quad \text{or} \quad f = \frac{E}{h}, \quad \text{or} \quad E = \frac{hc}{\lambda},$$

where:

- $c = 299{,}792{,}458$ m/s is the speed of light in vacuum and
- $h = 6.62606896(33) \times 10^{-34}$ J s $= 4.13566733(10) \times 10^{-15}$ eV s is Planck's constant.

Whenever electromagnetic waves exist in a medium with matter, their wavelength is decreased. Wavelengths of electromagnetic radiation, no matter what medium they are traveling through, are usually quoted in terms of the *vacuum wavelength*, although this is not always explicitly stated.

Generally, electromagnetic radiation is classified by wavelength into radio wave, microwave, terahertz (or sub-millimeter) radiation, infrared, the visible region is perceived as light, ultraviolet, X-rays and gamma rays. The behavior of EM radiation depends on its wavelength. When EM radiation interacts with single atoms and molecules, its behavior also depends on the amount of energy per quantum (photon) it carries.

Regions of the spectrum

The types of electromagnetic radiation are:

1. Gamma radiation
2. X-ray radiation
3. Ultraviolet radiation

4. Visible radiation

5. Infrared radiation

6. Terahertz radiation

7. Microwave radiation

8. Radio waves

Radio frequency

Radio waves generally are utilized by antennas of appropriate size with wavelengths ranging from hundreds of meters to about one millimeter. They are used for transmission of data, via modulation. Television, mobile phones, wireless networking, and amateur radio all use radio waves. The use of the radio spectrum is regulated by many governments through frequency allocation.

Microwaves

The super-high frequency (SHF) and extremely high frequency (EHF) of microwaves are on the short side of radio waves. Microwaves are waves that are typically short enough to employ tubular metal waveguides of reasonable diameter. Microwave energy is produced with klystron and magnetron tubes, and with solid state diodes such as Gunn and IMPATT devices. Microwaves are absorbed by molecules that have a dipole moment in liquids. In a microwave oven, this effect is used to heat food. Low-intensity microwave radiation is used in Wi-Fi, although this is at intensity levels unable to cause thermal heating.

Terahertz radiation

Terahertz radiation is a region of the spectrum between far infrared and microwaves. Until recently, the range was rarely studied and few sources existed for microwave energy at the high end of the band but applications such as imaging and communications are now appearing.

Infrared radiation

The infrared part of the electromagnetic spectrum covers the range from roughly 300 GHz to 400 THz (1 mm - 750 nm). It can be divided into three parts:

- **Far-infrared**, from 300 GHz to 30 THz (1 mm - 10 μm). The lower part of this range may also be called microwaves. This radiation is typically absorbed by so-called

rotational modes in gas-phase molecules, by molecular motions in liquids, and by phonons in solids.

- **Mid-infrared**, from 30 to 120 THz (10 - 2.5 μm). Hot objects can radiate strongly in this range, and human skin at normal body temperature radiates strongly at the lower end of this region. This range is sometimes called the *fingerprint region*.

- **Near-infrared**, from 120 to 400 THz (2,500 - 750 nm). Physical processes that are relevant for this range are similar to those for visible light.

Visible radiation

Visible light is the part of the EM spectrum to which the human eye is the most sensitive. Visible is typically absorbed and emitted by electrons in molecules and atoms that move from one energy level to another. The light which excites the human visual system is a very small portion of the electromagnetic spectrum. Electromagnetic radiation with a wavelength between 380 nm and 760 nm is detected by the human eye and perceived as visible light. Other wavelengths, especially near infrared (longer than 760 nm) and ultraviolet (shorter than 380 nm) are also sometimes referred to as light, especially when the visibility to humans is not relevant.

Ultraviolet radiation

The wavelength of UV rays is shorter than the violet end of the visible spectrum but longer than the X-ray. UV in the very shortest range (next to X-rays) is capable even of ionizing atoms.

X-rays

Hard X-rays have shorter wavelengths than soft X-rays. As they can pass through most substances with some absorption, X-rays can be used to 'see through' objects with thicknesses less than equivalent to a few meters of water.

Gamma rays

Gamma rays are used for the irradiation of food and seed for sterilization, and in medicine they are occasionally used in radiation cancer therapy. More commonly, gamma rays are used for diagnostic imaging in nuclear medicine, with an example being PET scans. The wavelength of gamma rays can be measured with high

accuracy by means of Compton scattering. Gamma rays are first and mostly blocked by Earth's magnetosphere then by the atmosphere.

15. Explain about transformers in EMF. [CO5-L2-April/May 2015]

In electrical power transformer, one alternating electrical source is applied to the primary winding and due to this, magnetizing current flowing through the primary winding which produces alternating flux in the core of transformer.
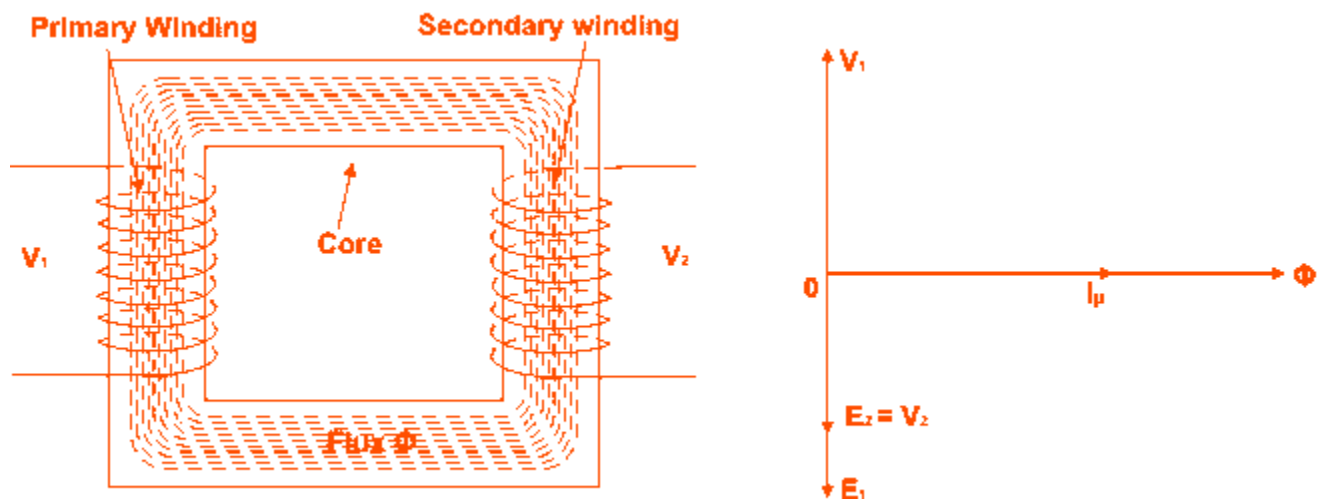
This flux links with both primary and secondary windings. As this flux is alternating in nature, there must be a rate of change of flux.

According to <u>Faraday's law of electromagnetic induction</u> if any coil or conductor links with any changing flux, there must be an induced emf in it.

As the <u>current</u> source to primary is sinusoidal, the flux induced by it will be also sinusoidal. Hence, the function of flux may be considered as a sine function. Mathematically, derivative of that function will give a function for rate of change of flux linkage with respect to time. This later function will be a cosine function since $d(\sin\theta)/dt = \cos\theta$.

So, if we derive the expression for rms value of this cosine wave and multiply it with number of turns of the winding, we will easily get the expression for rms value of induced emf of that winding. In this way, we can easily derive the **emf equation of transformer**.

Let's say, T is number of turns in a winding,

Φ_m is the maximum flux in the core in Wb.

As per Faraday's law of electromagnetic induction,

$$emf, \ e \ = \ -T\frac{d\phi}{dt}$$

Where φ is the instantaneous alternating flux and represented as,

$$\phi \ = \ \phi_m \sin 2\pi ft$$

$$Hence, \ e \ = \ -T\frac{d(\phi_m \sin 2\pi ft)}{dt}$$

$$\Rightarrow e \ = \ -T\phi_m \cos(2\pi ft) \ \times 2\pi f$$

$$\Rightarrow e \ = \ -T\phi_m \times 2\pi f \cos(2\pi ft)$$

As the maximum value of cos2πft is 1, the maximum value of induced emf e is,

$$e_m \ = \ T\phi_m \times 2\pi f$$

To obtain the rms value of induced counter emf, divide this maximum value of e by √2.

$$Then, \ E \ = \ \frac{2\pi}{\sqrt{2}} \times \phi_m fT$$

$$E \ = \ 4.44\phi_m fT \ Volts \ \ \left(Since \ \frac{2\pi}{\sqrt{2}} \ = \ 4.44 \right)$$

This is EMF equation of transformer.

If E_1 & E_2 are primary and secondary emfs and T_1 & T_2 are primary and secondary turns then, voltage ratio or turns ratio of transformer is,

$$\frac{E_1}{E_2} \ = \ \frac{4.44\phi_m fT_1}{4.44\phi_m fT_2} \ = \ \frac{T_1}{T_2}$$

$$\Rightarrow \frac{E_1}{E_2} \ = \ \frac{T_1}{T_2}$$

Transformation Ratio of Transformer

This constant is called **transformation ratio of transformer** , if $T_2 > T_1$, K > 1, then the transformer is step up transformer. If $T_2 < T_1$, K < 1, then the transformer is step down transformer.

Voltage Ratio of Transformer

This above stated ratio is also known as <u>voltage</u> **ratio of transformer** if it is expressed as ratio of the primary and secondary voltages of transformer.

Turns Ratio of Transformer

As the <u>voltage</u> in primary and secondary of transformer is directly proportional to the number of turns in the respective winding, the transformation ratio of transformer is sometime expressed in ratio of turns and referred as **turns ratio of transformer**.

16. Explain in detail about time harmonic fields. [CO5-L2-April/May 2015]

Time Harmonic Fields

One of the most important case of time varying electromagnetic field is the time harmonic (sinusoidal or co sinusoidal) time variation where the excitation of the source varies sinusoidally in time with a single frequency. For time-harmonic fields, phasor analysis can be applied to obtain single frequency steady state response. Since Maxwell's equations are linear differential equations, for source functions with arbitrary time dependence, electromagnetic fields can be determined by superposition. Periodic time functions can be expanded into Fourier series of harmonic sinusoidal components while transient non-periodic functions can be expressed as Fourier integrals. Field vectors that vary with space coordinates and are sinusoidal function of time can be represented in terms of vector phasors that depend on the space coordinates but not on time. For time harmonic case, the general time variation is $e^{j\omega t}$ and for a cosine reference, the instantaneous fields can be written as:

$$\vec{E}(x,y,z,t) = \text{Re}\left[\vec{E}(x,y,z)e^{j\omega t}\right] \qquad (5.42)$$

where $\vec{E}(x,y,z)$ is a vector phasor that contain the information on direction, magnitude and phase. The phasors in general are complex quantities. All time harmonic filed components can be written in this manner.

Thus we find that if the electric field vector $\vec{E}(x,y,z,t)$ is represented in the phasor form as $\vec{E}(x,y,z)$, then $\dfrac{\partial \vec{E}(x,y,z,t)}{\partial t}$ can be represented by the phasor $j\omega\vec{E}(x,y,z)$. The integral $\int \vec{E}(x,y,z,t)dt$ can be represented by the phasor $\dfrac{\vec{E}(x,y,z)}{j\omega}$. In the same manner, higher order derivatives and integrals with respect to *t* can be represented by multiplication and division of the phasor $\vec{E}(x,y,z)$ by higher power of $j\omega$. Considering the field phasors $(\vec{E},\vec{H})$ and source phasors $(\rho,\vec{J})$ in a simple linear isotropic medium, we can write the Maxwell's equations for time harmonic case in the phasor form as:

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \qquad (5.44a)$$

The time rate of change of $\vec{E}$ can be written as:

$$\frac{\partial \vec{E}(x,y,z,t)}{\partial t} = \mathrm{Re}\left[j\omega\vec{E}(x,y,z)e^{j\omega t} \right]$$

Similarly, the wave equations described in equation (5.32) can be written as:

$$\nabla^2 \vec{E} = \mu_0\varepsilon_0 (j\omega)^2 \vec{E} = -\omega^2 \mu_0\varepsilon_0 \vec{E}$$

or $\nabla^2 \vec{E} + k_0^2 \vec{E} = 0$ $\qquad (5.45a)$

3. $\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$ $\qquad (5.44b)$

4. $\nabla\cdot\vec{E} = \dfrac{\rho}{\omega}$ $\qquad (5.44c)$

5. $\nabla\cdot\vec{H} = 0$ $\qquad (5.44d)$

and in the same manner, for the magnetic field

$$\nabla^2 \vec{H} + k_0^2 \vec{H} = 0$$

where $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ is called the *wave number* .

17. Explain the boundary conditions for time varying fields.[CO5-L3]

Boundary conditions for time varying field

The relationship between electric flux density $\vec{D}$, electric field intensity $\vec{E}$, magnetic flux density $\vec{B}$ and magnetic field intensity $\vec{H}$ can be explained with the help of point and integral form of Maxwell's equations.The field equations postulated by Maxwell are valid at a point in a continuous medium. The maxwell's equations are useful in finding the conditions at the boundary of two different mediums. We can apply the concepts in linear, isotropic and homogeneous mediums. Consider the boundary between medium 1 having parameters μ_1, ε_1 and σ_1 and medium 2 having parameters μ_2, ε_2 and σ_2. The boundary conditions for time varying fields are the sme as for static fields.

Thus the the boundary conditions for static electric and magnetic fields as follows,

(i). The tangential component of electric field intensity $\vec{E}$ is continues across the boundary.

$$E_{tan1} = E_{tan2}$$

(ii). The tangential component of magnetic field intensity $\vec{H}$ is continues across the boundary except for perfect conductor

$$.H_{tan1} = H_{tan2}$$

At the surface of the boundary of perfect conductor

$$H_{tan1} - H_{tan2} = K$$

(iii). The normal component of electric flux density $\vec{D}$ is continues across the boundary If the surface charge density is zero.

$$D_{N1} = D_{N2}$$

If the surface charge density is non-zero, then

$$D_{N1} - D_{N2} = \rho_s$$

(iv). The normal component of magnetic flux density $\vec{B}$ is continues across the boundary

$$B_{N1} = B_{N2}$$

18. Derive the electromagnetic wave equations and their solutions. [CO5-H2]

One of the most fundamental equations to all of Electromagnetics is the wave equation, which shows that all waves travel at a single speed - the speed of light. On this page we'll derive it from Ampere's and Faraday's Law.

We assume we are in a source free region - so no charges or currents are flowing. We want to determine how Electric and Magnetic Fields propagate through the region.

To start, let me throw out a vector identity, which is basically a mathematical manipulation that is true for allvector fields:

$$\nabla \times \nabla \times \vec{H} = \nabla(\nabla \cdot \vec{H}) - \nabla^2\vec{H} \dots\dots\dots\dots.1$$

The left side of Equation 1 is simply the curl of the curl of a vector field. On the right side, I can define the terms for you in the next couple of equations. The first term on the right side of Equation [1] is known as the "gradient of the divergence". However, since we know the divergence of the fields in question will be zero because we are in a source free region. Hence, this term is zero $\nabla(\nabla \cdot \vec{H}) = 0$

Substitute the above in equation 1

$$\nabla \times \nabla \times \vec{H} = -\nabla^2\vec{H} \dots\dots\dots\dots.2$$

Sameway for electric field

$$\nabla \times \nabla \times \vec{E} = -\nabla^2\vec{E} \dots\dots\dots\dots.3$$

Faraday"s law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

We know that

$$\vec{B} = \mu\vec{H}$$

$$\nabla \times \vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \dots\dots\dots\dots\dots\dots 4$$

Taking curl on both sides

$$\nabla \times \nabla \times \vec{E} = -\mu\frac{\partial(\nabla \times \vec{H})}{\partial t} \dots\dots\dots 5$$

We can rewrite the left side of equation [5] (the curl of the curl of E) with the help of Equation [3]

$$\nabla \times \nabla \times \vec{E} = -\nabla^2 \vec{E} = -\mu \frac{\partial(\nabla \times \vec{H})}{\partial t} \dots \dots \dots \dots 6$$

From ampere circuit law

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

In charge free region

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \dots \dots \dots \dots \dots .7$$

Substitiute equation 7 in 6

$$-\nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} \left(\frac{\partial \vec{D}}{\partial t} \right) \dots \dots \dots \dots 8$$

We know that

$$\vec{D} = \varepsilon \vec{E}$$

$$-\nabla^2 \vec{E} = -\mu \varepsilon \frac{\partial}{\partial t} \left(\frac{\partial \vec{E}}{\partial t} \right) \dots \dots \dots \dots 9$$

$$\nabla^2 \vec{E} = \mu \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \dots \dots \dots \dots 10$$

Equation [10] is known as the **Wave Equation** It is actually 3 equations, since we have an x-, y- and z- component for the **E** field.

To break down and understand Equation [10], let's imagine we have an E-field that exists in source-free region. Suppose we only have an E-field that is polarized in the x-direction, which means that Ey=Ez=0 (the y- and z- components of the E-field are zero). Further, let's assume that the field is travelling in the z-direction, and there is no variation in the x- and y-directions (this means the partial derivatives with respect to x- and y- are zero). Then Equation [10] simplifies to:

$$\nabla^2 E_x = \mu \varepsilon \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu \varepsilon \frac{\partial^2 E_x}{\partial t^2}$$

Ampere circuit law in source free region

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} \dots \dots \dots \dots \dots \dots \dots \dots 11$$

Taking curl on both sides

$$\nabla \times \nabla \times \vec{H} = \varepsilon \frac{\partial (\nabla \times \vec{E})}{\partial t} \dots \dots \dots \dots \dots \dots \dots \dots 12$$

From equation 2

$$\nabla \times \nabla \times \vec{H} = -\nabla^2 \vec{H} = \varepsilon \frac{\partial (\nabla \times \vec{E})}{\partial t} \dots \dots \dots \dots \dots 13$$

Substitute Faraday"s law in equation13

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$-\nabla^2 \vec{H} = -\varepsilon \frac{\partial}{\partial t}\left(\frac{\partial \vec{B}}{\partial t}\right) \dots \dots \dots \dots \dots 14$$

We know that

$$\vec{B} = \mu \vec{H}$$

$$\nabla^2 \vec{H} = \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} \dots \dots \dots \dots 15$$

$$\nabla^2 \begin{bmatrix} \vec{E} \\ \vec{D} \\ \vec{H} \\ \vec{B} \end{bmatrix} = \mu\varepsilon \frac{\partial^2}{\partial t^2} \begin{bmatrix} \vec{E} \\ \vec{D} \\ \vec{H} \\ \vec{B} \end{bmatrix}$$